Physique mesoscopique des electrons et des photons -
Structures fractales et quasi-periodiques

Eric Akkermans
Physics-Technion

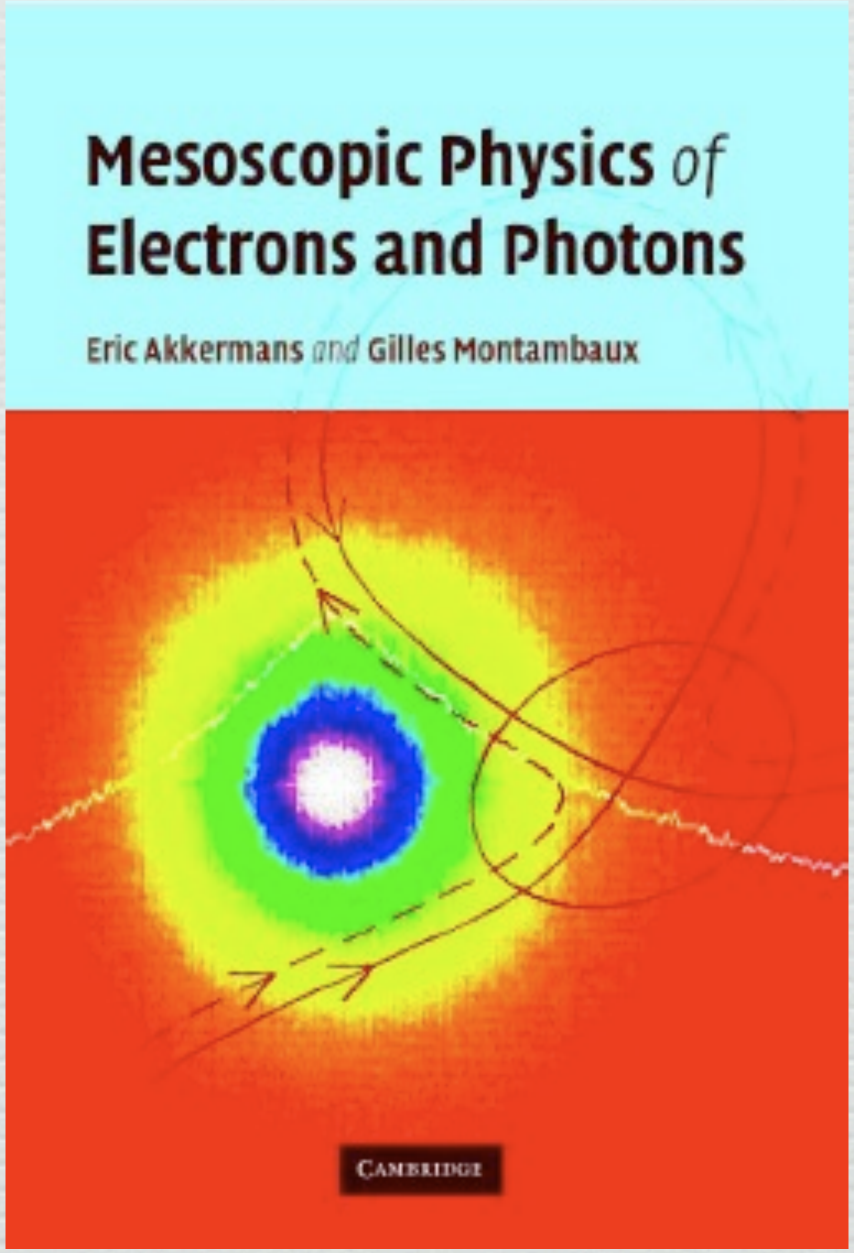
Aux frontieres de la physique mesoscopique,
Mont Orford Quebec, Canada,
Septembre 2013

Part 1

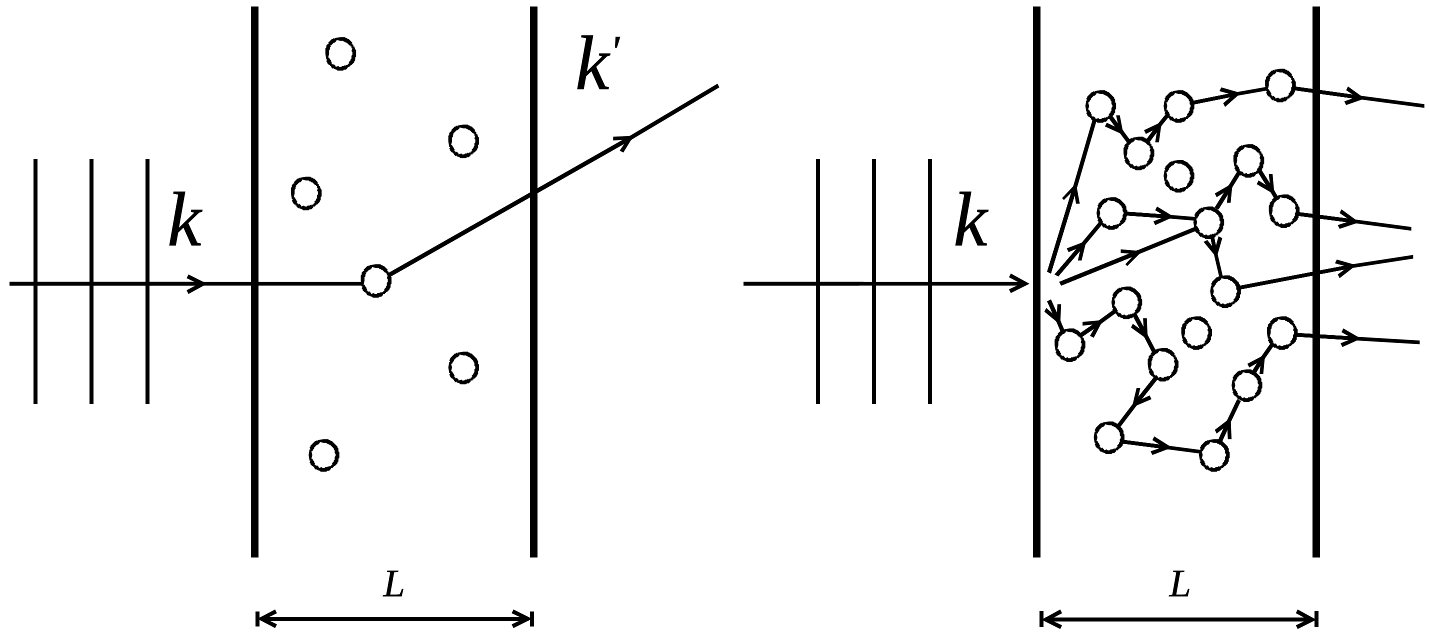
What is quantum in mesoscopic physics ?
Transport and interferences

(E.A., G. Montambaux)

more details in:



Multiple scattering of electrons



2 characteristic lengths:

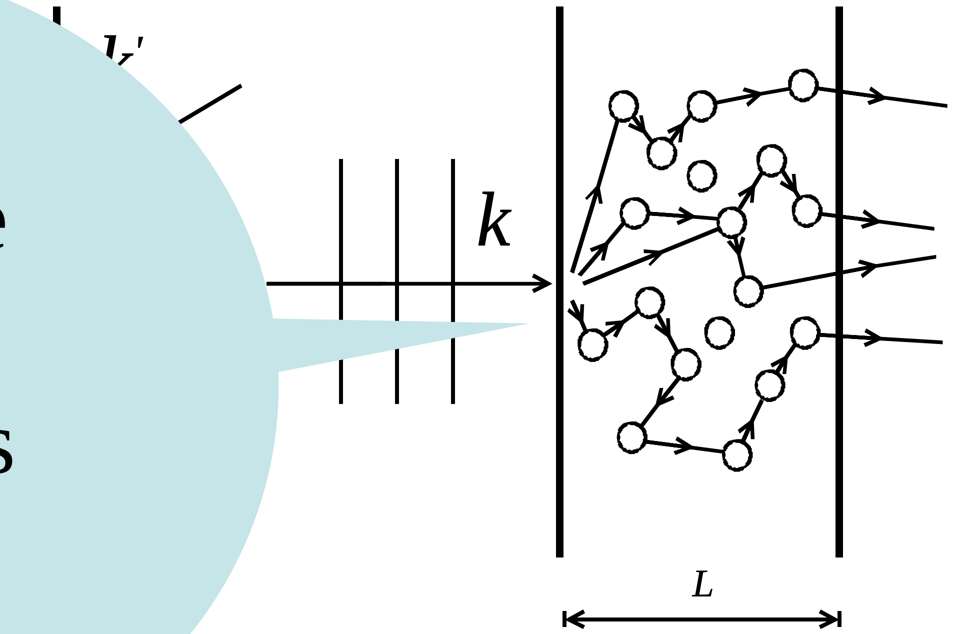
Wavelength: $\lambda_F = k_F^{-1}$

Elastic mean free path: l (Disorder - Origin ?)

Weak disorder $\lambda_F \ll l$: independent scattering events

Multiple scattering of electrons

We shall be interested only by this limit



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Some “canonical” mesoscopic effects

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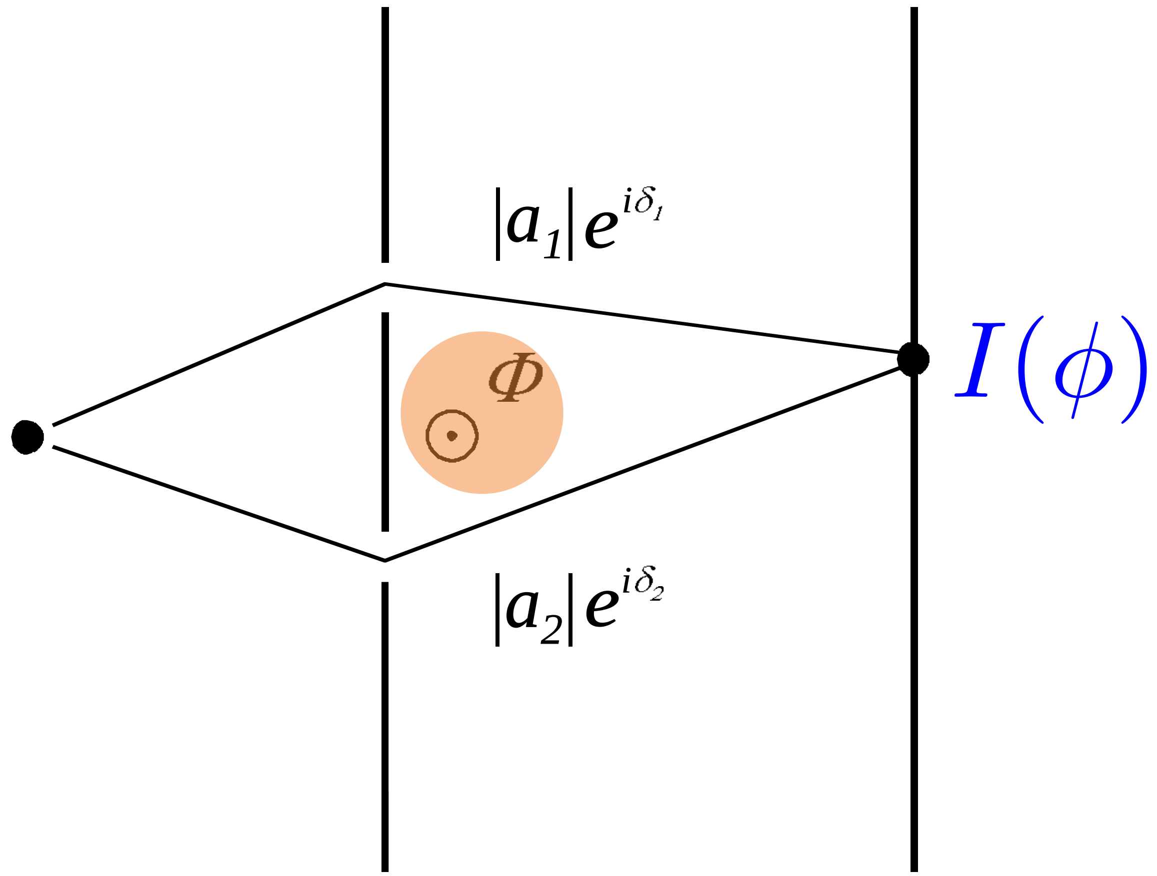
The Aharonov-Bohm effect

Aharonov-Bohm (1959)

The setup : Young slits

No magnetic field on the electrons : no Lorentz force and no orbital motion.



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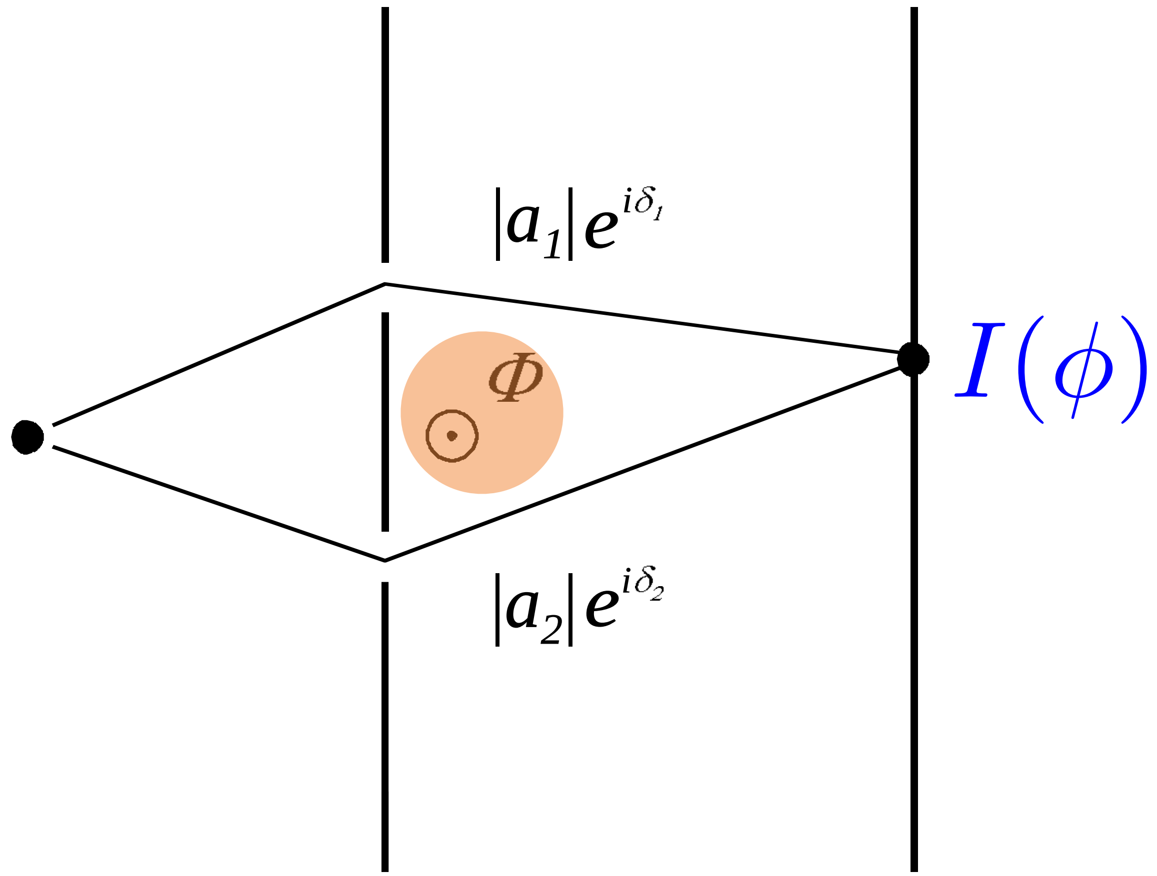


The quantum amplitudes $a_{1,2} = |a_{1,2}|e^{i\delta_{1,2}}$ have phases:

$$\delta_1 = \delta_1^{(0)} - \frac{e}{\hbar}\int_1 \mathbf{A}.d\mathbf{l} \quad \text{and} \quad \delta_2 = \delta_2^{(0)} - \frac{e}{\hbar}\int_2 \mathbf{A}.d\mathbf{l}$$

The intensity $I(\phi)$ is given by

$$I(\phi) = |a_1 + a_2|^2 = |a_1|^2 + |a_2|^2 + 2|a_1 a_2| \cos(\delta_1 - \delta_2)$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\delta_1 - \delta_2)$$

The phase difference $\Delta\delta(\phi) = \delta_1 - \delta_2$ is modulated by the magnetic flux ϕ :

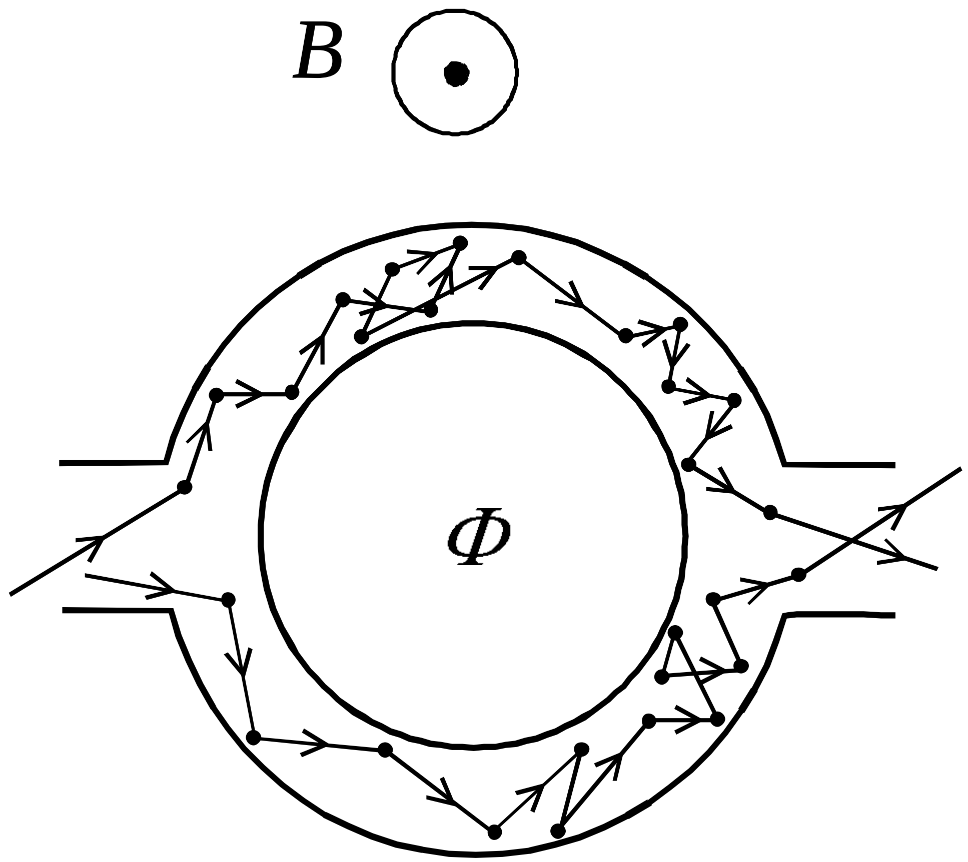
$$\Delta\delta(\phi) = \delta_1^{(0)} - \delta_2^{(0)} + \frac{e}{\hbar} \oint \mathbf{A}.d\mathbf{l} = \Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0}$$

where $\phi_0 = h/e$ is the quantum of magnetic flux.

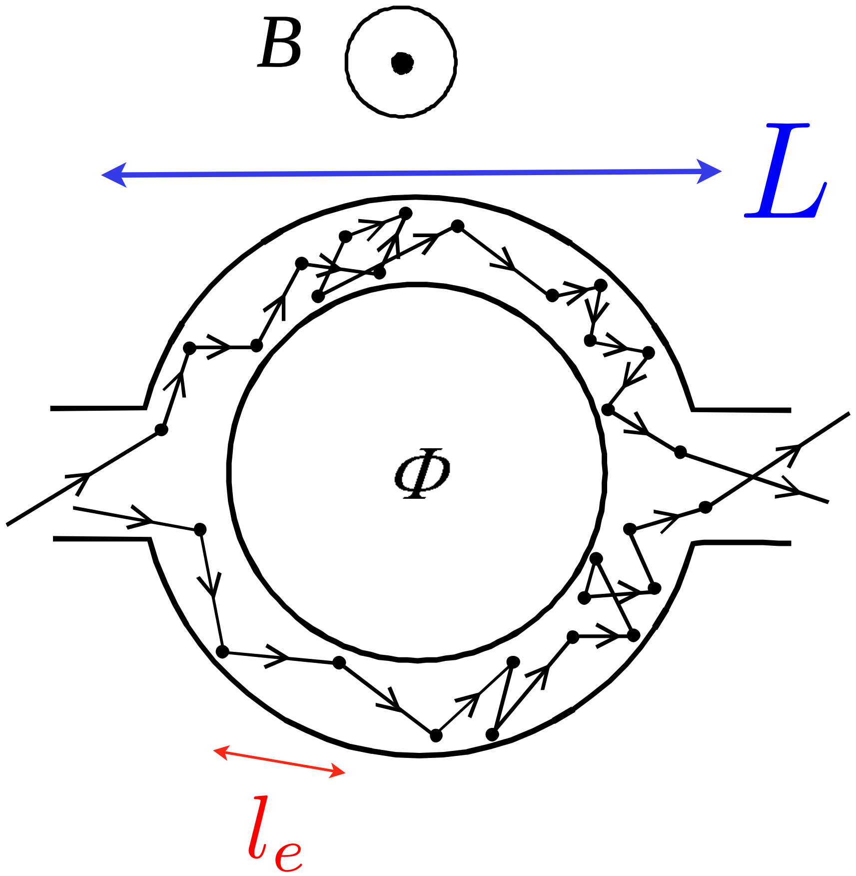
There is a continuous change of the state of interference:

Aharonov-Bohm effect (1959)

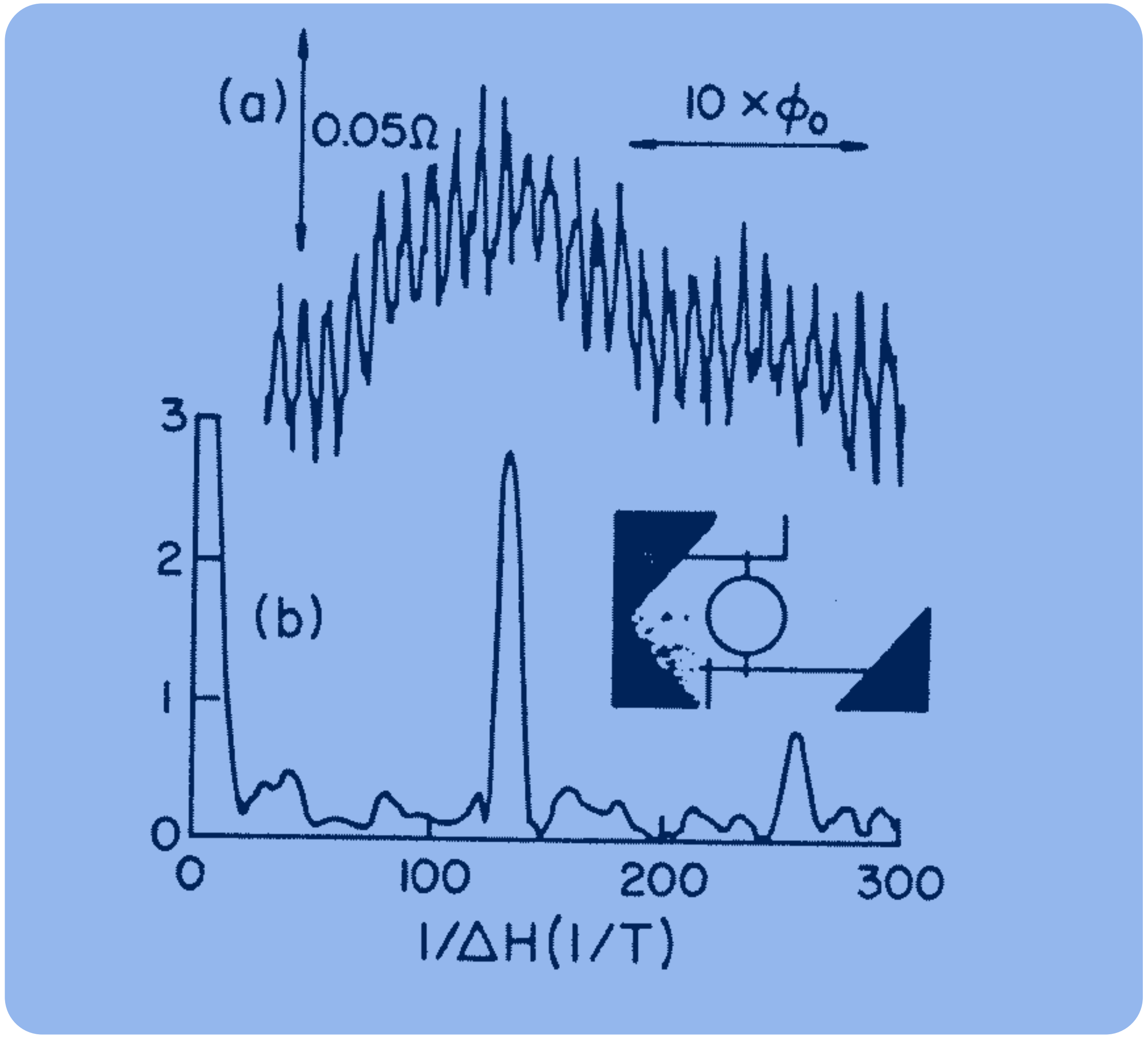
Aharonov-Bohm effect in disordered metals



Implementation in metals : the conductance $G(\phi)$ is the analog of the intensity.



elastic mean free path



Webb et al. 1985

$$G(\phi) = G_0 + \delta G \cos(\Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0})$$

Phase coherent effects subsist in ***disordered*** metals.

Reconsider the Drude theory.

Phase coherence and effect of disorder

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Quantum coherence: gas of quantum particles in a finite volume

Quantum states of the gas are coherent superposition of single particle states and they extend over the total volume (*ex. superconductivity, superfluidity, free electron gas*).

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Quantum coherence: gas of quantum particles in a finite volume

Quantum states of the gas are coherent superposition of single particle states and they extend over the total volume (*ex. superconductivity, superfluidity, free electron gas*).

For the electron gas, coherence disappears at non zero temperature so that we can use a classical description of transport and thermodynamics

Vanishing of quantum coherence results from the existence of incoherent and irreversible processes associated to the coupling of electrons to their surrounding (additional degrees of freedom) :

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Chaotic dynamical systems (large recurrence times, Feynman chain)

Vanishing of quantum coherence results from the existence of incoherent and irreversible processes associated to the coupling of electrons to their surrounding (additional degrees of freedom) :

Coupling to a bath of excitations : thermal excitations of the lattice (phonons)

Chaotic dynamical systems (large recurrence times, Feynman chain)

Impurities with internal degrees of freedom (magnetic impurities)

Electron-electron interactions,....

The understanding of decoherence is difficult.

It is a great challenge in quantum mesoscopic physics.

The phase coherence length L_ϕ accounts in a generic way for decoherence processes.

The observation of coherent effects requires

$$L << L_\phi$$

Averaging over disorder ?

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Expect to wash up interference effects

Average coherence and multiple scattering

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Phase coherence leads to interference effects for a *given realization of disorder.*

The *Webb* experiment corresponds to a fixed configuration of disorder.

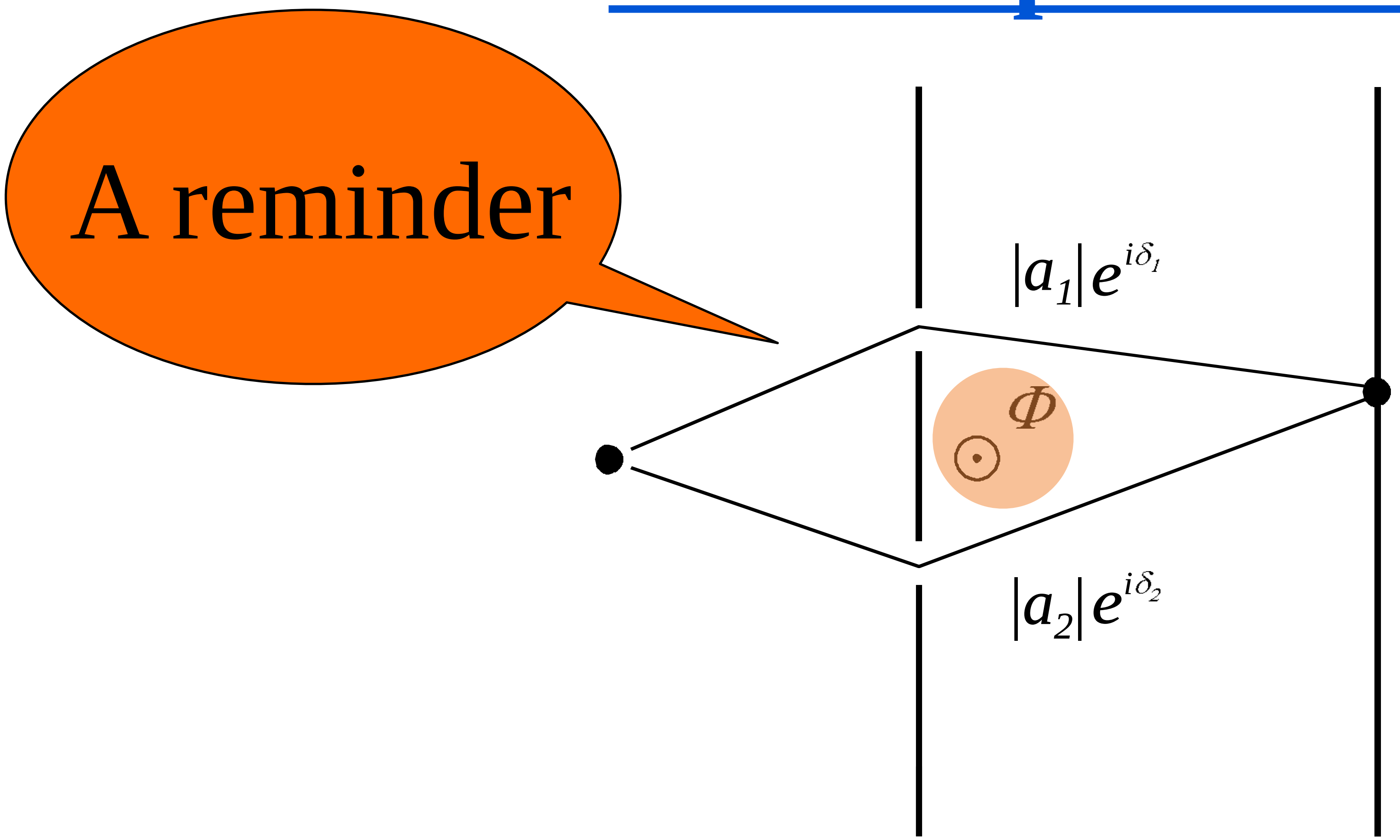
Averaging over disorder ⟹ vanishing of the Aharonov-Bohm effect

$$G(\phi) = G_0 + \delta G \cos(\Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0})$$

$$\Longrightarrow \langle G(\phi) \rangle = G_0$$

Disorder seems to erase coherent effects....

The setup : Young slits



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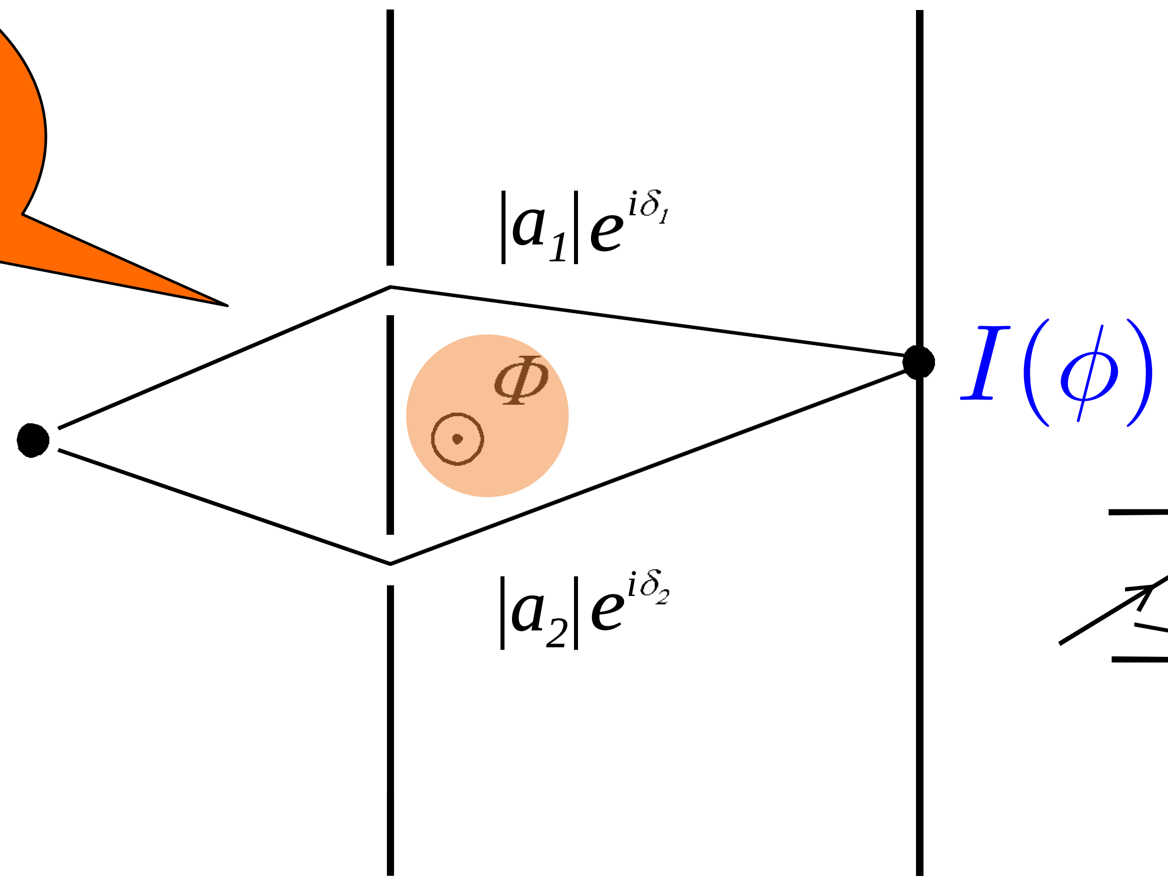
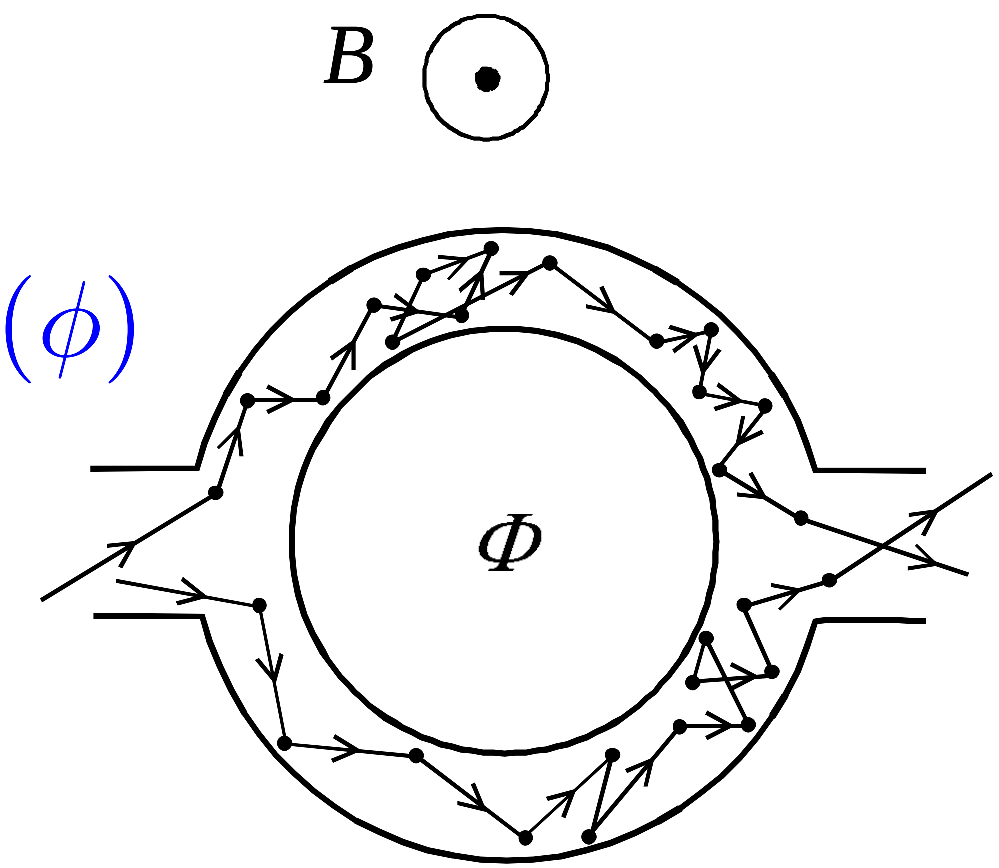
A reminder





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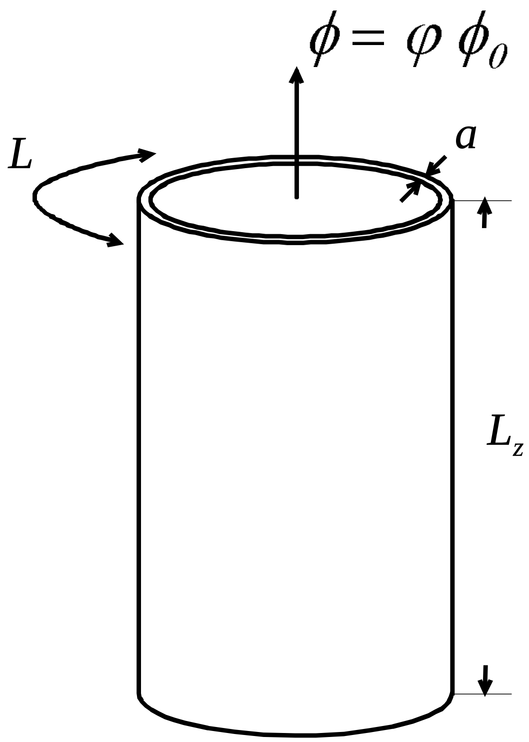
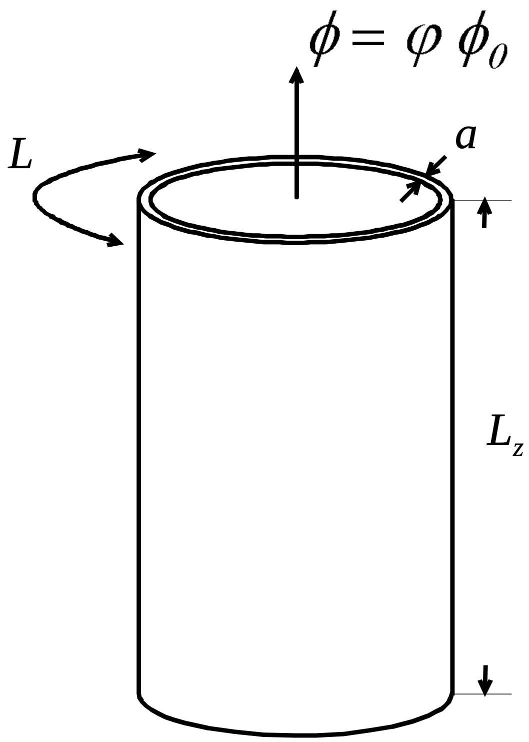
The Sharvin[2] experiment



Experiment analogous to that of *Webb* but performed on a hollow cylinder of height larger than L_ϕ pierced by a Aharonov-Bohm flux.
Ensemble of rings identical to those of *Webb* but incoherent between themselves.

The Sharvin² experiment



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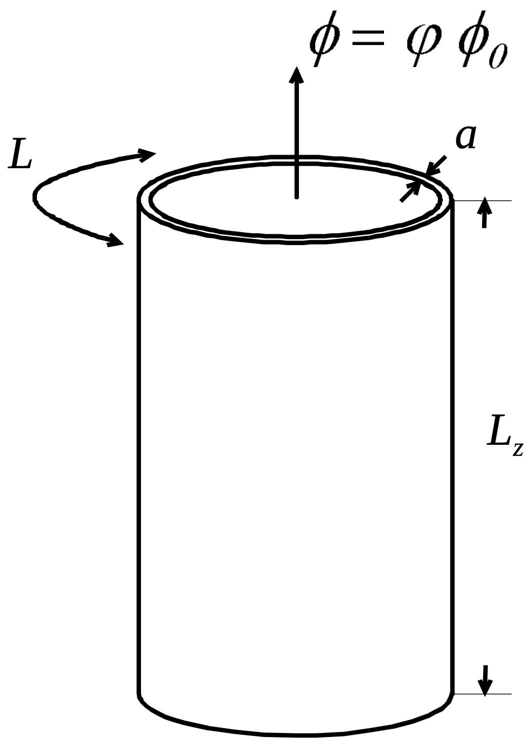
The signal modulated at ϕ_0 *disappears*

The Sharvin^2 experiment



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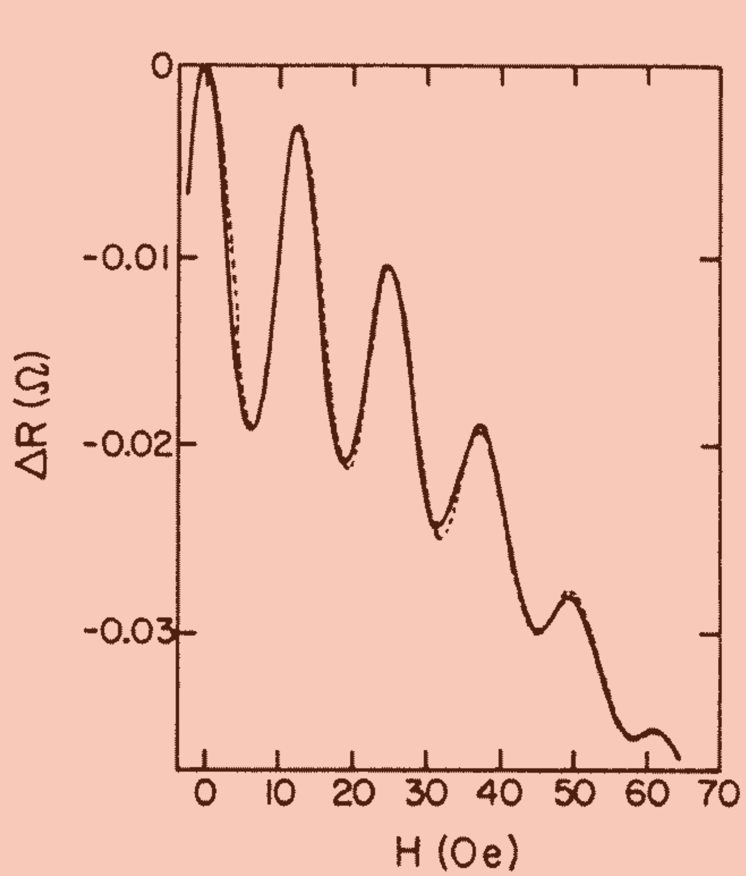


The signal modulated at ϕ_0 *disappears* but, instead, it appears a new contribution modulated at $\phi_0/2$

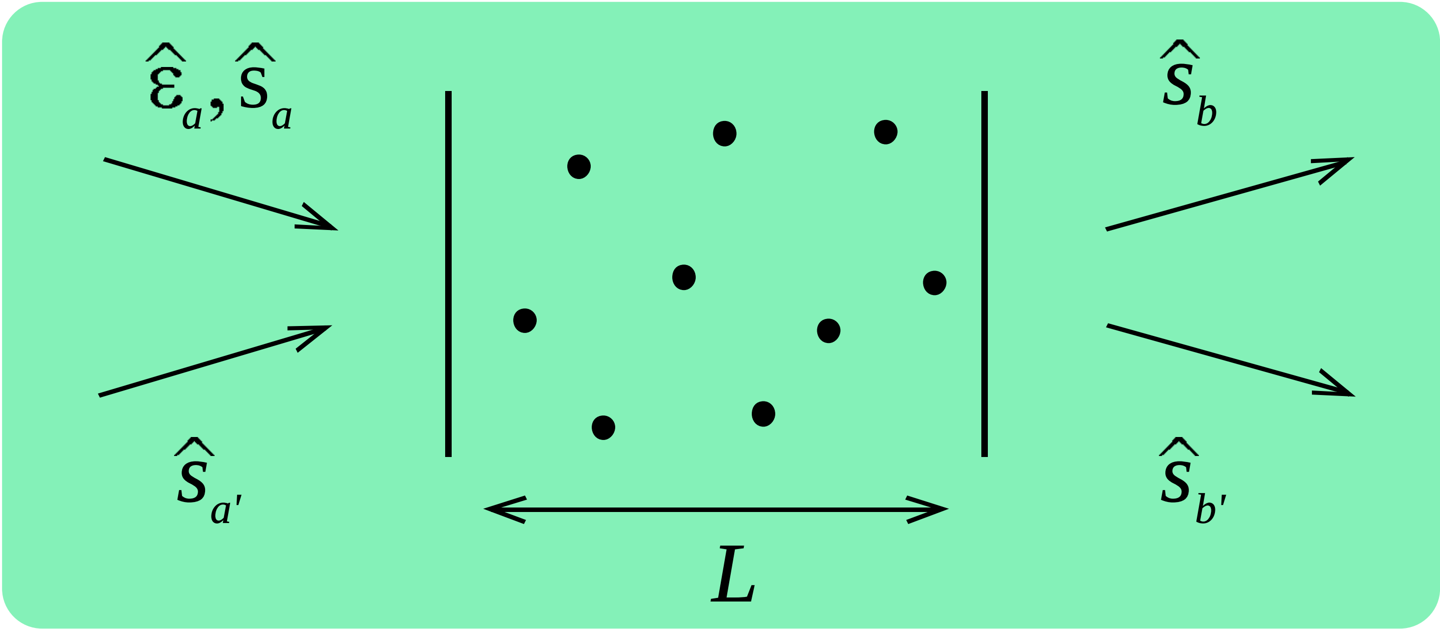
After all, disorder does not seem to erase coherent effects, but to modify them....

Some “canonical” mesoscopic effects

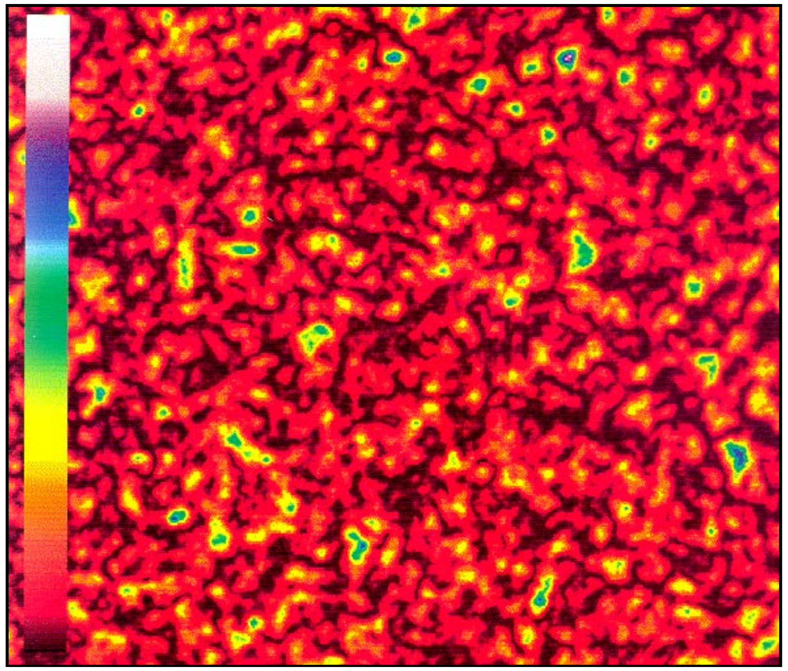
Coherent backscattering in optics

An analogous problem: *Speckle patterns in optics*

Consider the elastic multiple scattering of light transmitted through a fixed disorder configuration.

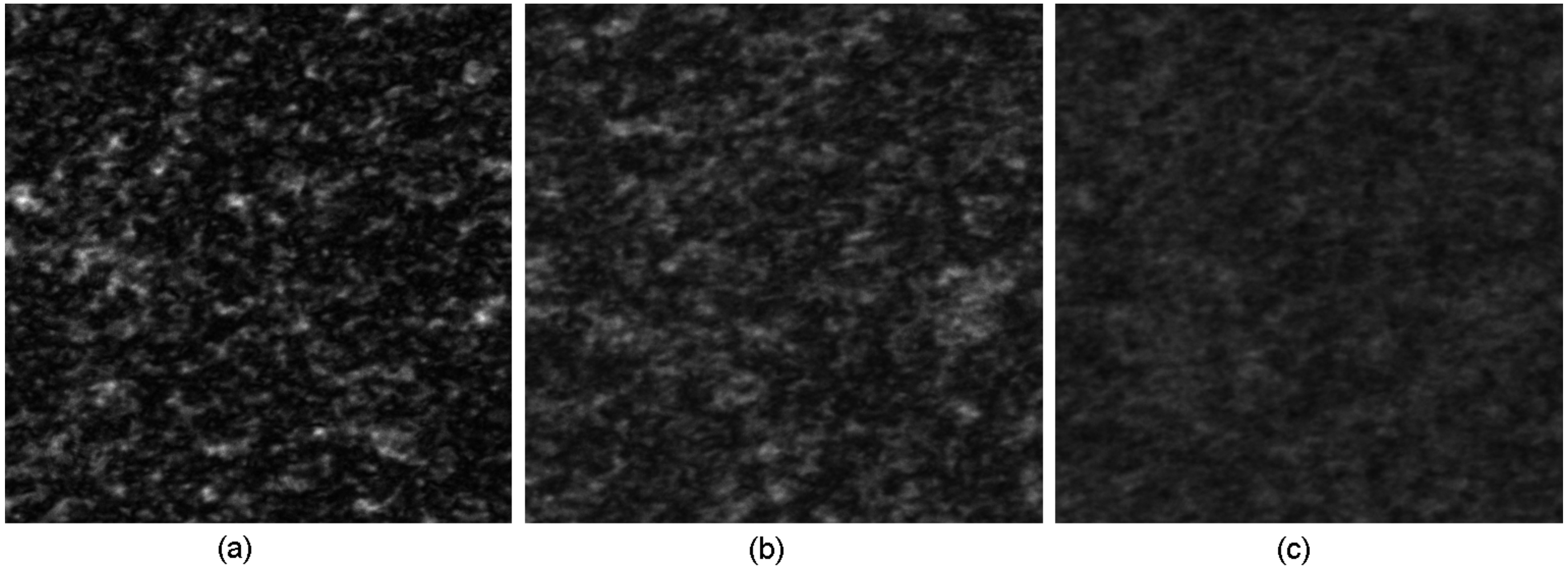


Outgoing light builds a speckle pattern *i.e.*, an interference picture:



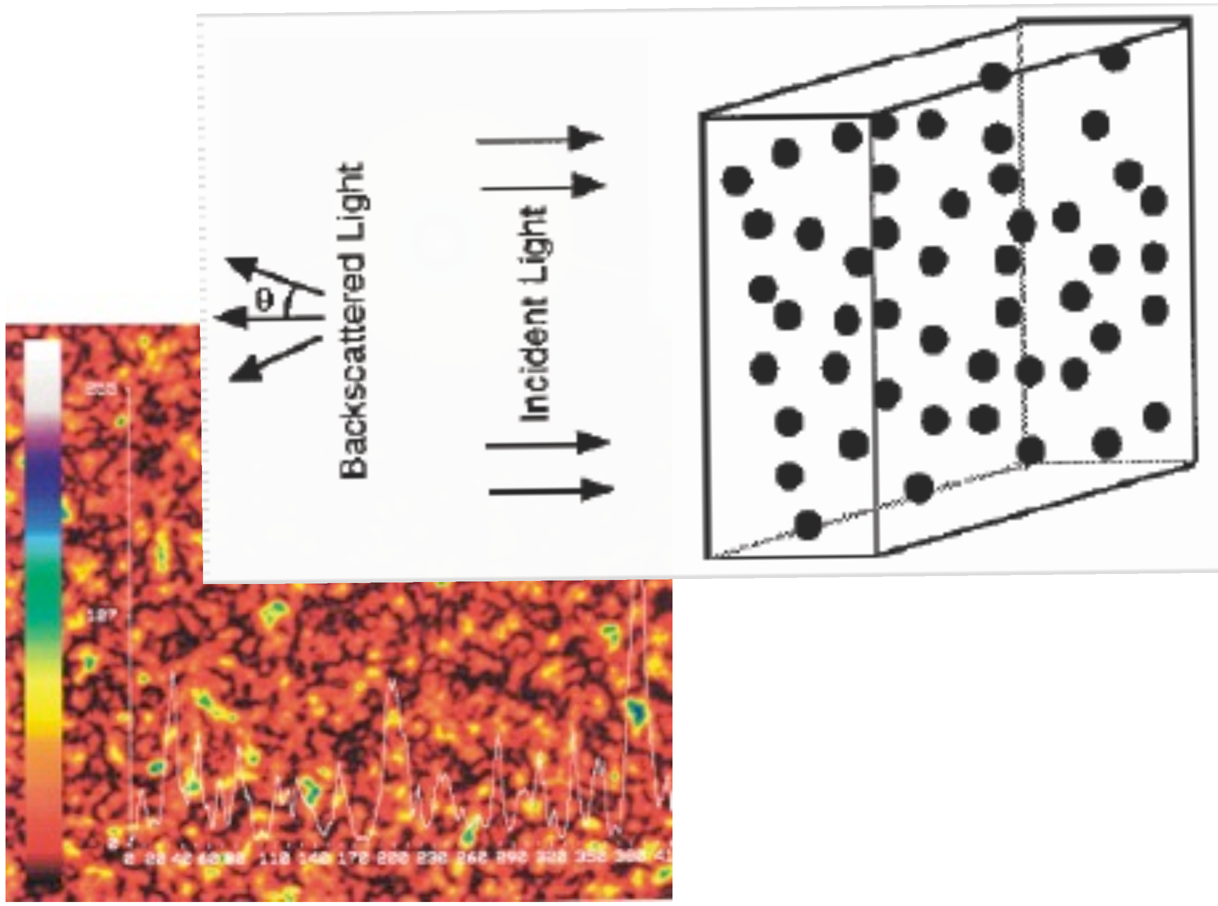
Averaging over disorder erases the speckle pattern:

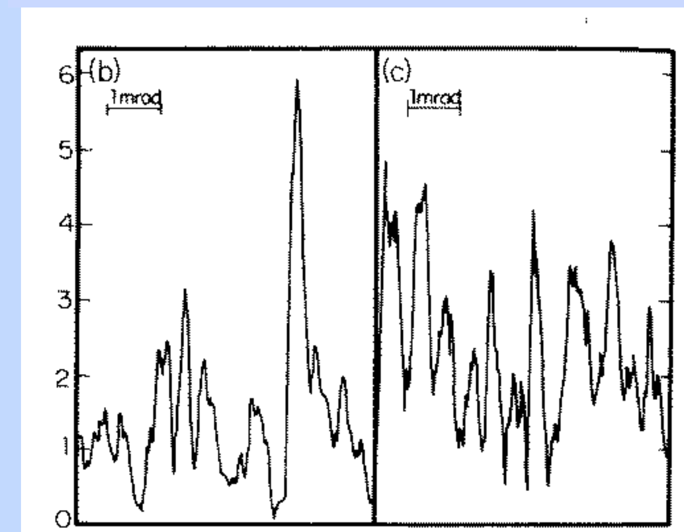
Integration over the motion of the scatterers leads to self-averaging



Time averaging

Does it erase all interferences ?





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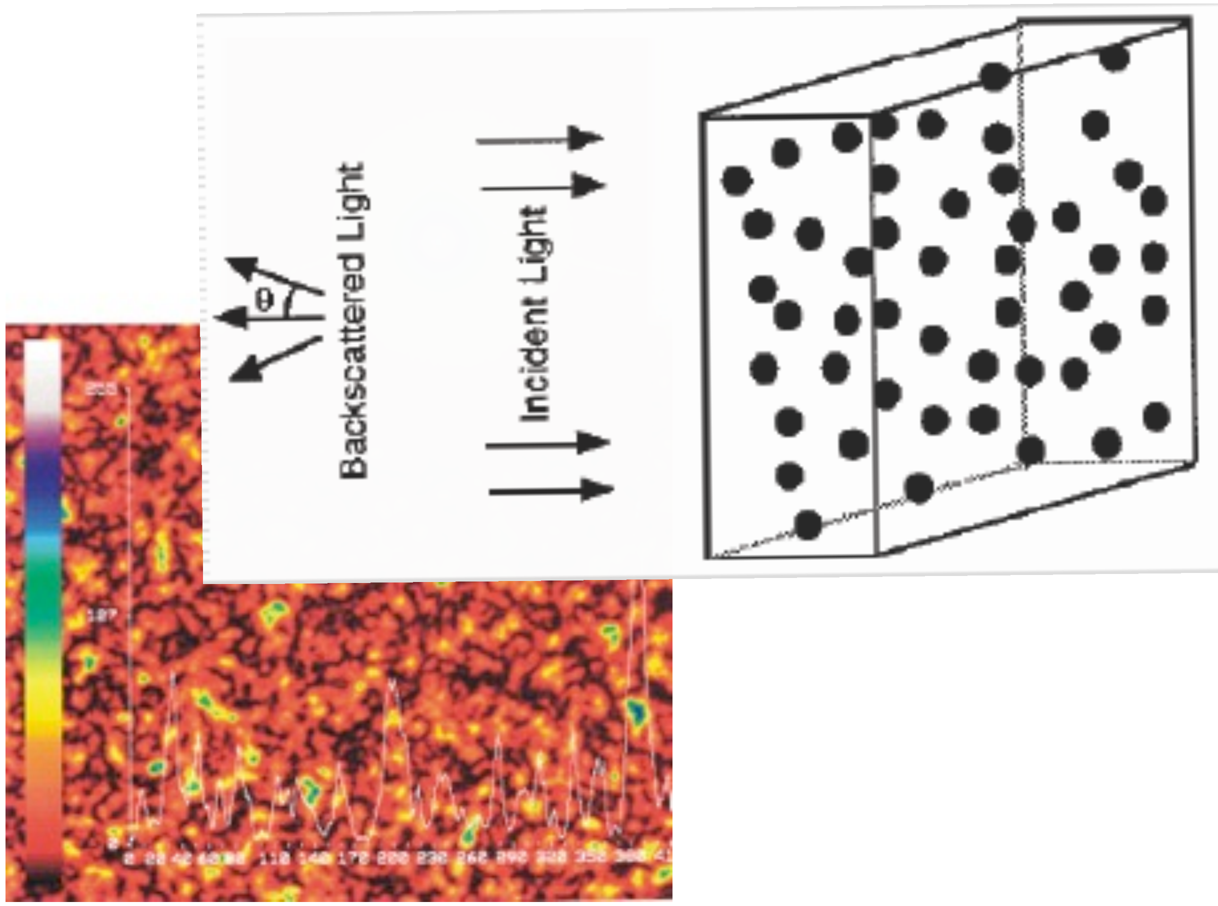
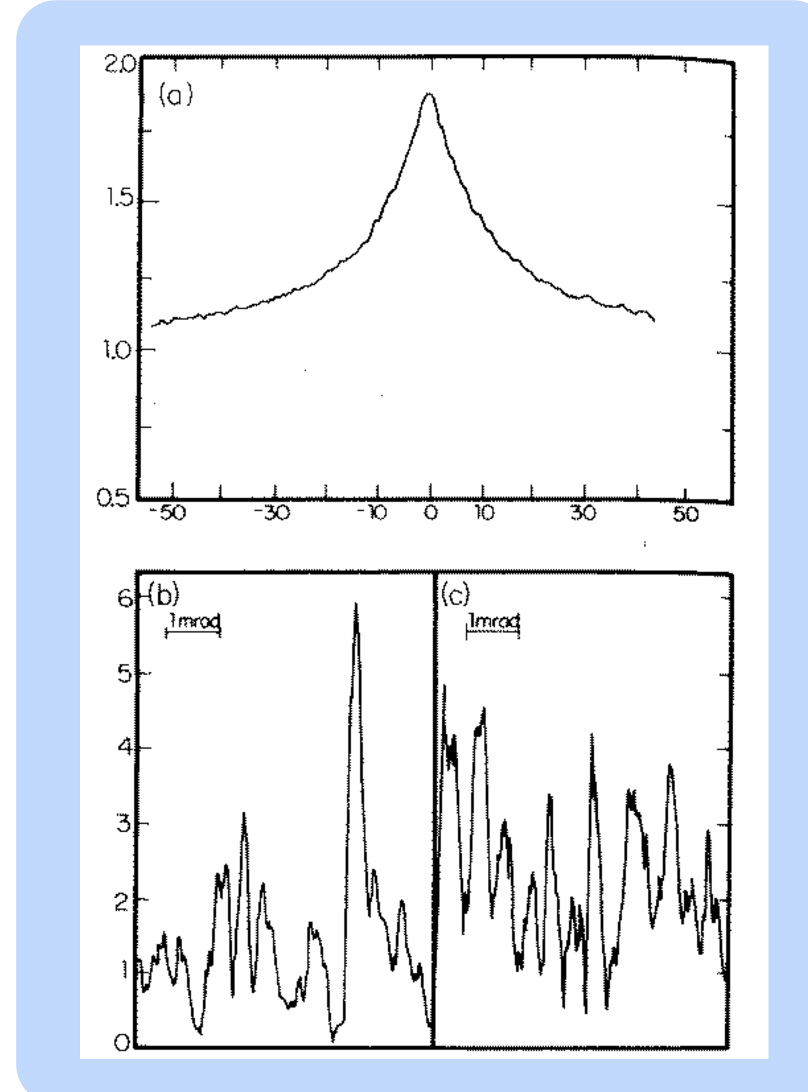
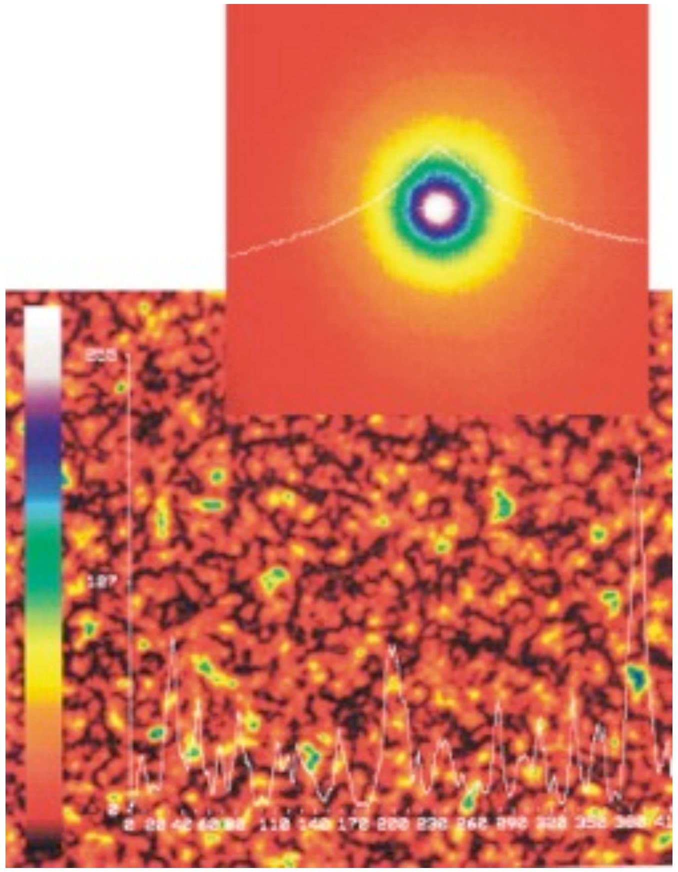


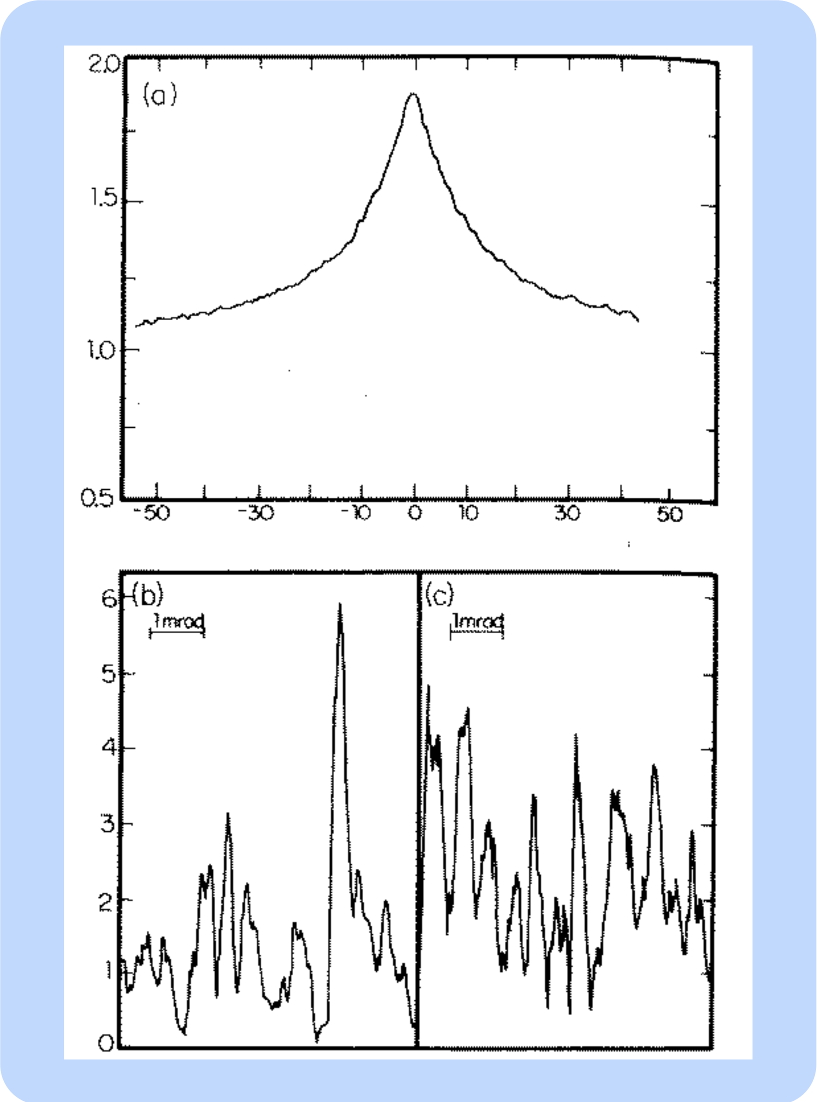


Averaging over disorder does not produce incoherent intensity only, but also an angular dependent part, the coherent backscattering, which is a coherence effect. We may conclude:

Elastic disorder is not related to decoherence : disorder does not destroy phase coherence and does not introduce irreversibility.

Does it erase all interferences ?





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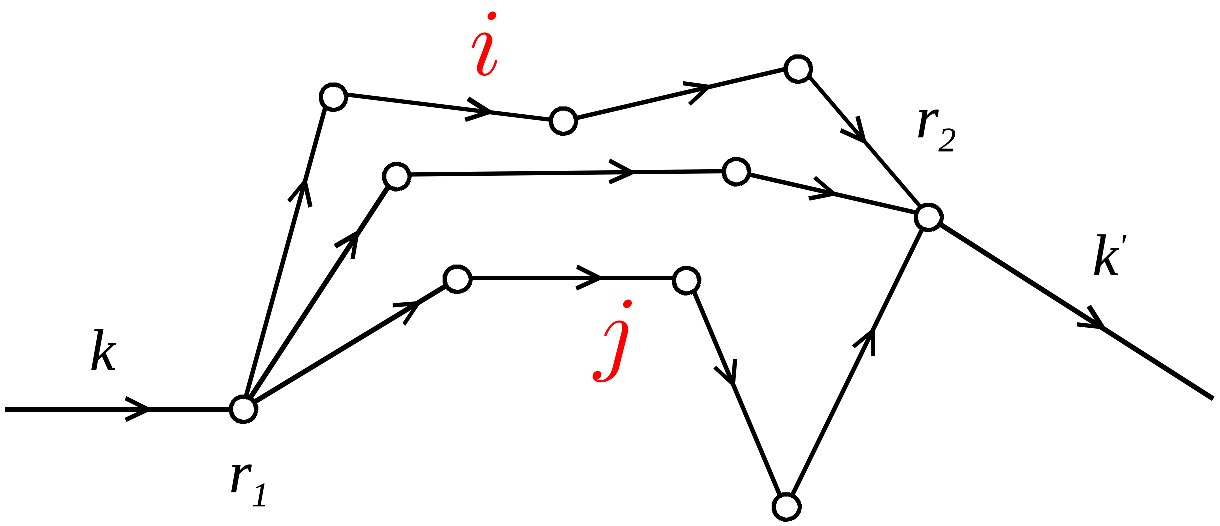
How to understand average coherent effects ?



Complex amplitude $A(\mathbf{k},\mathbf{k}')$ associated to the multiple scattering of a wave (electron or photon) incident with a wave vector $\mathbf{k}$ and outgoing with $\mathbf{k}'$

$$A(\mathbf{k},\mathbf{k}') = \sum_{\mathbf{r_1},\mathbf{r_2}} f(\mathbf{r_1},\mathbf{r_2}) e^{i(\mathbf{k}.\mathbf{r_1} - \mathbf{k}'.\mathbf{r_2})}$$

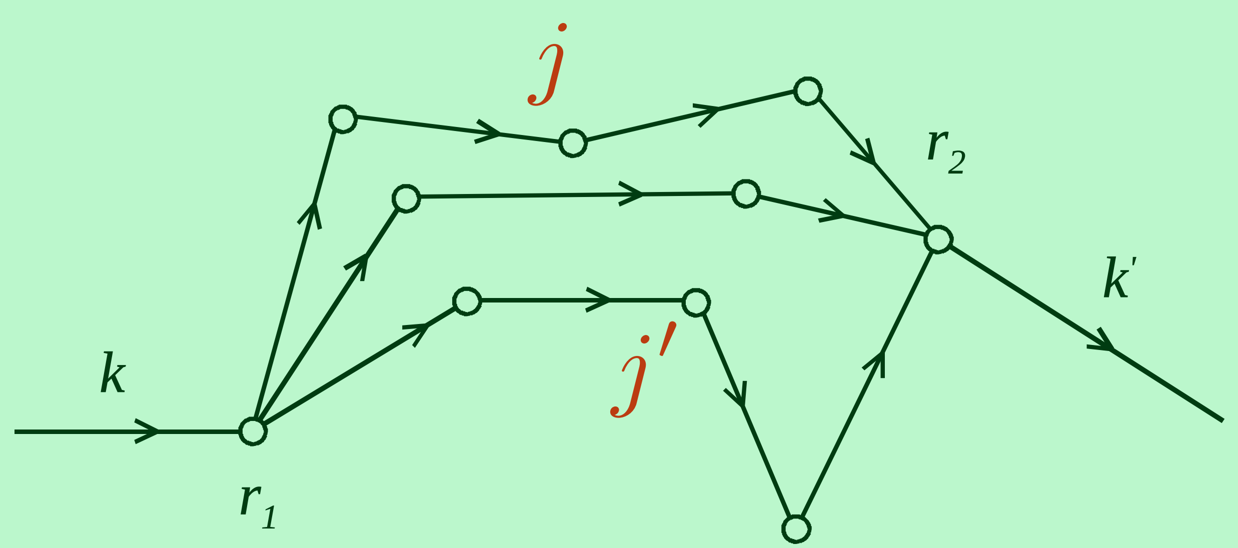
the complex amplitude $f(\mathbf{r_1},\mathbf{r_2}) = \sum_j |a_j| e^{i\delta_j}$ describes the propagation of the wave between $\mathbf{r_1}$ and $\mathbf{r_2}$.

The corresponding intensity is

$$|A(\mathbf{k},\mathbf{k'})|^2 = \sum_{\mathbf{r_1},\mathbf{r_2}} \sum_{\mathbf{r_3},\mathbf{r_4}} f(\mathbf{r_1},\mathbf{r_2}) f^*(\mathbf{r_3},\mathbf{r_4}) e^{i(\mathbf{k}.\mathbf{r_1}-\mathbf{k'}.\mathbf{r_2})} e^{-i(\mathbf{k}.\mathbf{r_3}-\mathbf{k'}.\mathbf{r_4})}$$

with

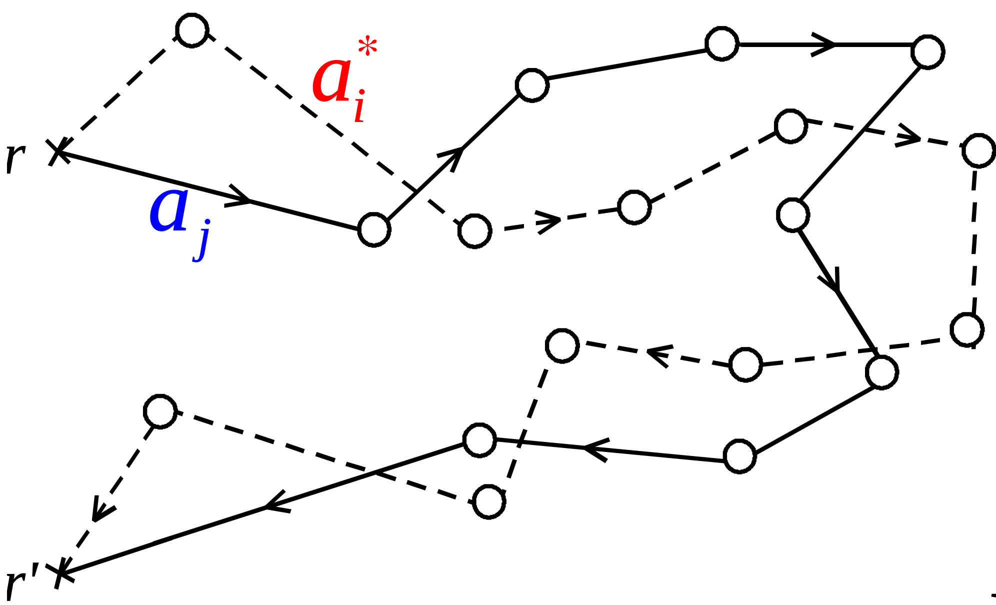
$$f(\mathbf{r_1},\mathbf{r_2}) f^*(\mathbf{r_3},\mathbf{r_4}) = \sum_{j,j'} a_j(\mathbf{r_1},\mathbf{r_2}) a^*_{j'}(\mathbf{r_3},\mathbf{r_4}) = \sum_{j,j'} |a_j||a_{j'}| e^{i(\delta_j - \delta_{j'})}$$



On average over disorder, most contributions to $f\,f^*$ disappear since the dephasing $\delta_j - \delta_{j'} \gg 1$

The only remaining contributions to the intensity correspond to terms with zero dephasing, *i.e.*, to identical trajectories.

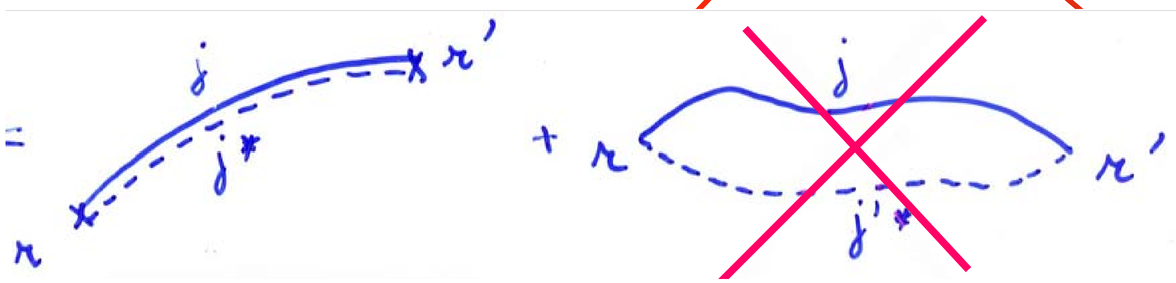
Quantum probability for propagation between two points



$$P(r,r') = \sum_{i,j} \overline{a_i^*(r,r')\, a_j(r,r')}$$

$$P(r,r') = \sum_j \overline{\left| a_j(r,r') \right|^2} + \sum_{i \neq j} \overline{a_i^*(r,r') a_j(r,r')}$$

$$a_i^* a_j = |a_i||a_j| e^{i(\delta_i - \delta_j)}$$

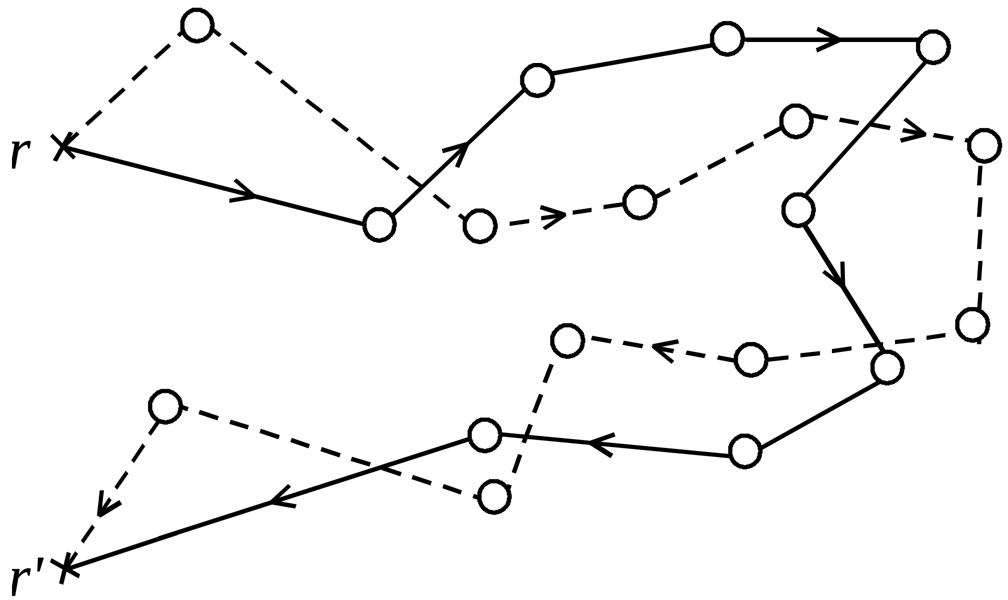


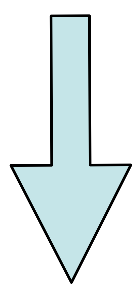
$$\delta_i - \delta_j \gg 1$$

vanishes on average

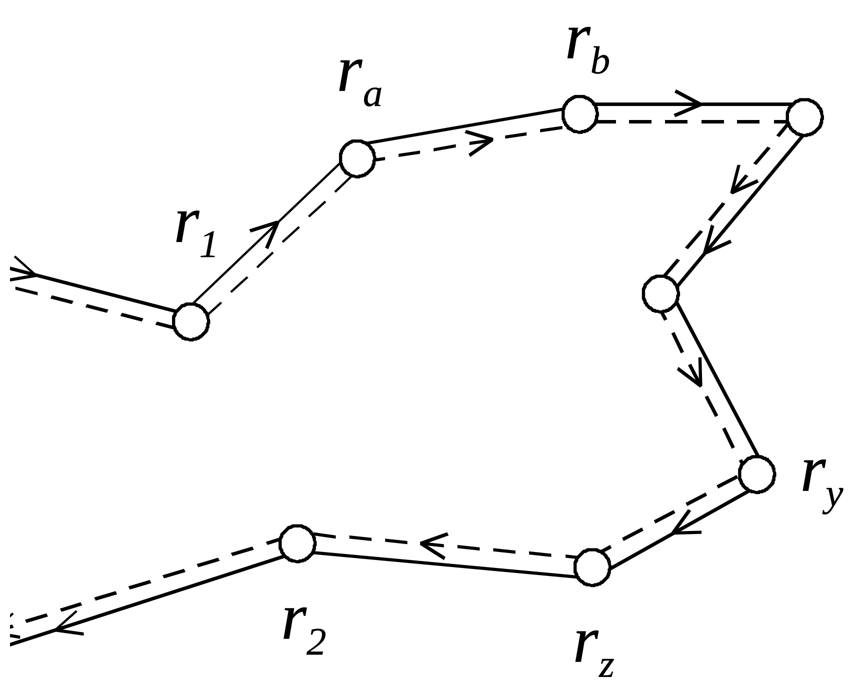
Incoherent propagation !

Some useful design





Disorder
averaging



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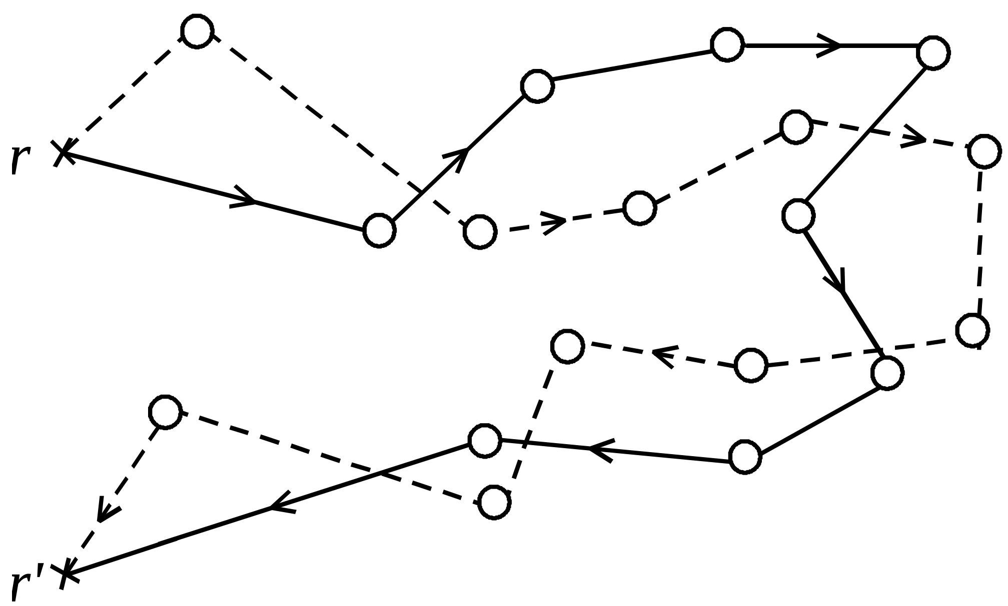
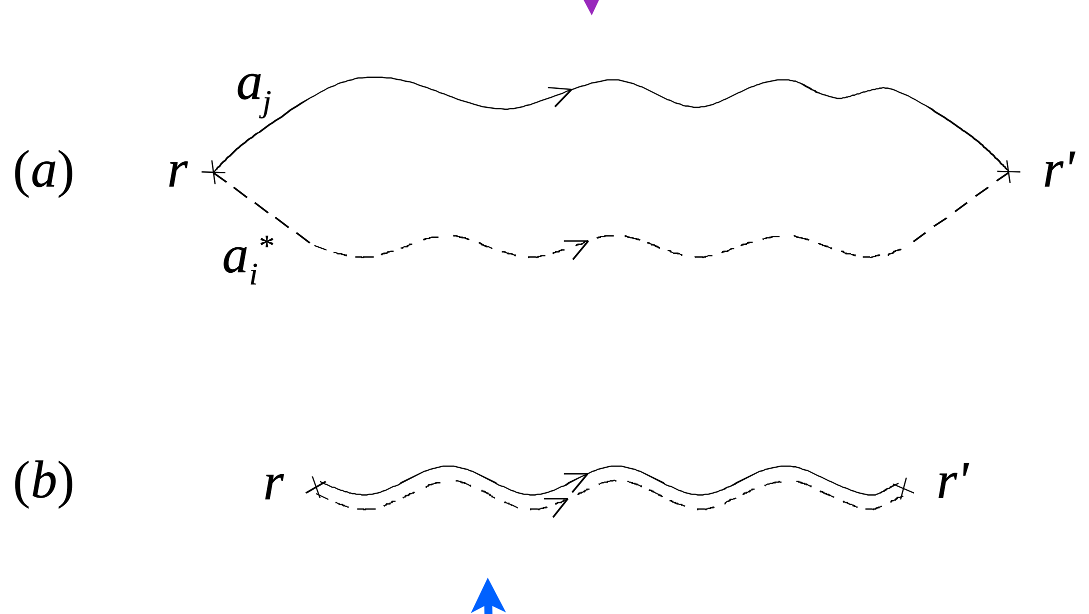
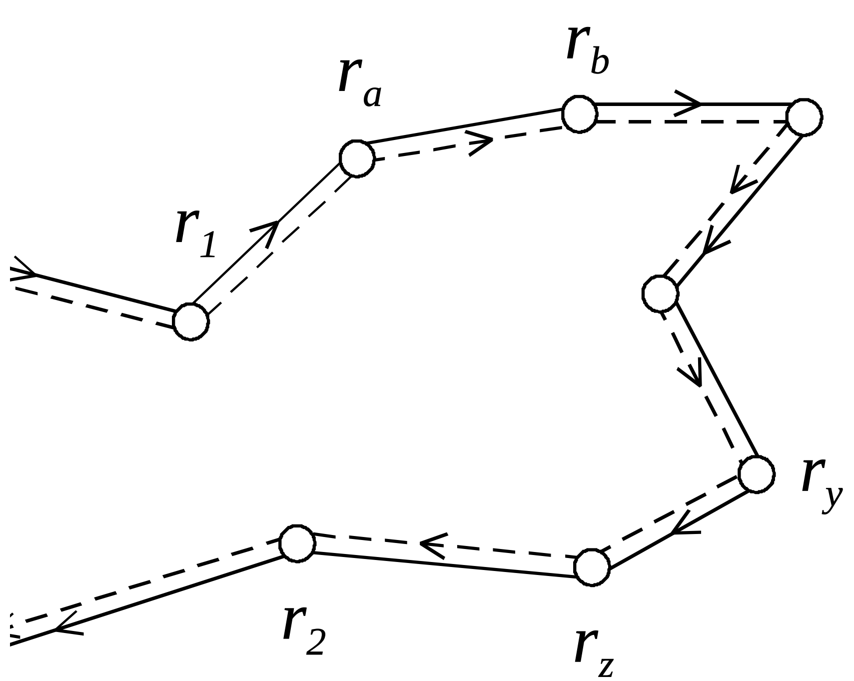


Disorder averaging

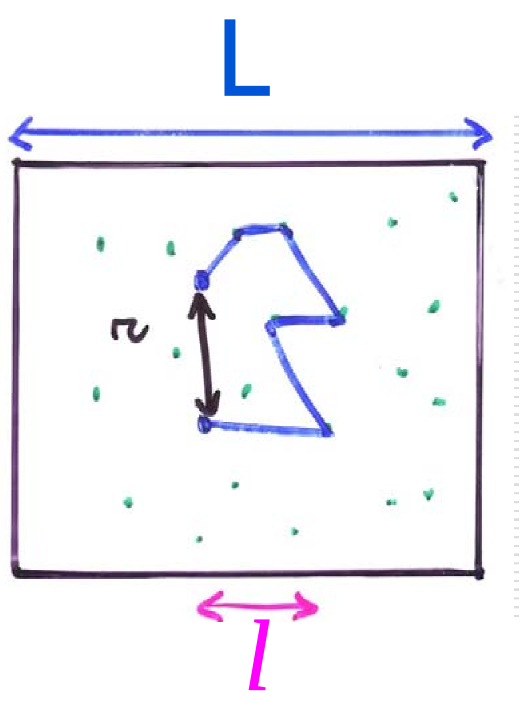




To a good approximation, the incoherent contribution obeys a classical diffusion equation

$$\left(\frac{\partial}{\partial t} - D\Delta\right)P(r,r',t) = \delta(r-r')\delta(t) \quad \Leftrightarrow \quad \left(-i\omega + Dq^2\right)P(q,\omega) = 1$$

Incoherent electrons diffuse in the conductor with a *diffusion coefficient D (Drude theory)*



$$\langle r^2 \rangle = 2d\, Dt$$

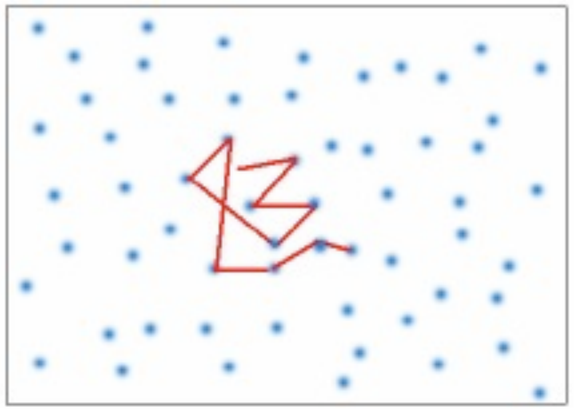
space dimensionality

Thouless time

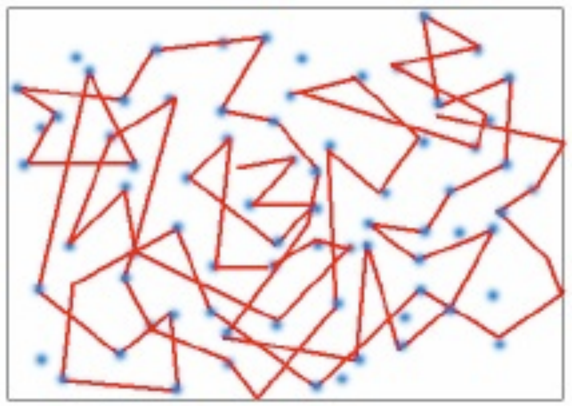
$$L^2 = D\tau_D$$

$l \ll L$

$t \ll \tau_D$

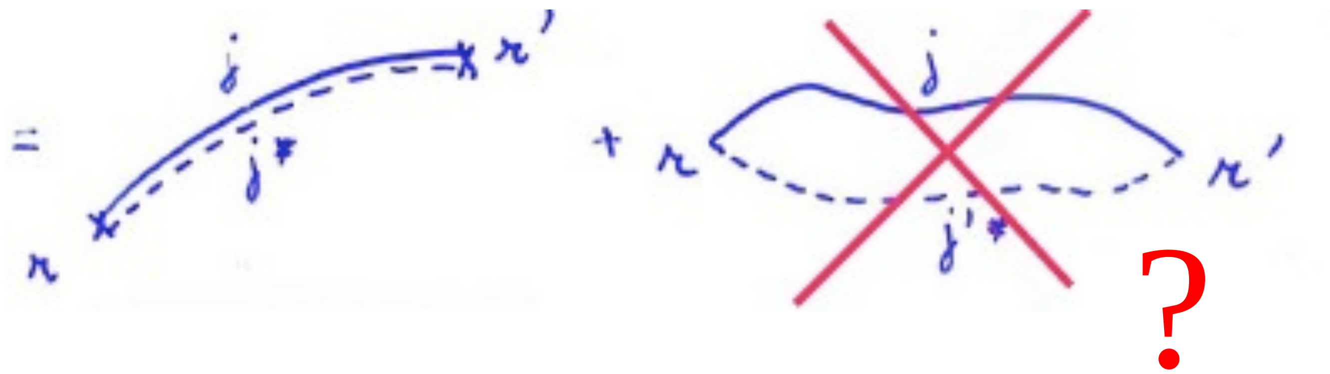


$t \gg \tau_D$



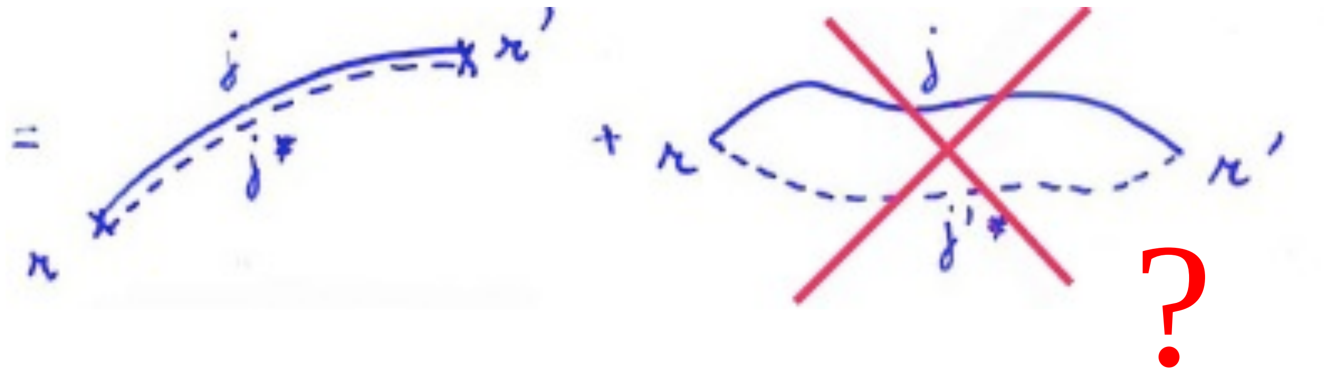
Beyond incoherent diffusion (qualitative description)

Coherent effects



What is the first correction *i.e.*, with the *smallest phase shift* ?

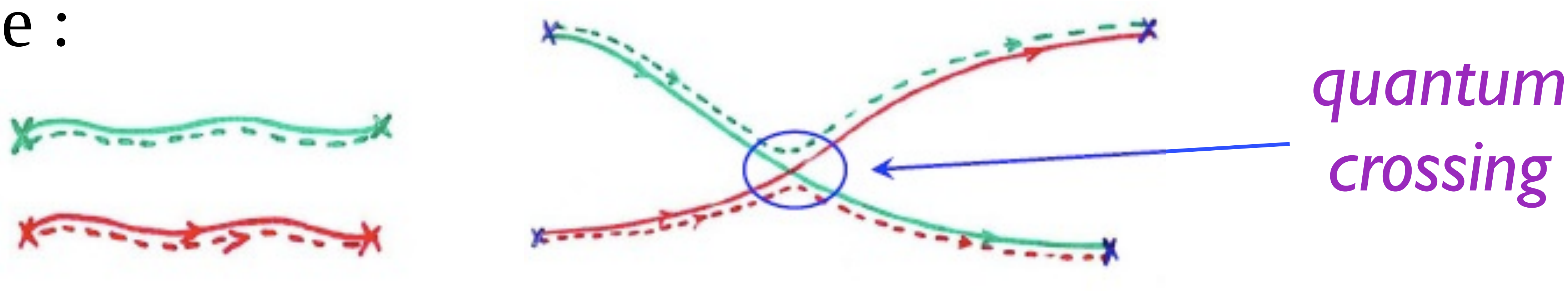
Coherent effects



What is the first correction *i.e.*, with the *smallest phase shift* ?

When amplitude paths cross

Example :



Classical diffusion

Exchange of amplitudes

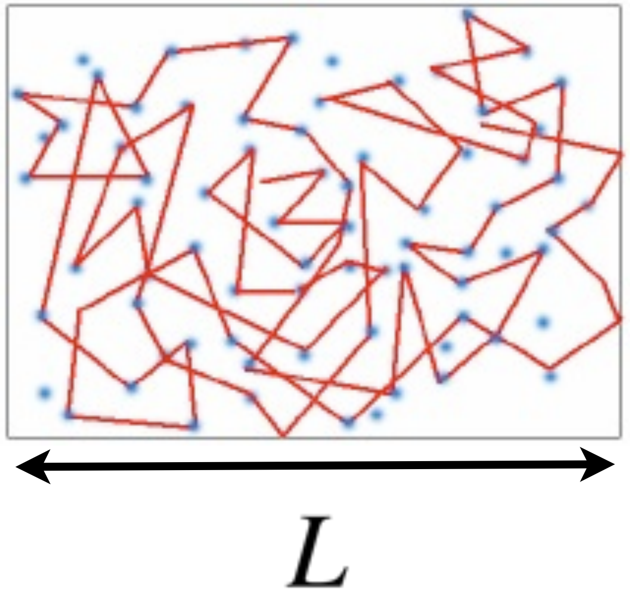
Occurrence of a *quantum crossing* after a time t for a electron diffusing in a volume L^d

$$p_\times(t) = \frac{\lambda_F^{d-1} v_F t}{L^d}$$

v_F : Fermi velocity

The time spent by a diffusing electron is $\tau_D = L^2/D$ so that

$$p_\times(\tau_D) = \frac{\lambda_F^{d-1} v_F \tau_D}{L^d} \equiv \frac{1}{g}$$



$$g = \frac{D}{v_F \lambda_F^{d-1}} L^{d-2}$$

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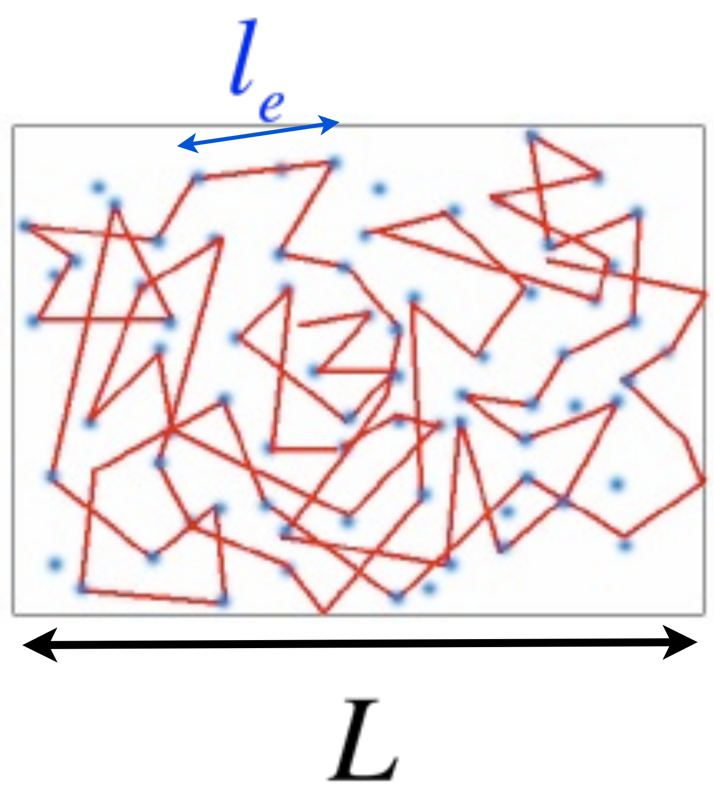
Physical meaning of this parameter ?

Electrical conductance of a metal

A metal can be modeled as a quantum gas of electrons scattered by an elastic disorder l_e .

Classically, the conductance of a cubic sample of size L^d is given by Ohm's law: $G = \sigma L^{d-2}$ where σ is the conductivity.



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Classically, the conductance of a cubic sample of size L^d is given by Ohm's law: $G = \sigma L^{d-2}$ where σ is the conductivity.

$$g = \frac{l_e}{3\lambda^{d-1}} L^{d-2} = G_{cl}/(e^2/h)$$

G_{cl} is the classical electrical conductance so that

$$G_{cl}/(e^2/h) \gg 1$$

A direct consequence: quantum corrections to electrical transport

Classical transport : $G_{cl} = g \times \frac{e^2}{h} \, \mathrm{with} \, g \gg 1$

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Quantum corrections: Δ

What does it mean ?

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Classical transport : $G_{cl} = g \times \frac{e^2}{h}$ with $g \gg 1$

Independent of the microscopic (and often unknown) disorder - Depends only on the geometry

Quantun

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A direct consequence: quantum corrections to electrical transport

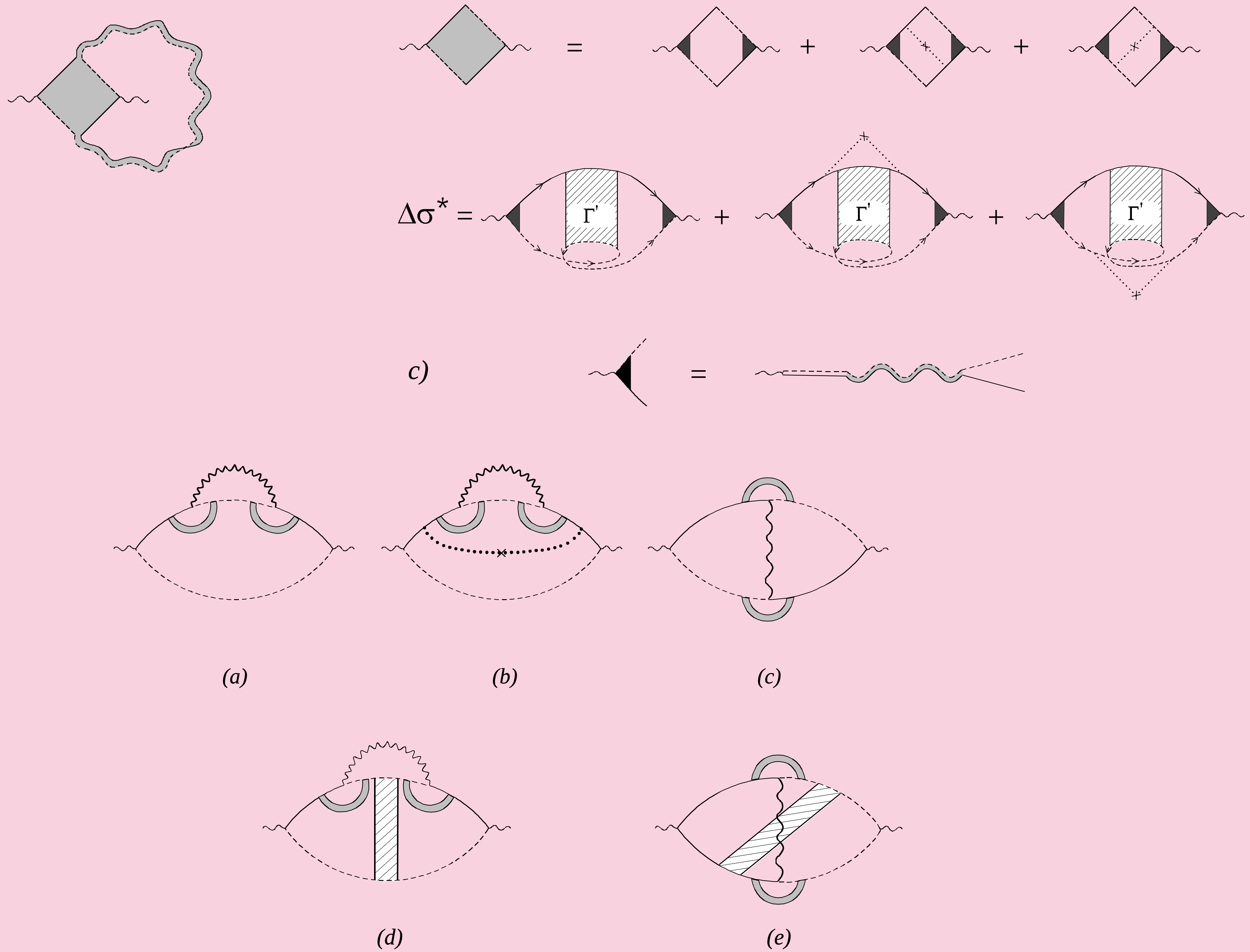
Cla... $G_{cl} = g \times \frac{e^2}{h}$ with $g \gg 1$

Not that simple ! We wish to obtain precise numbers... Need to sum up Feynman diagrams.

Quantum correc...

so that $\Delta G = \# \frac{e^2}{h}$ is <u>universal</u>

A direct consequence: quantum corrections to electrical



Complexity of a quantum mesoscopic system

Elastic disorder does not break phase coherence

and it does not introduce irreversibility

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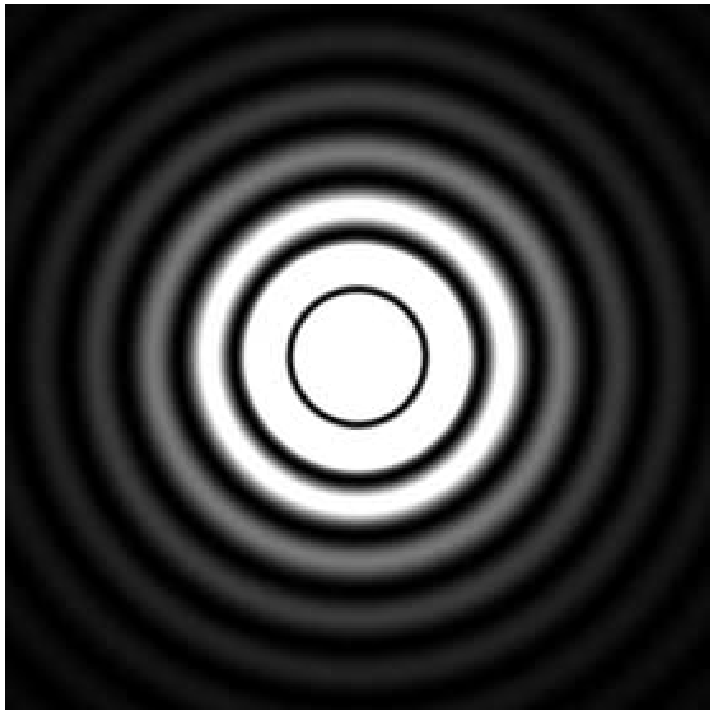
Disorder introduces randomness and complexity:

All symmetries are lost, there are no good quantum numbers.

Exemple: speckle patterns in optics

Diffraction through a circular aperture: order in interference



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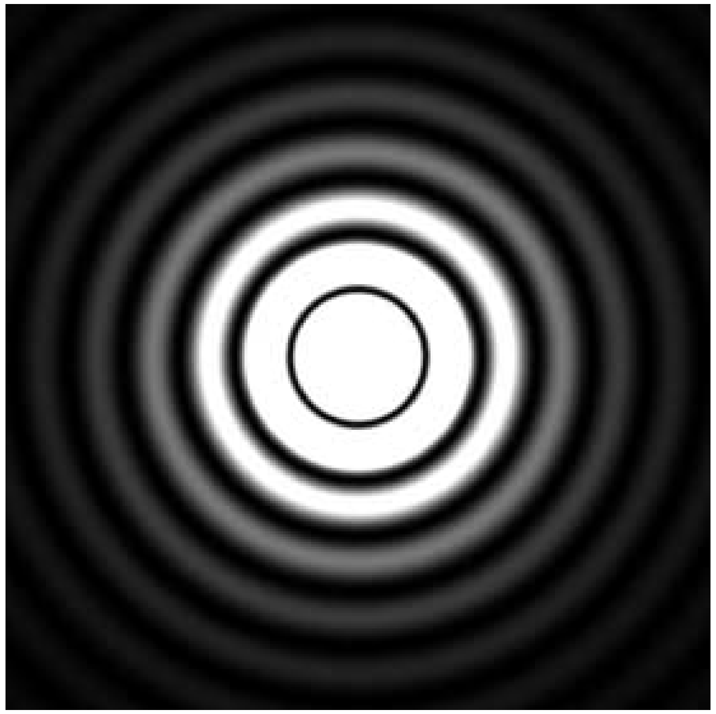

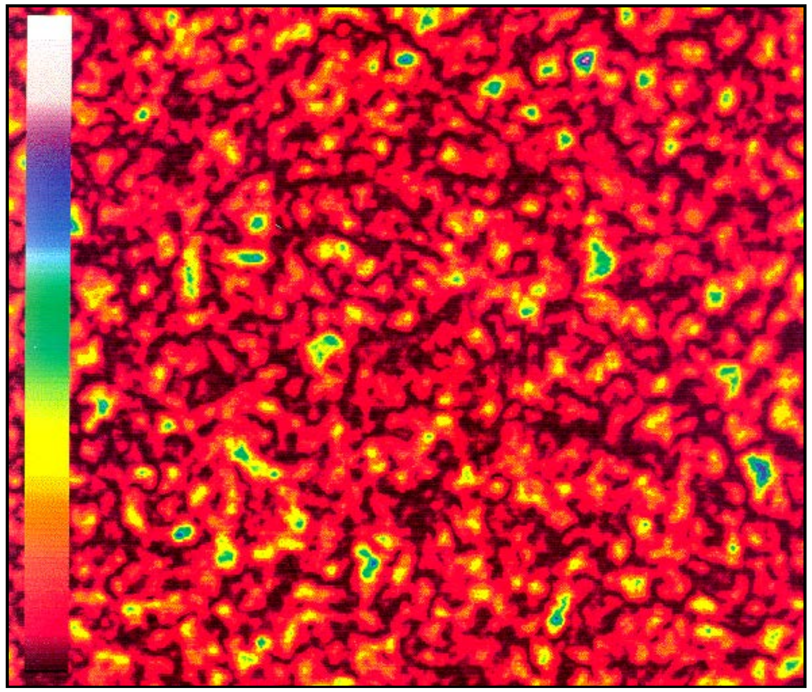

Transmission of light through a disordered suspension: complex system

Mesoscopic quantum systems

- Most (all ?) quantum systems are complex
- Complexity (randomness) and decoherence are separate and independent notions.
- Complexity: loss of symmetries (good quantum numbers)
- Decoherence: irreversible loss of quantum coherence $L \gg L_\varphi$

A mesoscopic quantum system is a coherent complex quantum system with $L \leq L_\varphi$

Phase coherence and self-averaging: universal fluctuations.

Classical limit : $L \gg L_\varphi$

The system is a collection of $N = (L/L_\varphi)^d \gg 1$ statistically independent subsystems.

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Law of large numbers: any macroscopic observable is equal with probability one to its average value.

The system performs an average over realizations of the disorder.

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For $L \ll L_{\varphi}$, we expect deviations from self-averaging which reflect the underlying quantum coherence.

Quantum conductance fluctuations

Classical self-averaging limit : $\frac{\delta G}{\overline{G}} = \frac{1}{N} = \left(\frac{L_\varphi}{L}\right)^{d/2}$

where $\delta G = \sqrt{\overline{G^2} - \overline{G}^2}$ and $\overline{G} = \sigma L^{d-2}$

$\overline{\cdots}$ is the average over disorder.

$$\delta G^2 \propto L^{d-4}$$

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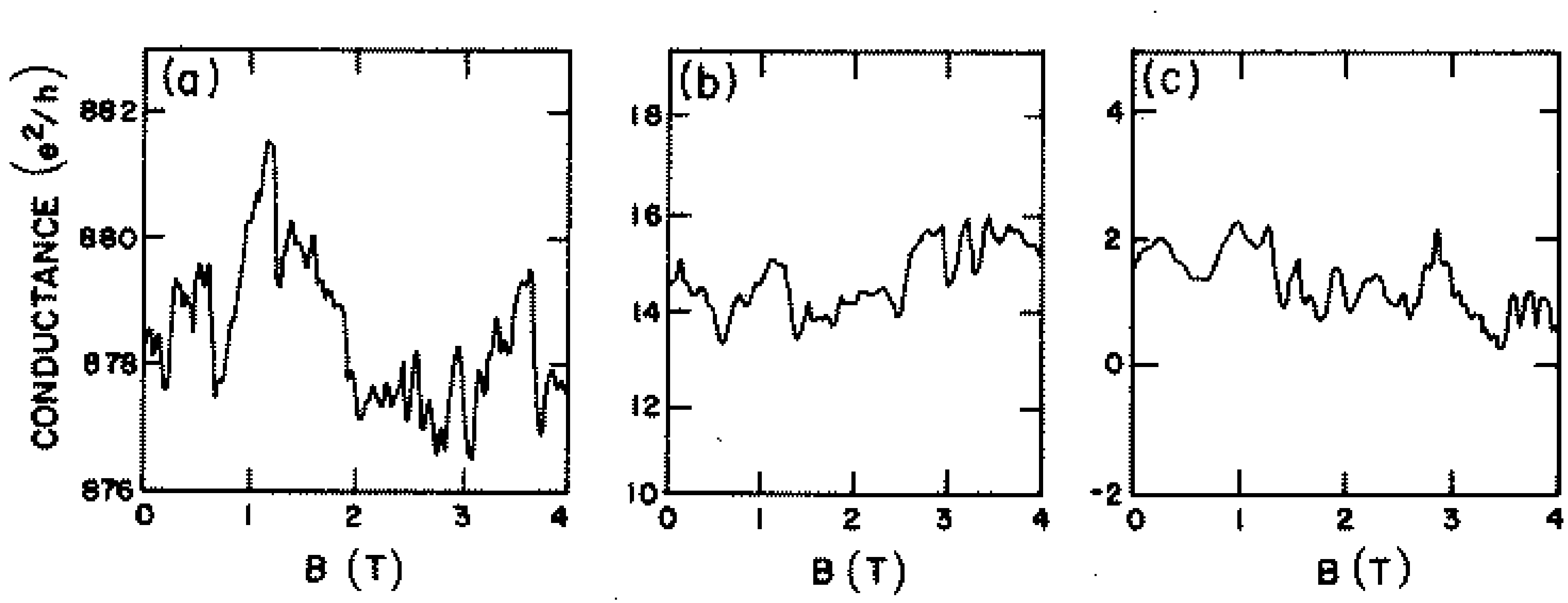
In the mesoscopic limit, the electrical conductance is not self-averaging.



Gold ring

Si-MOSFET

NUMERICS ON
THE ANDERSON MODEL

Summary : key ideas and concepts in quantum mesoscopic physics

1. Classical diffusion (long range)

2. Disorder does not wash out interference effects

3. Local (spatially) quantum corrections (quantum crossings) are propagated over long distances by means of classical diffusion.

3. Complexity (no good quantum numbers) is distinct from decoherence.