

# Quantum information processing with superconducting circuits

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## Sherbrooke's circuit QED theory group

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INTRIQ

# Motivations: A physicist point of view

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(Really) Long term goals:

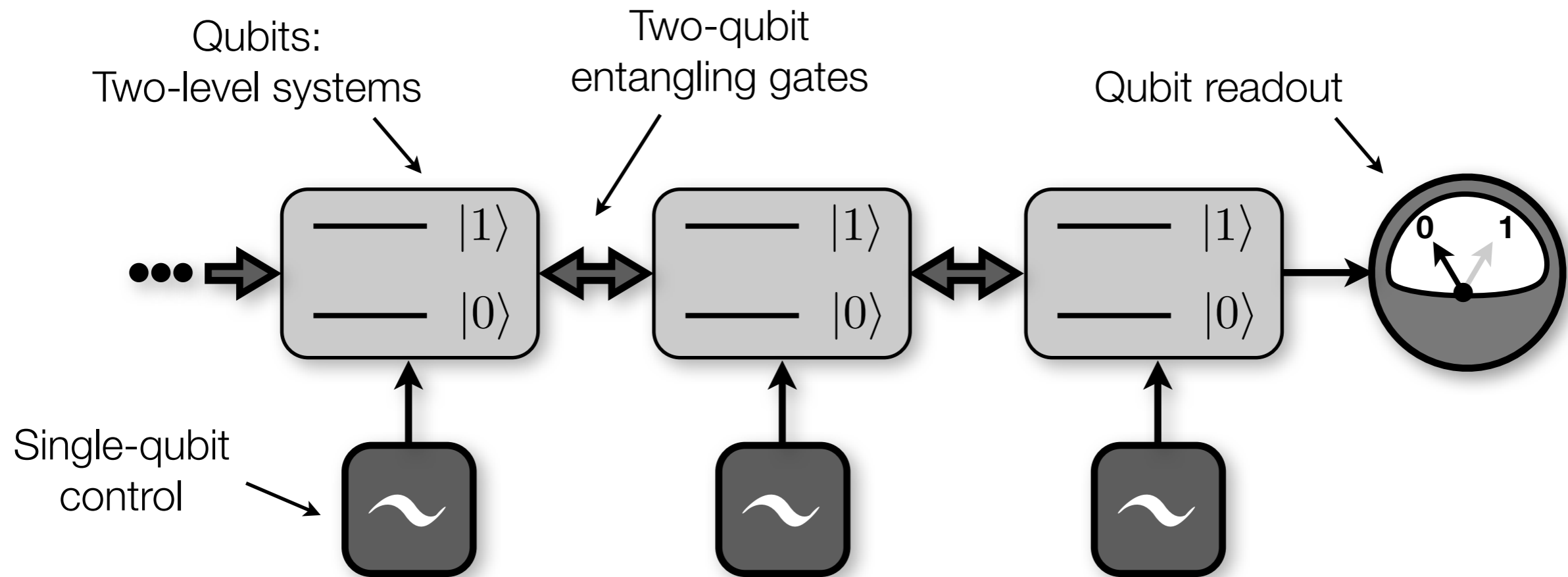
- Large scale quantum computation
- Solve computationally hard (and interesting!) problems

Current goals:

- Quantum mechanics of big objects
- Understanding decoherence and quantum measurement
- Control of open quantum system
- Does quantum mechanics break down at large scales?

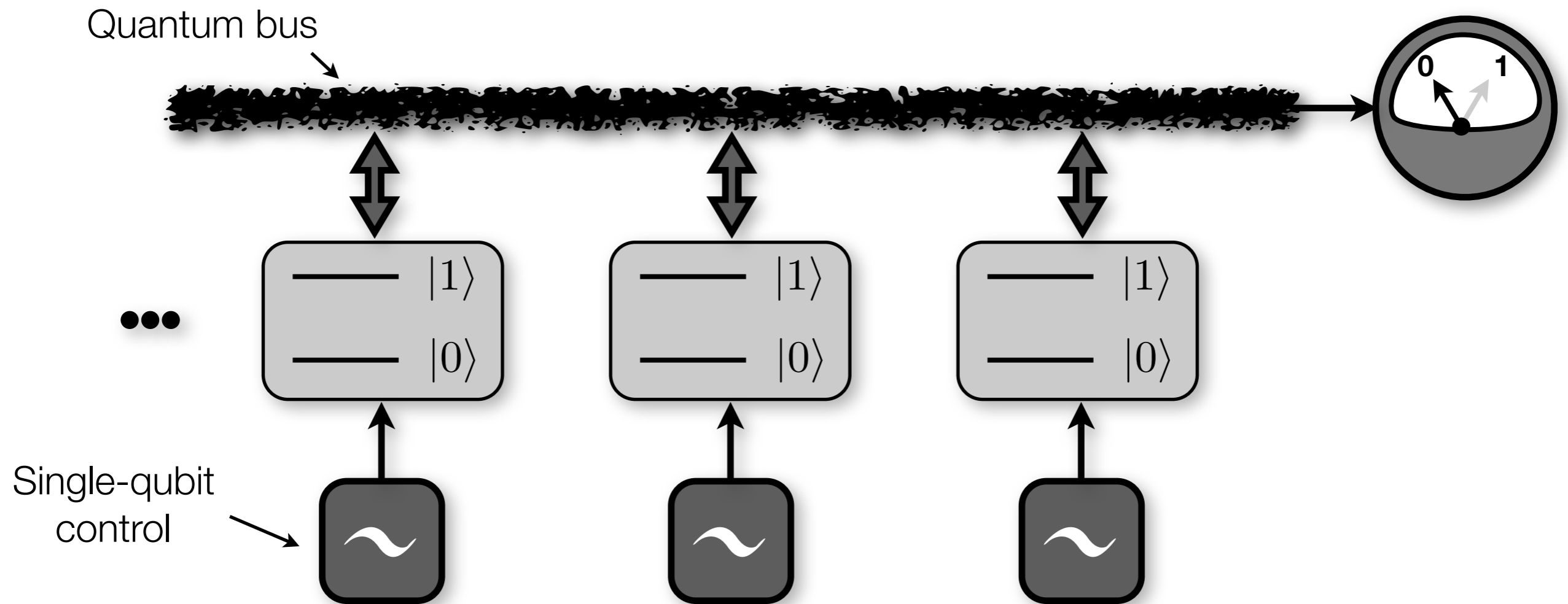
# Quantum information processing: the challenge

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- Conflicting requirements: *long* coherence, *fast* control and readout

# Quantum information processing: the challenge

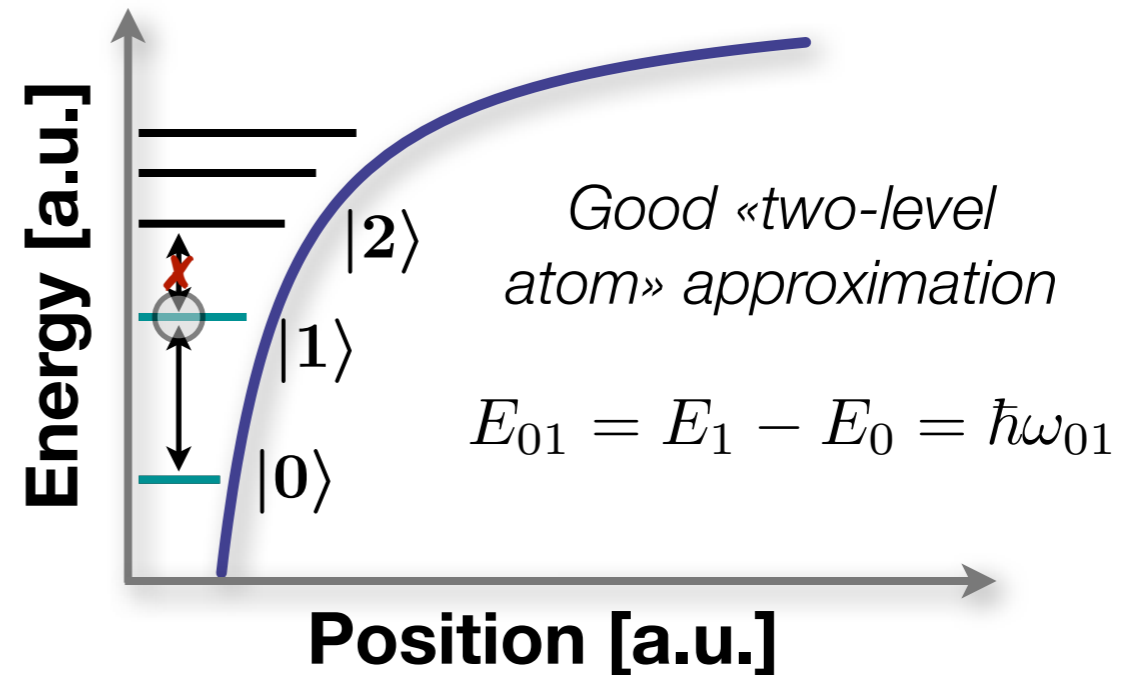
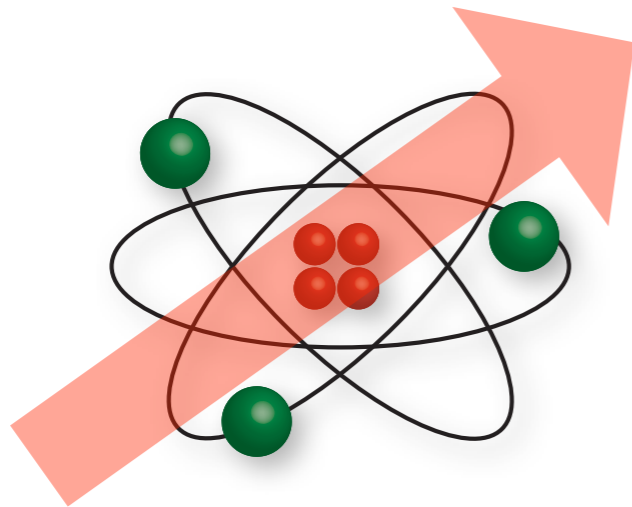


- Conflicting requirements: *long* coherence, *fast* control and readout



**Artificial atoms**

# Nature's atoms



- Control internal state by shining laser tuned at the transition frequency

$$H = -\vec{d} \cdot \vec{E}(t) \quad \text{with} \quad E(t) = E_0 \cos \omega_{01} t$$

- Hyperfine levels of  ${}^9\text{Be}_+$  have long decay and coherence times

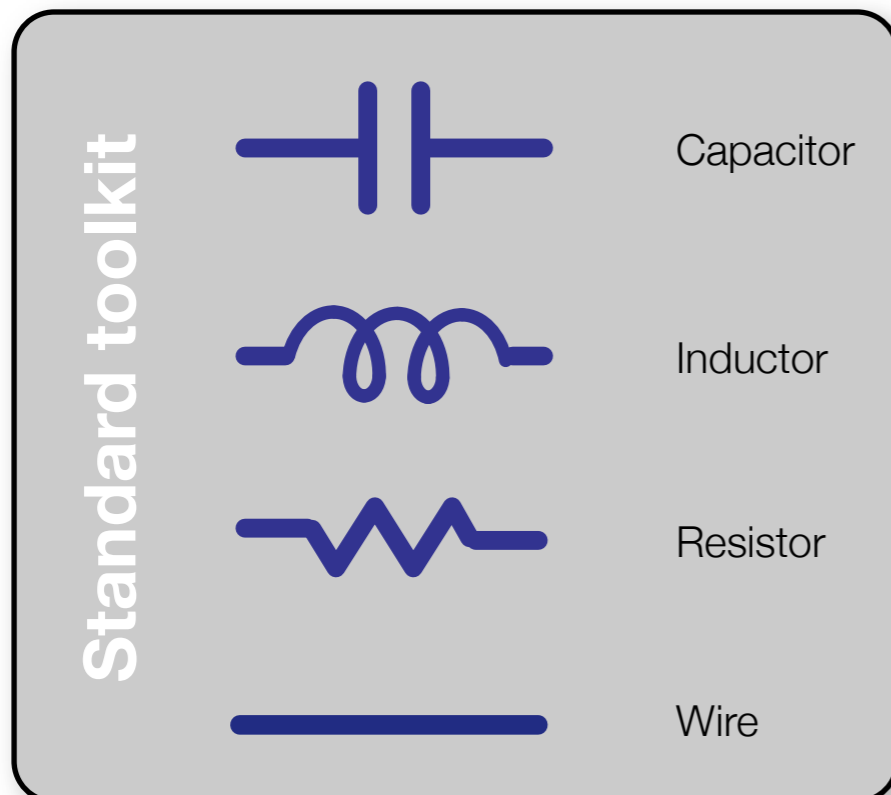
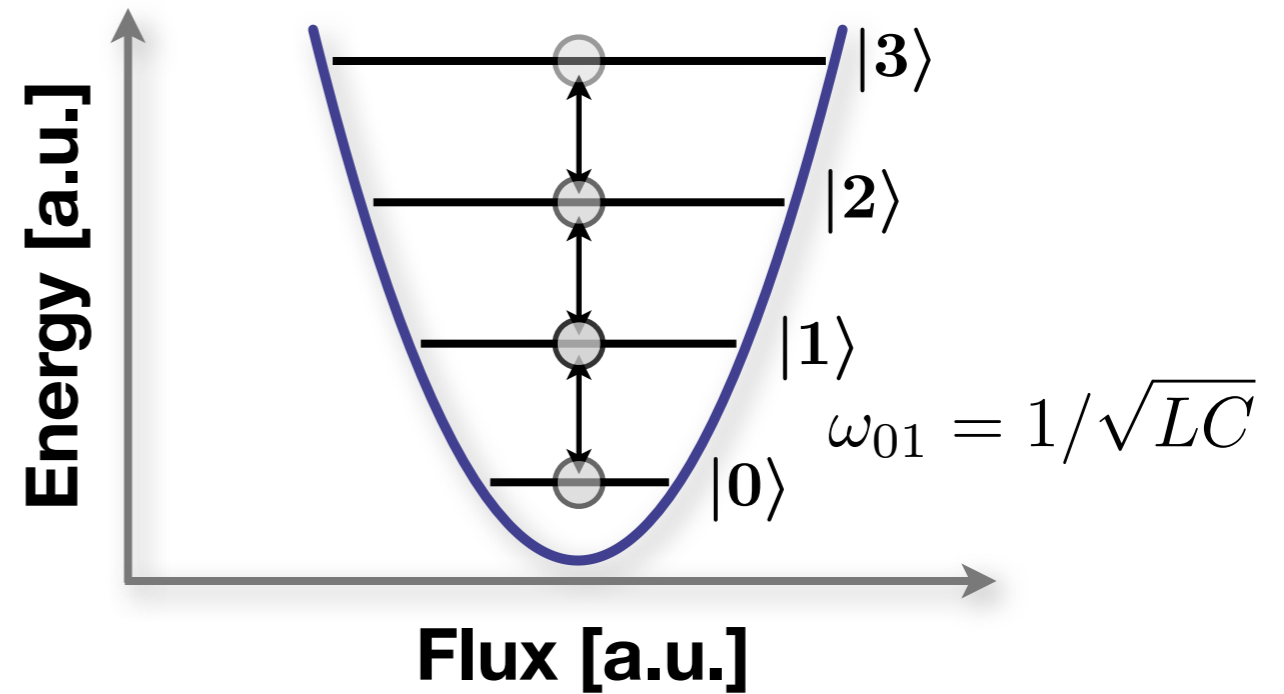
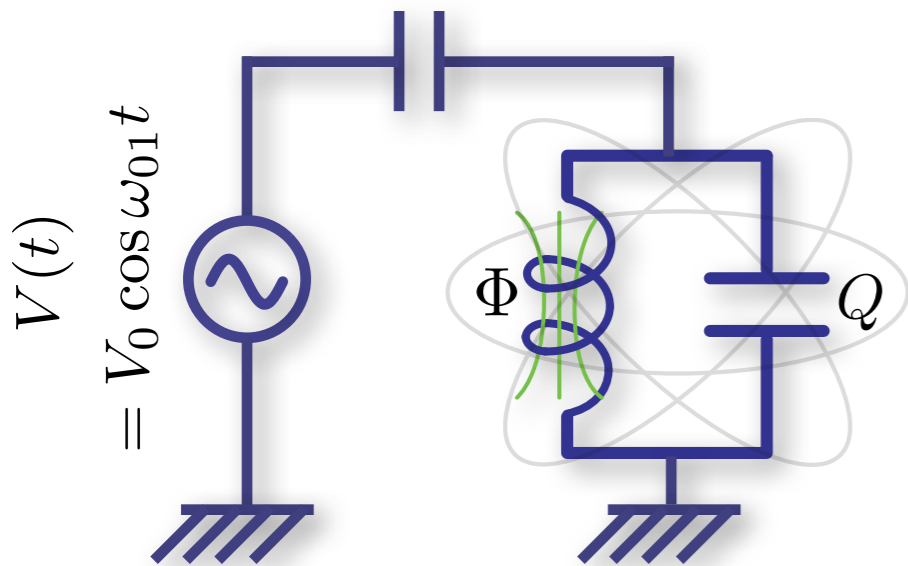
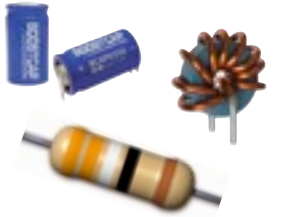
$$T_1 \sim \text{a few years} \quad T_2 \gtrsim 10 \text{ seconds}$$

- Reasonably short  $\pi$ -pulse time

$$T_\pi \sim 5 \mu\text{s}$$

- Low error per gates:  $\sim 0.48\%$

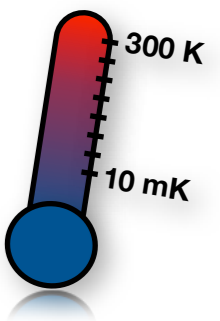
# Artificial atoms: a toolkit



- Initialization to the ground state is simple

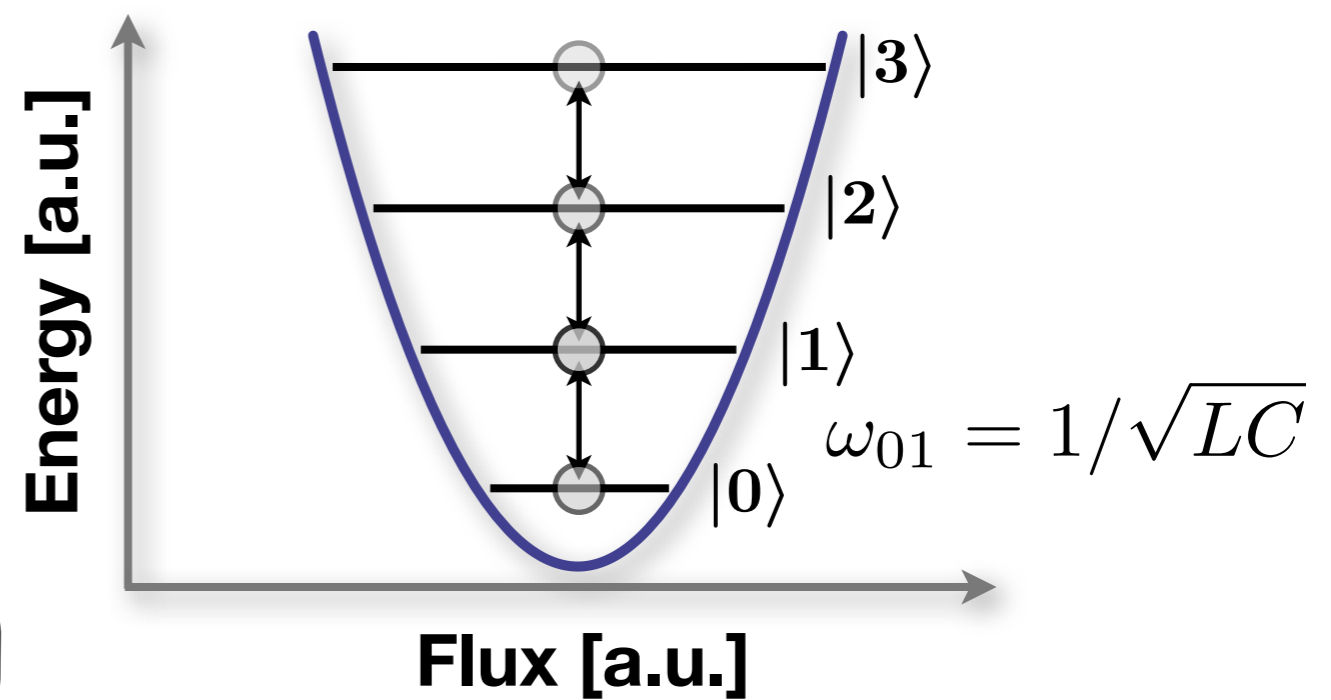
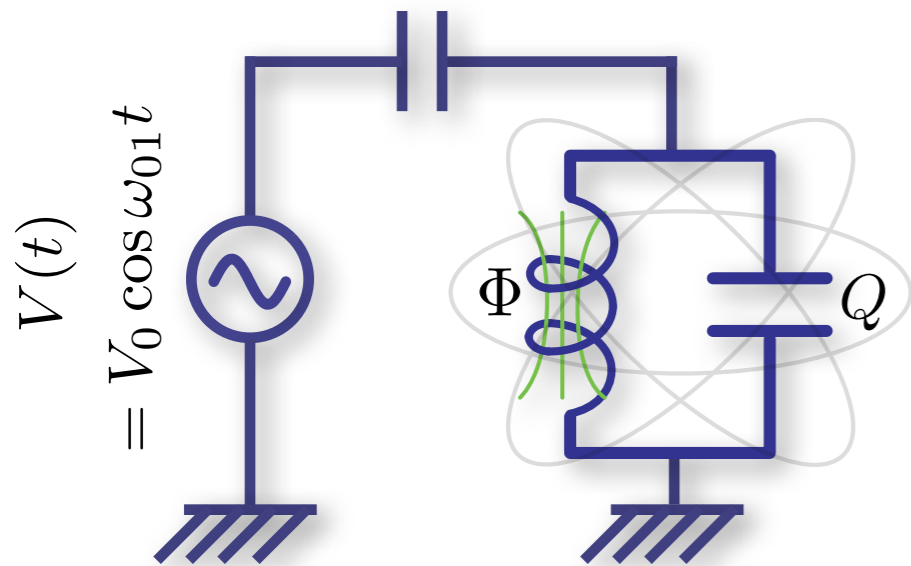
$$\omega_{01} = 1/\sqrt{LC} \sim 10 \text{ GHz}$$

$$\sim 0.5 \text{ K}$$

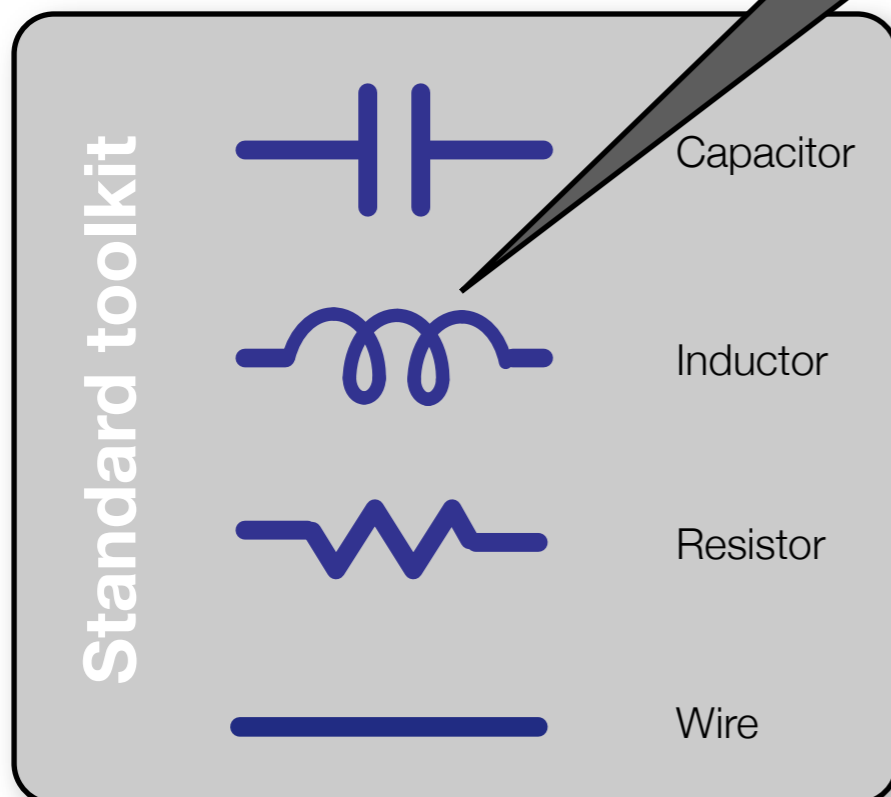


- Not a good «two-level» atom...

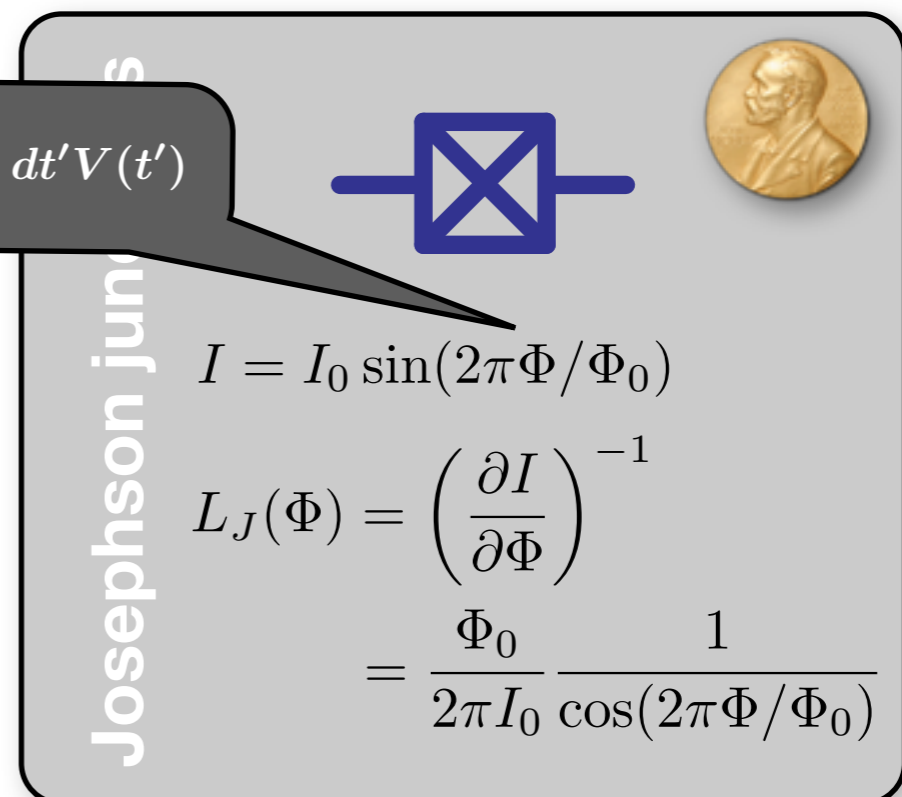
# Artificial atoms: potential shaping



$$I = \Phi/L$$

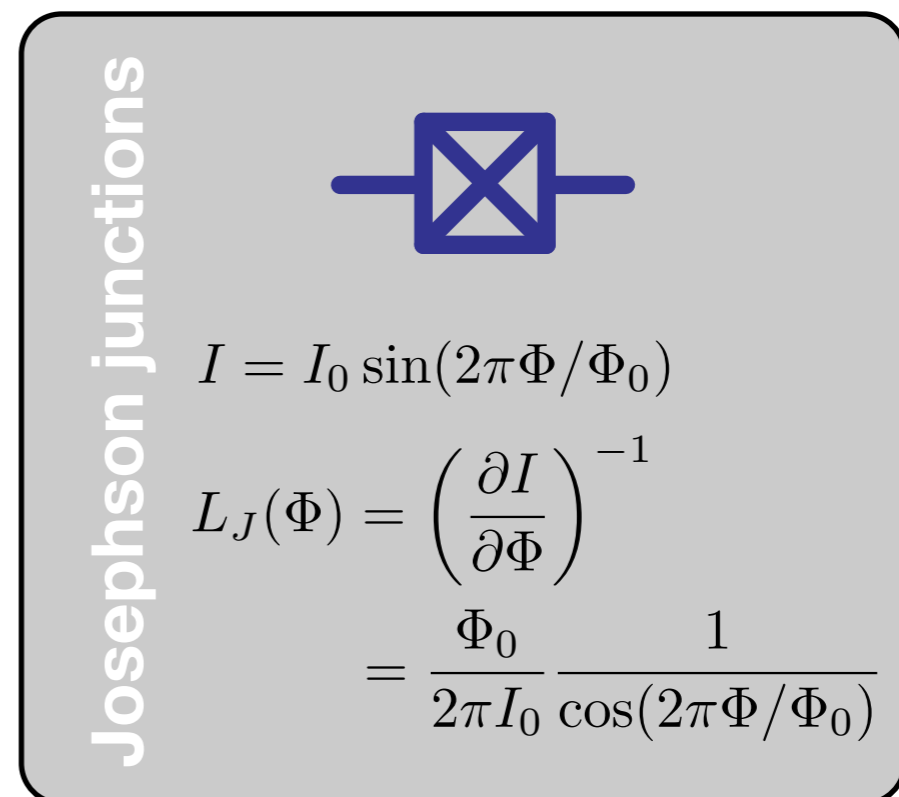
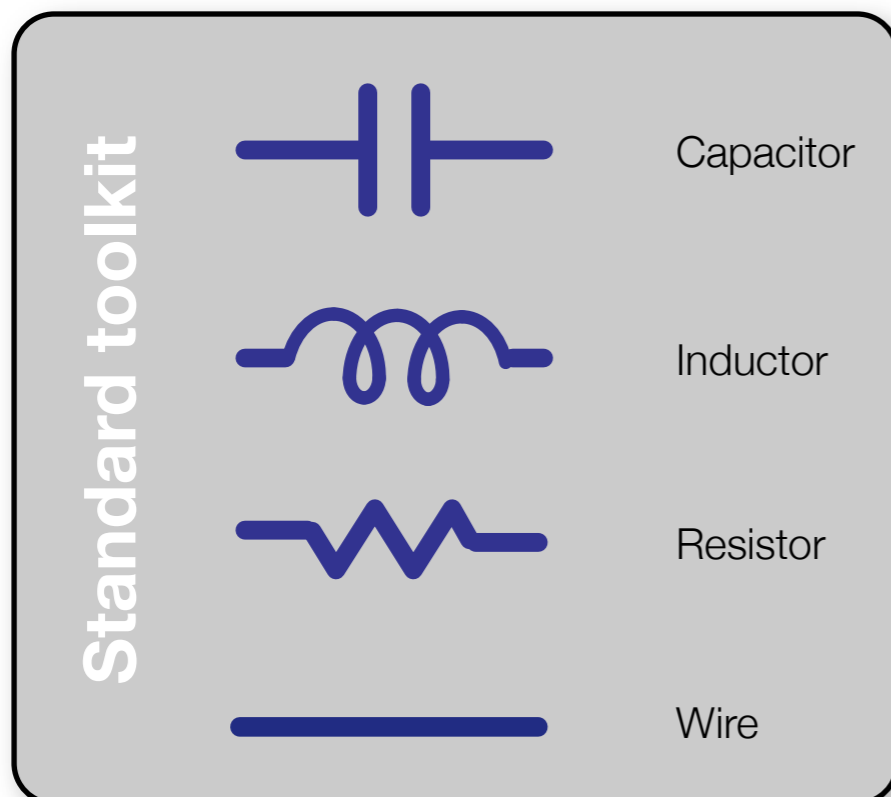
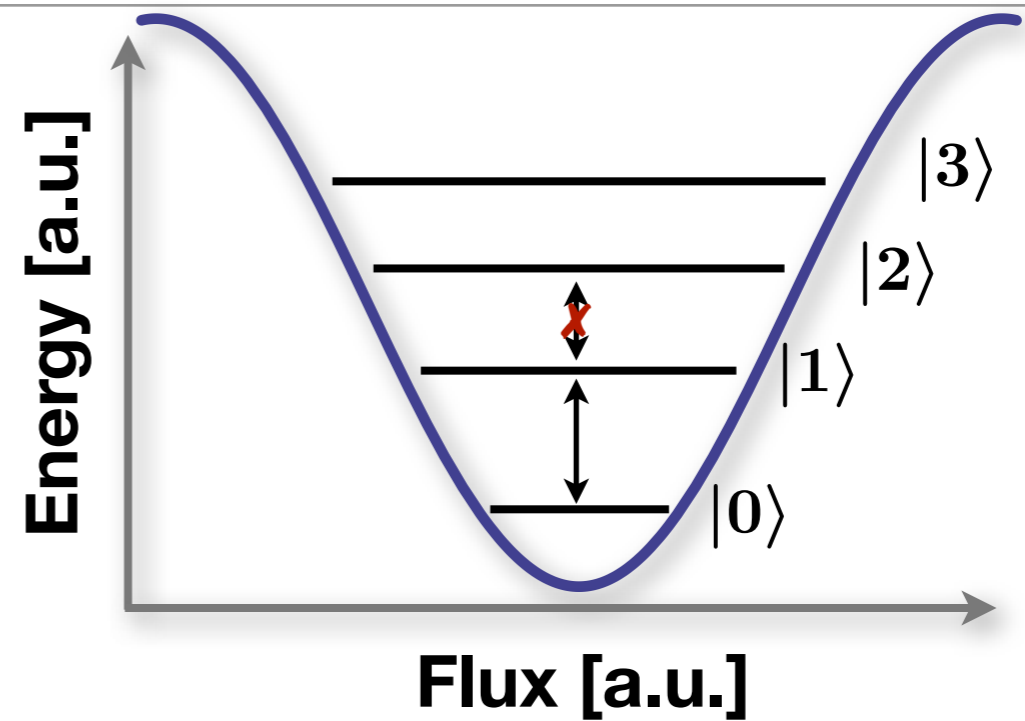
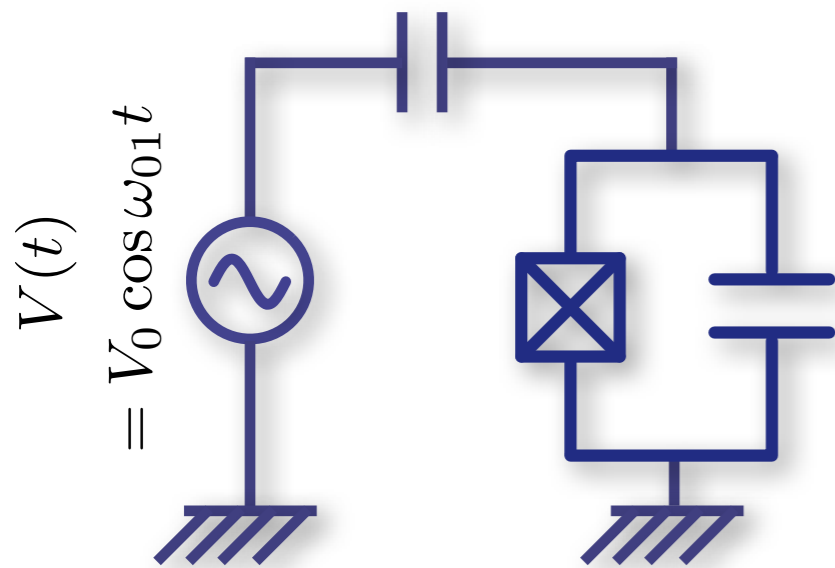


$$\Phi(t) = \int_{-\infty}^t dt' V(t')$$

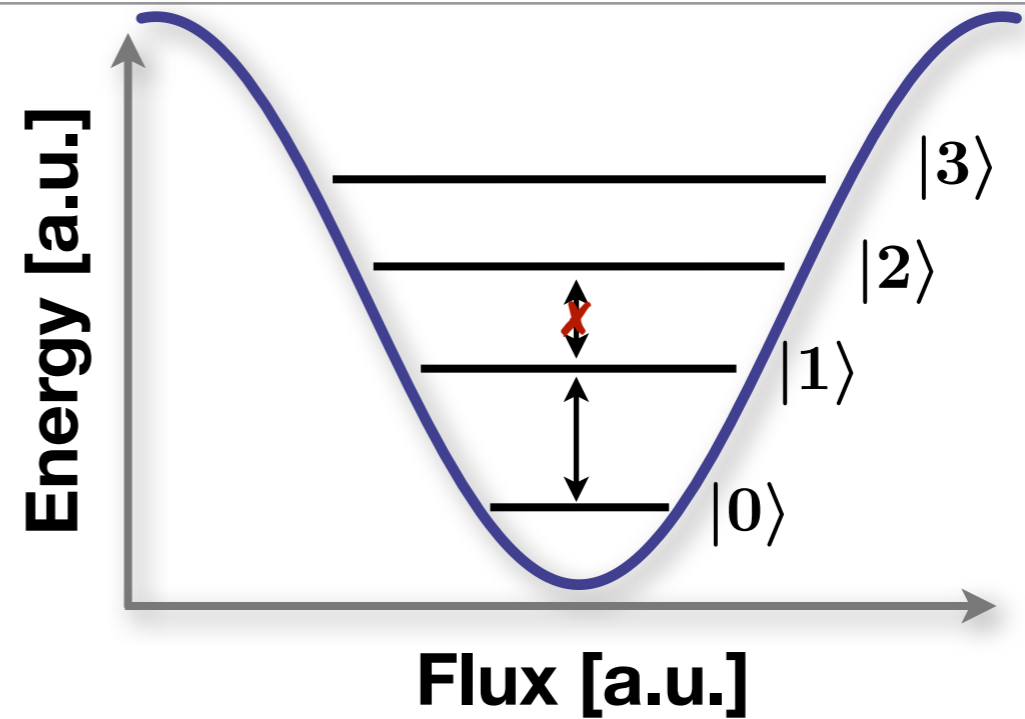
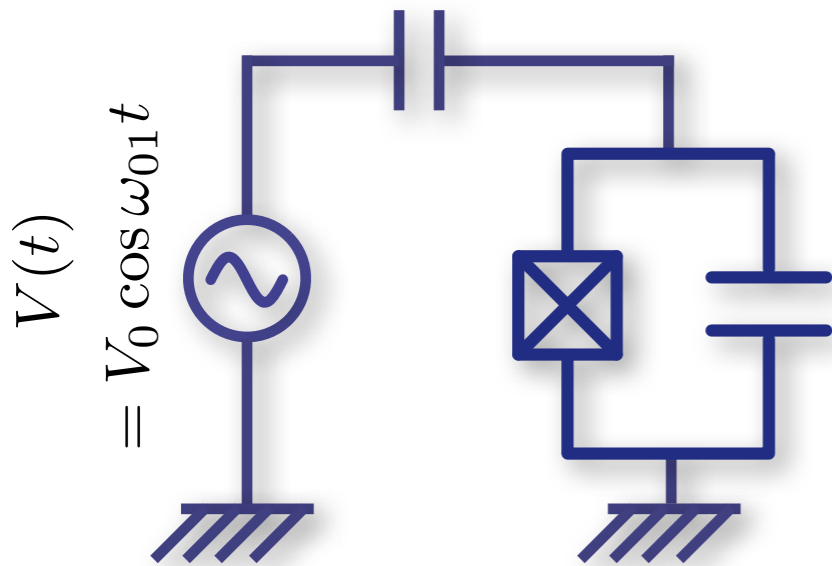




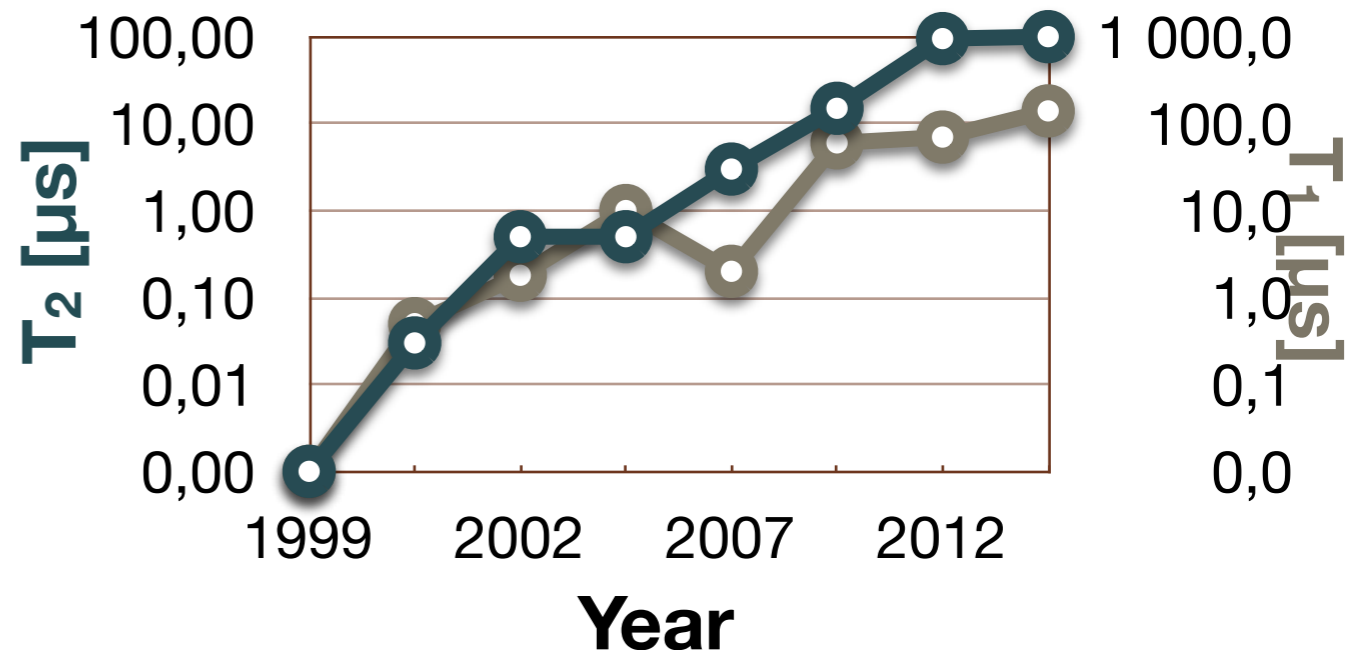
# Artificial atoms: potential shaping



# Artificial atoms: fast and coherent



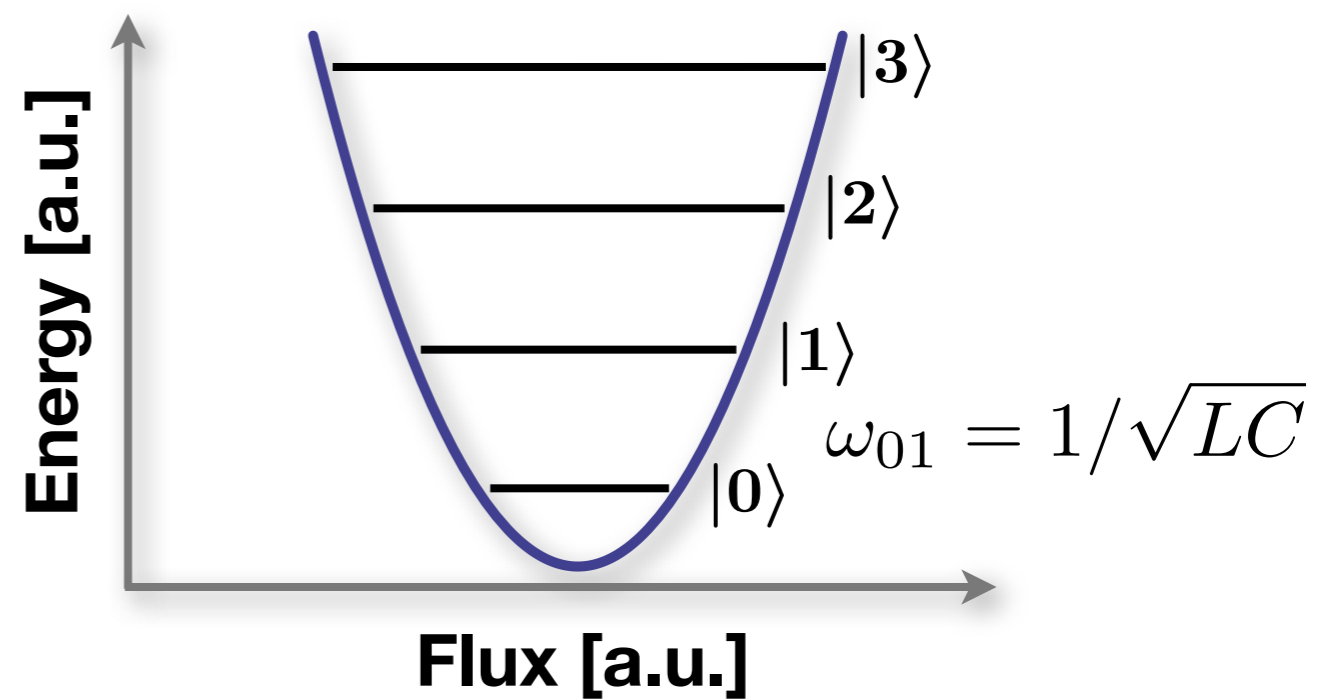
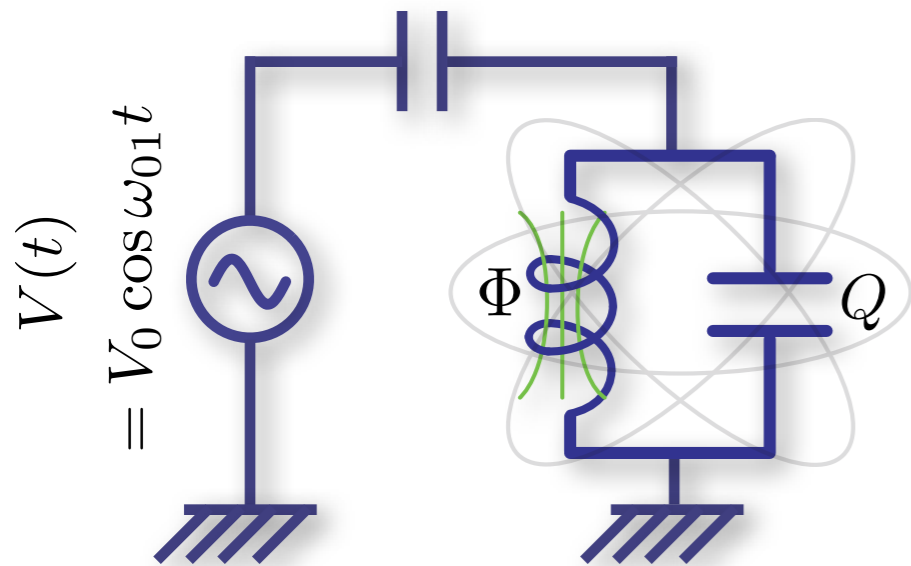
- Very short  $\pi$ -pulse time  
 $T_\pi \sim 4 - 20$  ns
- Big improvements in relaxation and dephasing times in last 10 years
- Error per gates of 0.2%, similar to trapped ion results



Low error per gates: E. Magesan *et al*, Phys. Rev. Lett. **109**, 080505 (2012)

Long  $T_1$  and  $T_2$ : H. Paik *et al*, Phys. Rev. Lett. **107**, 240501 (2011)

# Back to basic: the harmonic oscillator



$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{1}{2} C \omega_{01}^2 \hat{\Phi}^2$$

**Kinetic energy**      **Potential energy**

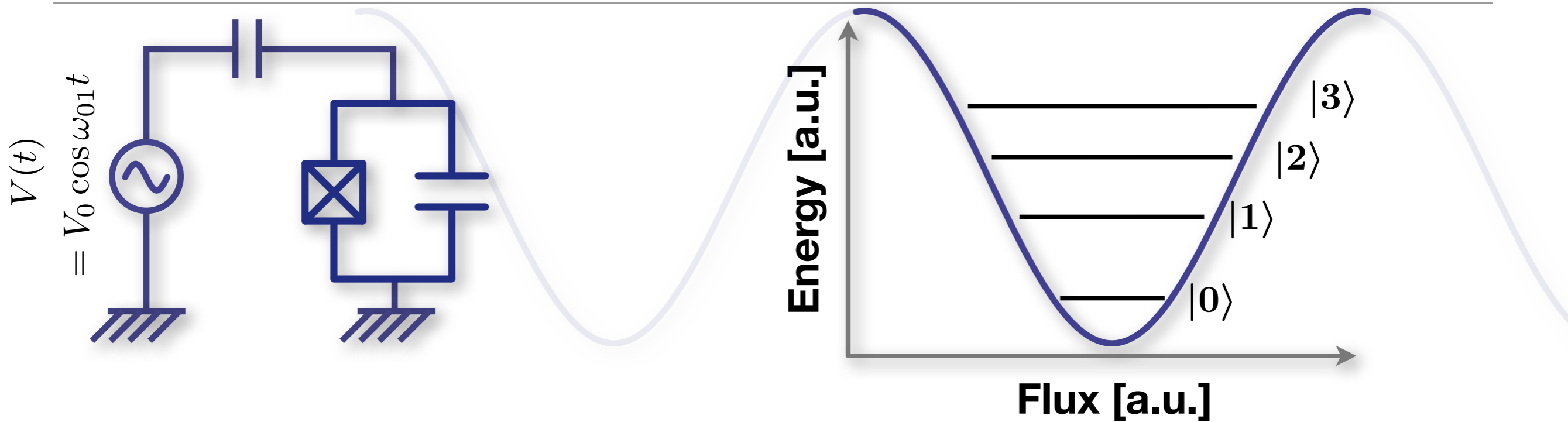
$$\hat{Q} = i \sqrt{\frac{\hbar}{2Z_0}} (\hat{a}^\dagger - \hat{a})$$

$$\hat{\Phi} = \sqrt{\frac{\hbar Z_0}{2}} (\hat{a}^\dagger + \hat{a})$$

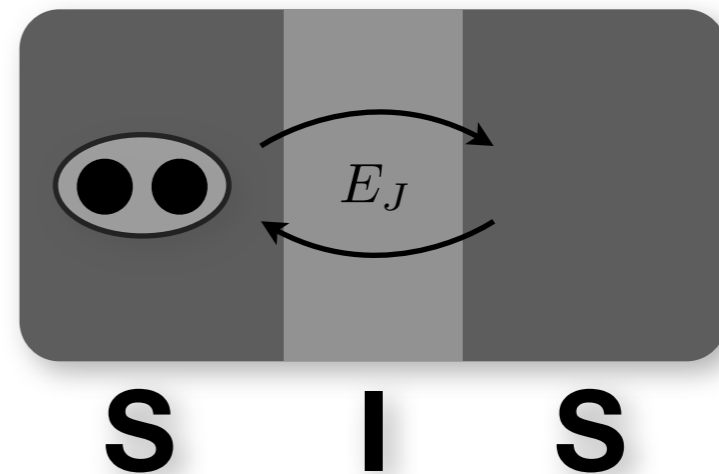
$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\hat{H} = \hbar \omega_{01} (\hat{a}^\dagger \hat{a} + 1/2)$$

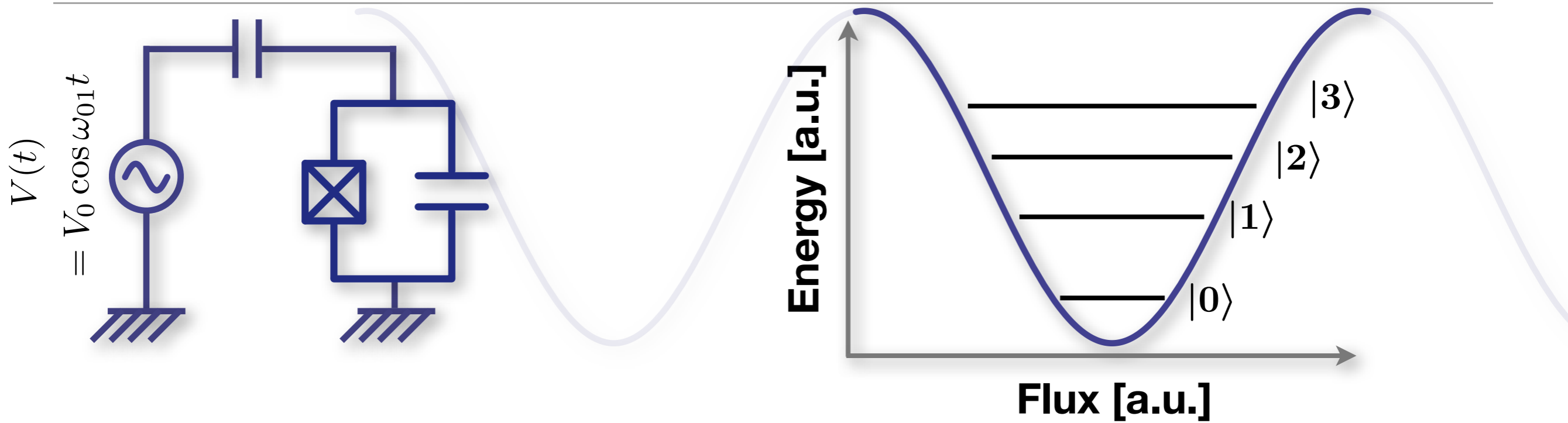
# Josephson energy



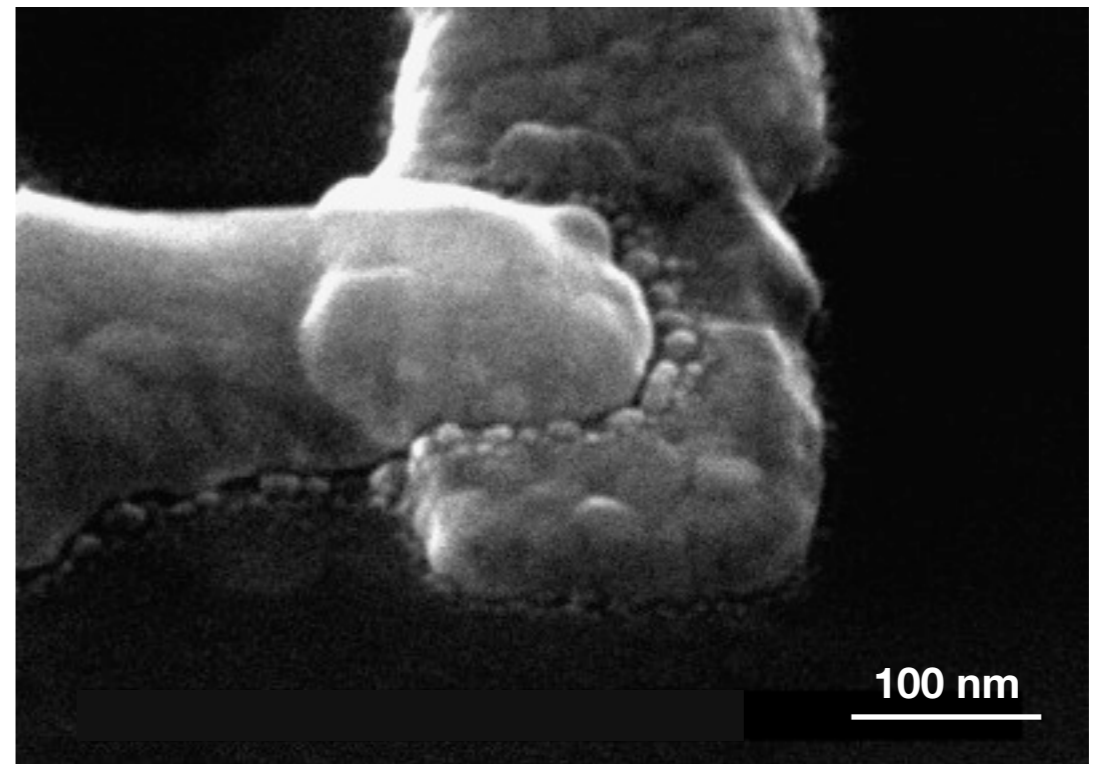
$$\begin{aligned}
 E &= \int dt V(t) I(t) \\
 &= \int dt \left( \frac{d\Phi}{dt} \right) \left( I_0 \sin \frac{2\pi\Phi}{\Phi_0} \right) \\
 &= -E_J \cos \frac{2\pi\Phi}{\Phi_0}
 \end{aligned}$$



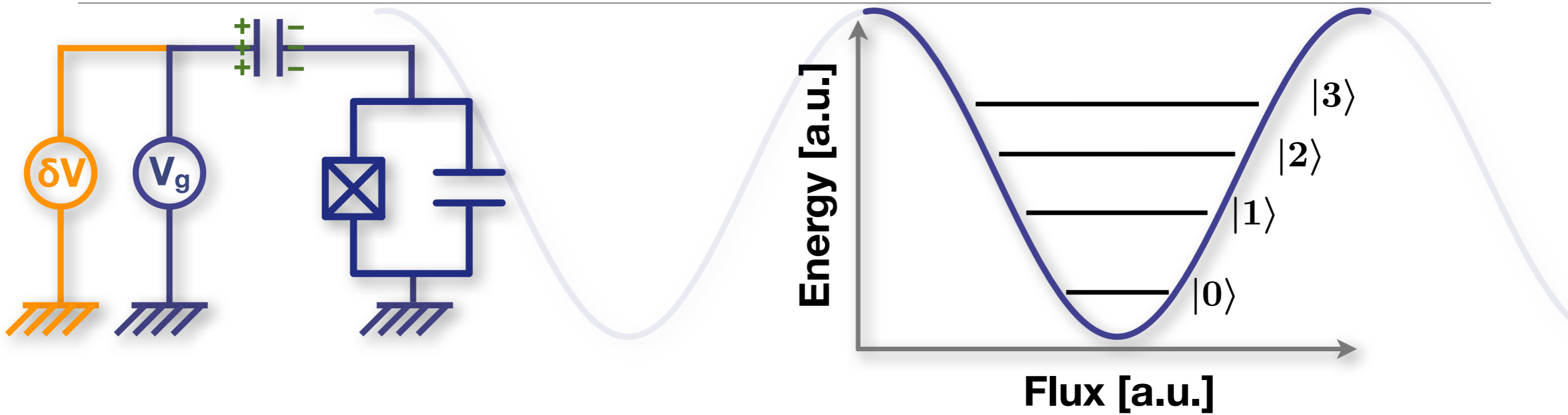
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$$\begin{aligned} E &= \int dt V(t) I(t) \\ &= \int dt \left( \frac{d\Phi}{dt} \right) \left( I_0 \sin \frac{2\pi\Phi}{\Phi_0} \right) \\ &= -E_J \cos \frac{2\pi\Phi}{\Phi_0} \end{aligned}$$



# Hamiltonian of a superconducting qubit



$$\begin{aligned} H &= \frac{\hat{Q}^2}{2C} - E_J \cos \frac{2\pi\hat{\Phi}}{\Phi_0} \\ &= \frac{(2e)^2}{2C} \hat{n}^2 - E_J \cos \hat{\phi} \\ &= 4E_C (\hat{n} - n_g)^2 - E_J \cos \hat{\phi} \end{aligned}$$

**Optimal choice  
of  $E_J/E_C$ ?**

# Interlude: eigenvalues and eigenstates

Find  $E_k$  and  $|\psi_k\rangle$  satisfying  $\hat{H}|\psi_k\rangle = E_k|\psi_k\rangle$

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos \hat{\phi}$$

In the phase basis,  $\hat{\phi}|\phi\rangle = \phi|\phi\rangle$ , need to solve:

$$\langle\phi|\hat{H}|\psi_k\rangle = E_k\langle\phi|\psi_k\rangle = E_k\psi_k(\phi)$$

$$\begin{aligned} \hat{n} &\rightarrow -i\frac{\partial}{\partial\phi} \\ \left(\hat{P} \rightarrow -i\hbar\frac{\partial}{\partial x}\right) &\Rightarrow \left[4E_C\left(-i\frac{\partial}{\partial\phi} - n_g\right)^2 - E_J \cos\phi\right]\psi_k(\phi) = E_k\psi_k(\phi) \end{aligned}$$

$$\text{DSolve}\left[4 E_C \left(-\psi_k'[\phi] + i n_g \psi_k[\phi] + n_g^2 \psi_k[\phi]\right) - E_J \text{Cos}[\phi] \psi_k[\phi] == E_k \psi_k[\phi], \psi_k[\phi], \phi\right]$$

$$\psi_k[\phi] = e^{\frac{1}{2} i \phi n_g} \left( C[1] \text{MathieuC}\left[\frac{E_k}{E_C} - 3 n_g^2, -\frac{E_J}{2 E_C}, \frac{\phi}{2}\right] + C[2] \text{MathieuS}\left[\frac{E_k}{E_C} - 3 n_g^2, -\frac{E_J}{2 E_C}, \frac{\phi}{2}\right] \right)$$

# Interlude: eigenvalues and eigenstates

Find  $E_k$  and  $|\psi_k\rangle$  satisfying  $\hat{H}|\psi_k\rangle = E_k|\psi_k\rangle$

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos \hat{\phi}$$

In the charge basis,  $\hat{n}|n\rangle = n|n\rangle$ ,  $H$  takes the form:

$$H = \sum_{n=-\infty}^{\infty} \left[ 4E_C(n - n_g)^2 |n\rangle\langle n| - \frac{E_J}{2} (|n\rangle\langle n+1| + |n+1\rangle\langle n|) \right]$$

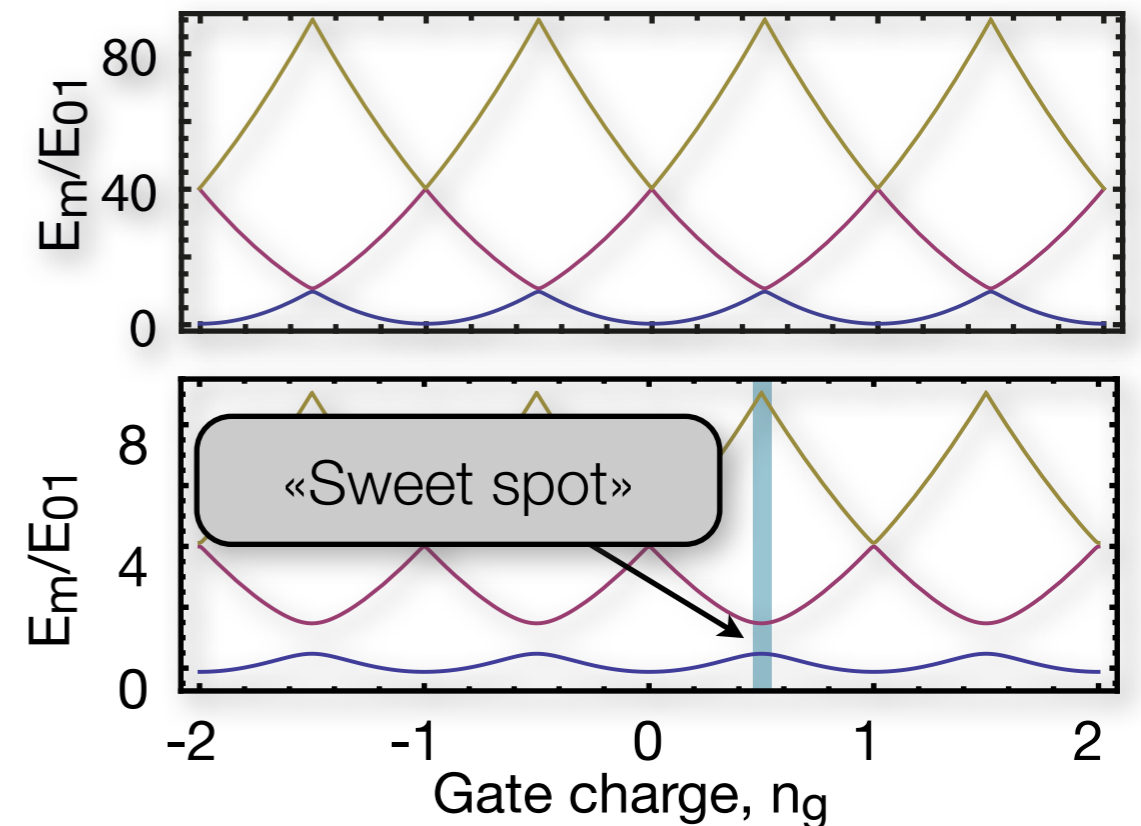
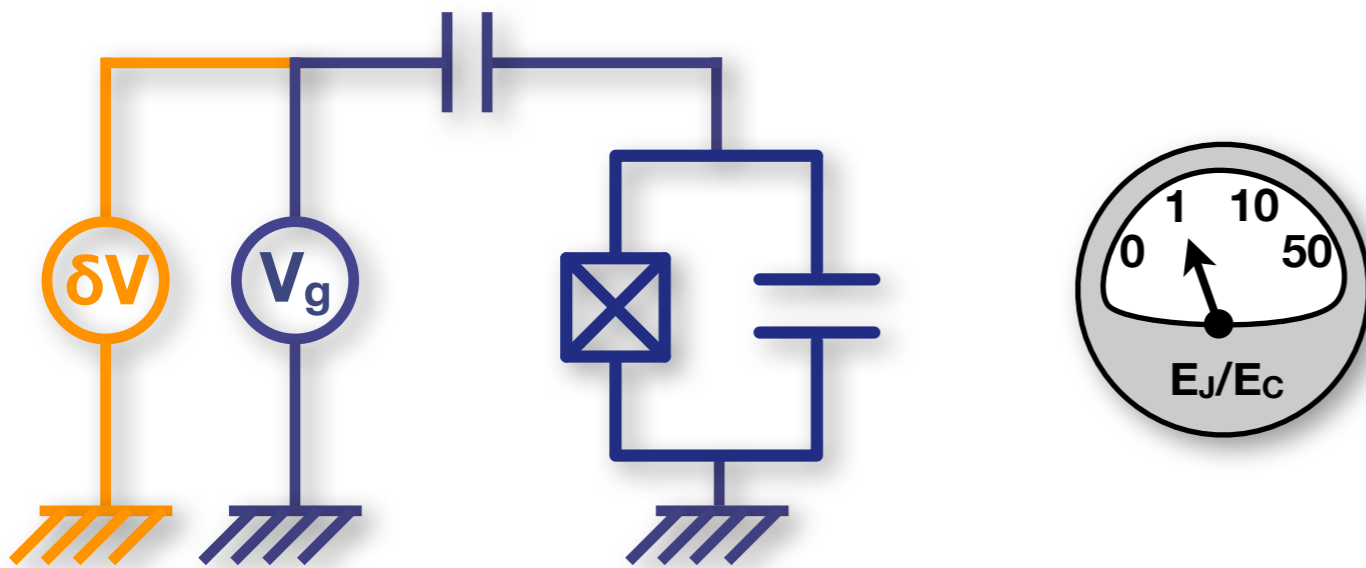
Truncating the Hilbert space:

$$\mathbf{H} = \begin{pmatrix} 4 E_C (-2 - n_g)^2 & -\frac{E_J}{2} & 0 & 0 & 0 \\ -\frac{E_J}{2} & 4 E_C (-1 - n_g)^2 & -\frac{E_J}{2} & 0 & 0 \\ 0 & -\frac{E_J}{2} & 4 E_C n_g^2 & -\frac{E_J}{2} & 0 \\ 0 & 0 & -\frac{E_J}{2} & 4 E_C (1 - n_g)^2 & -\frac{E_J}{2} \\ 0 & 0 & 0 & -\frac{E_J}{2} & 4 E_C (2 - n_g)^2 \end{pmatrix}$$

`{evalues, evector} = Eigensystem[H];`

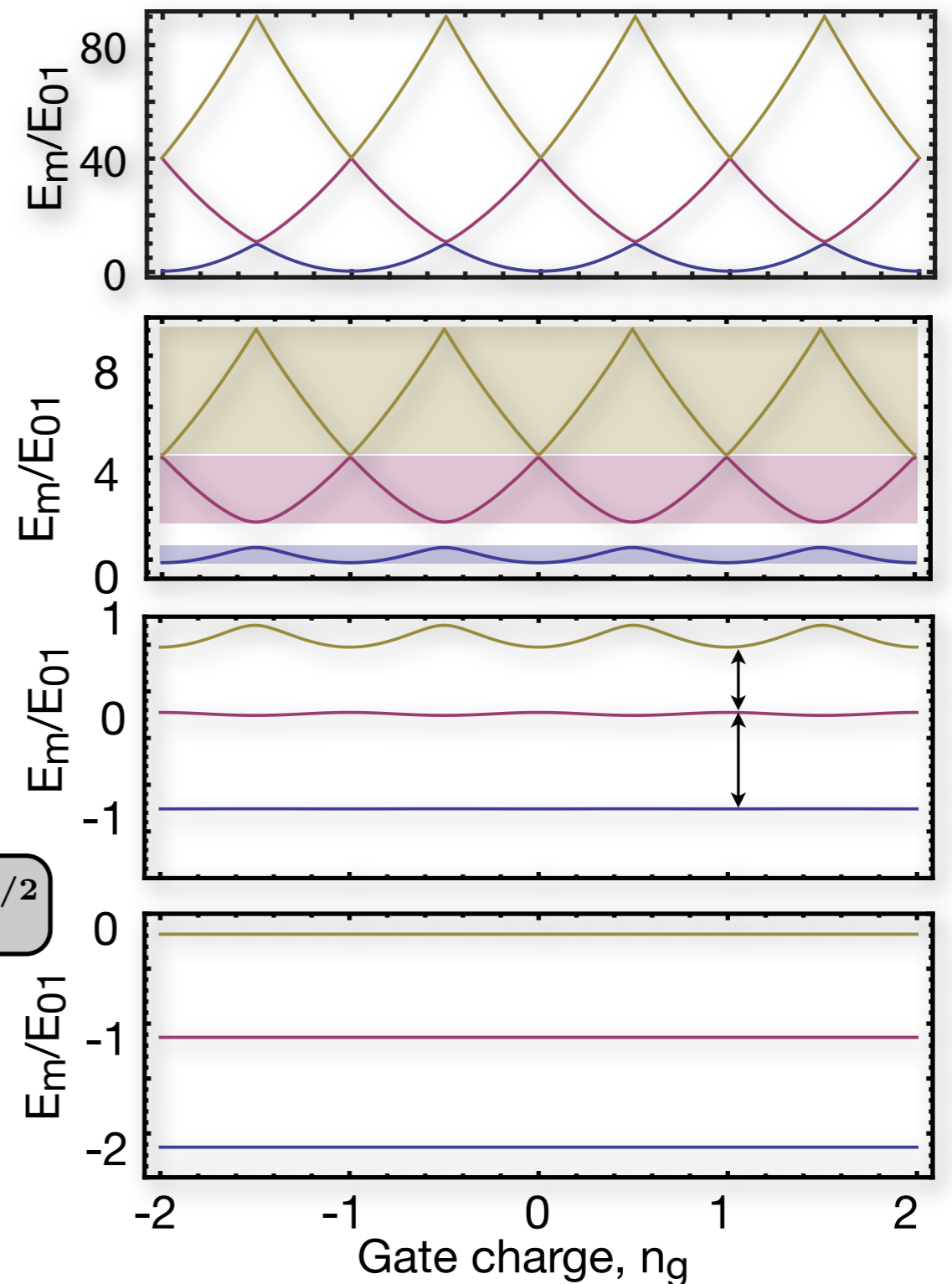
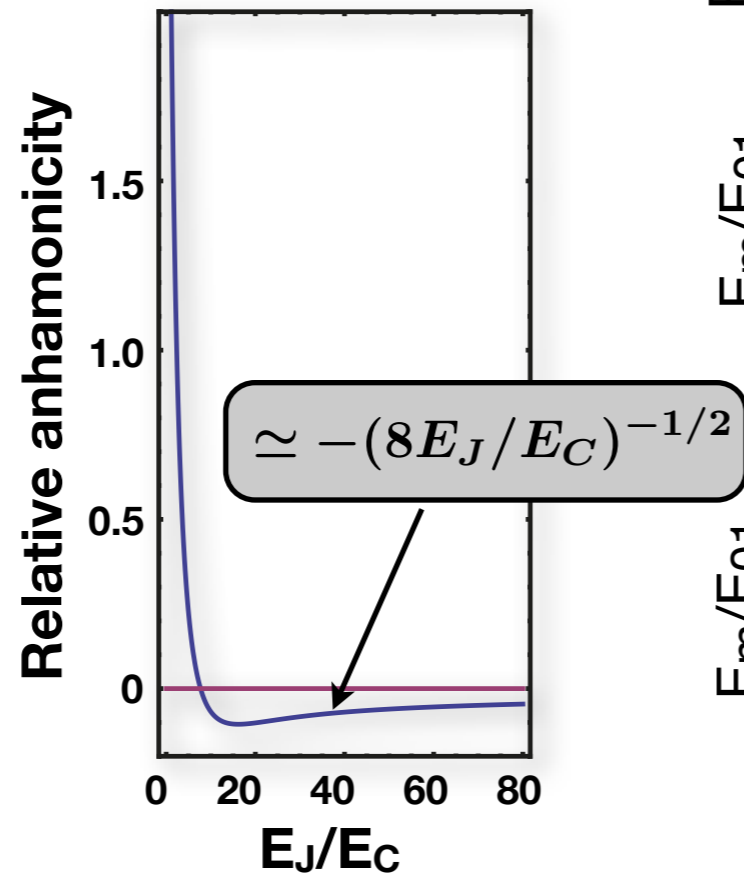
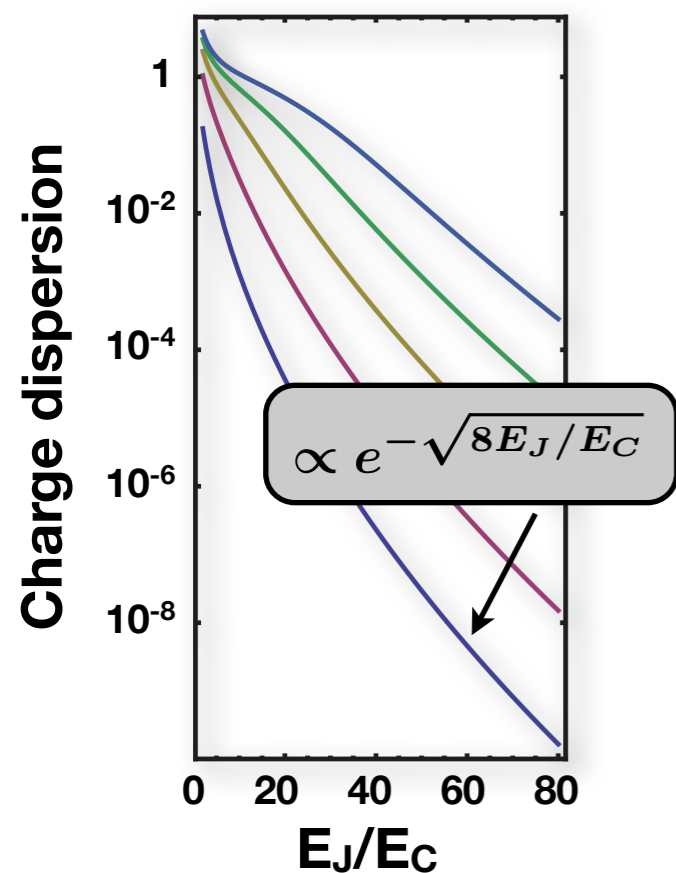
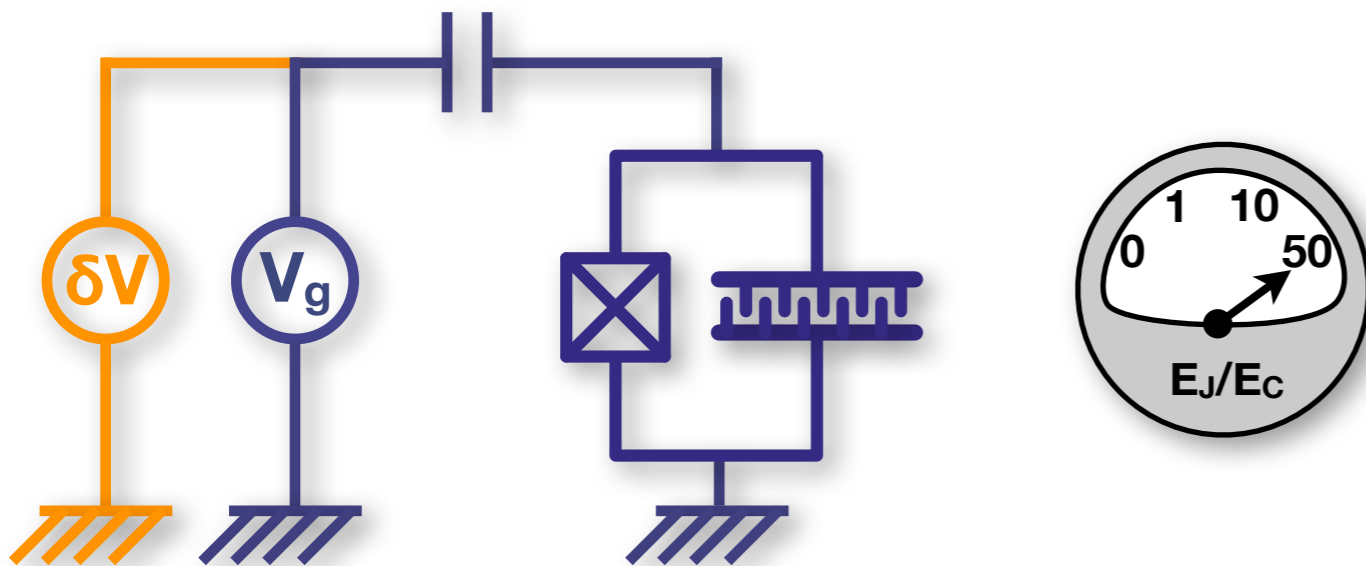


# Superconducting qubits: transmon regime

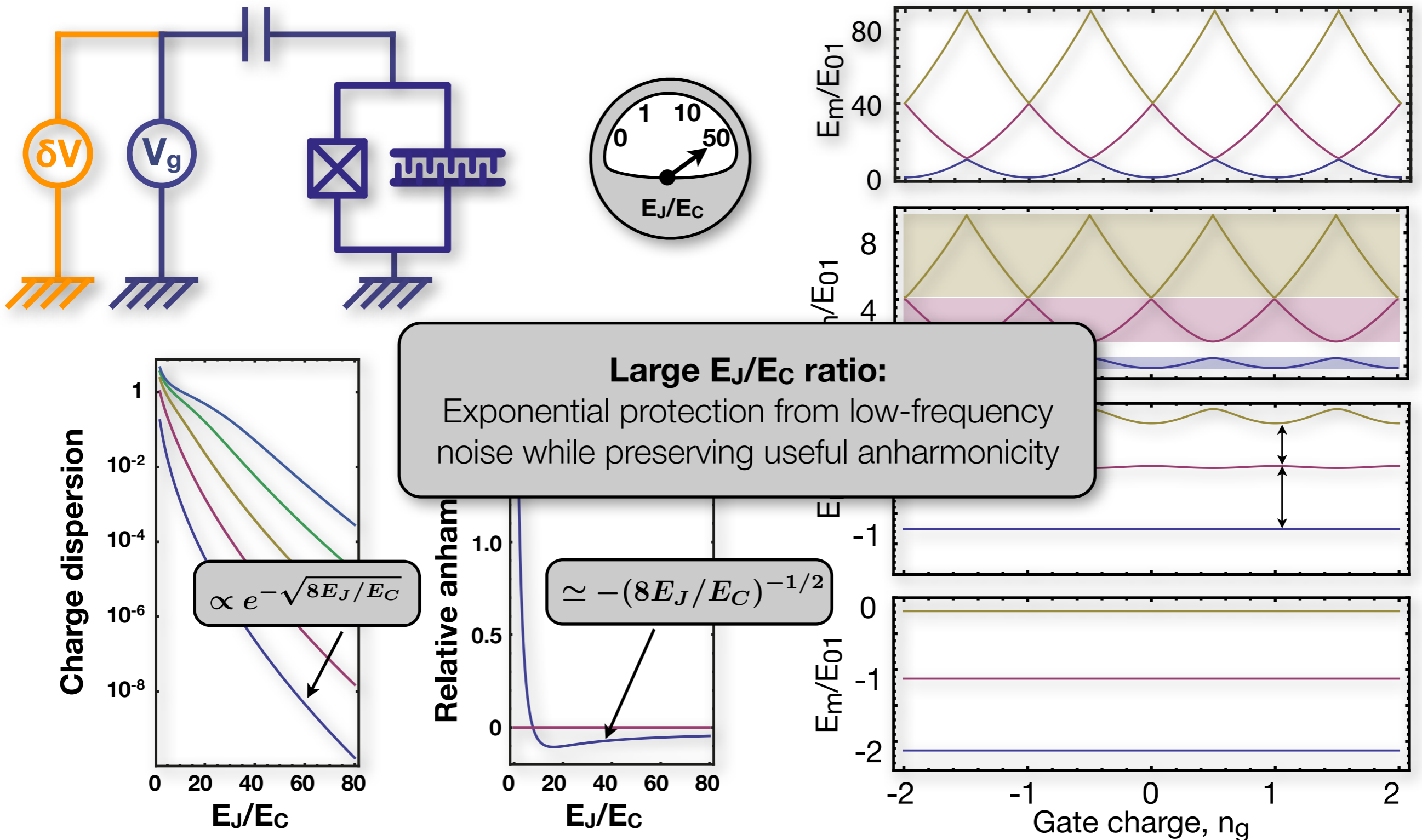


Small variations in  $n_g$  yields large variation in energy:  
dephasing due to charge noise

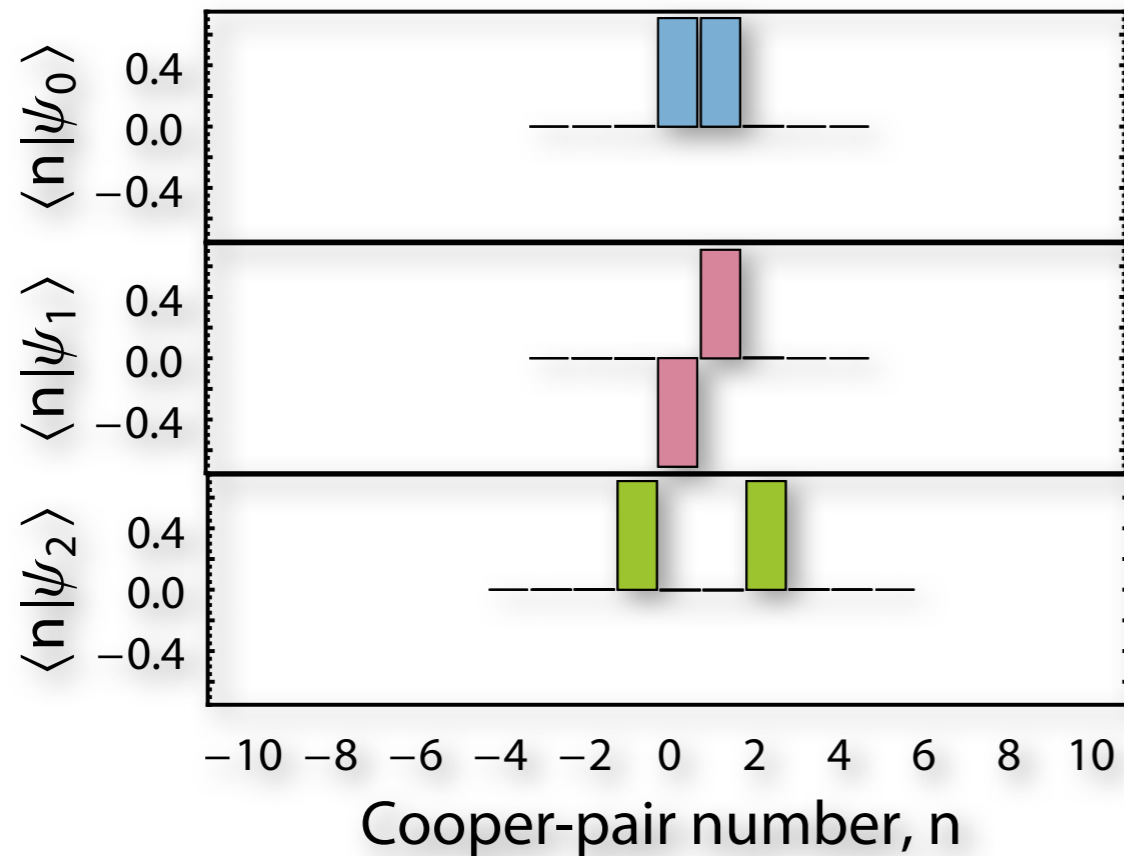
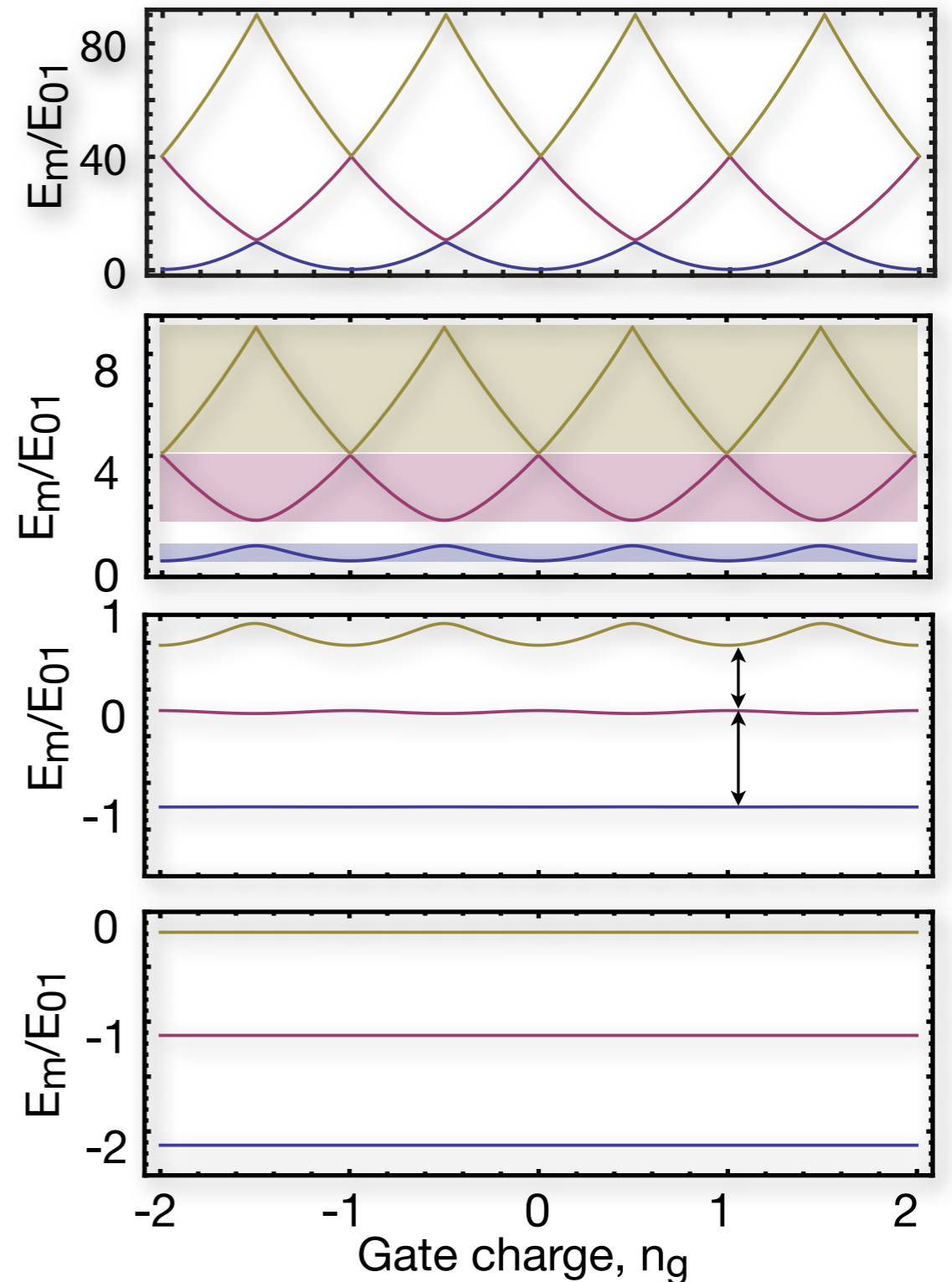
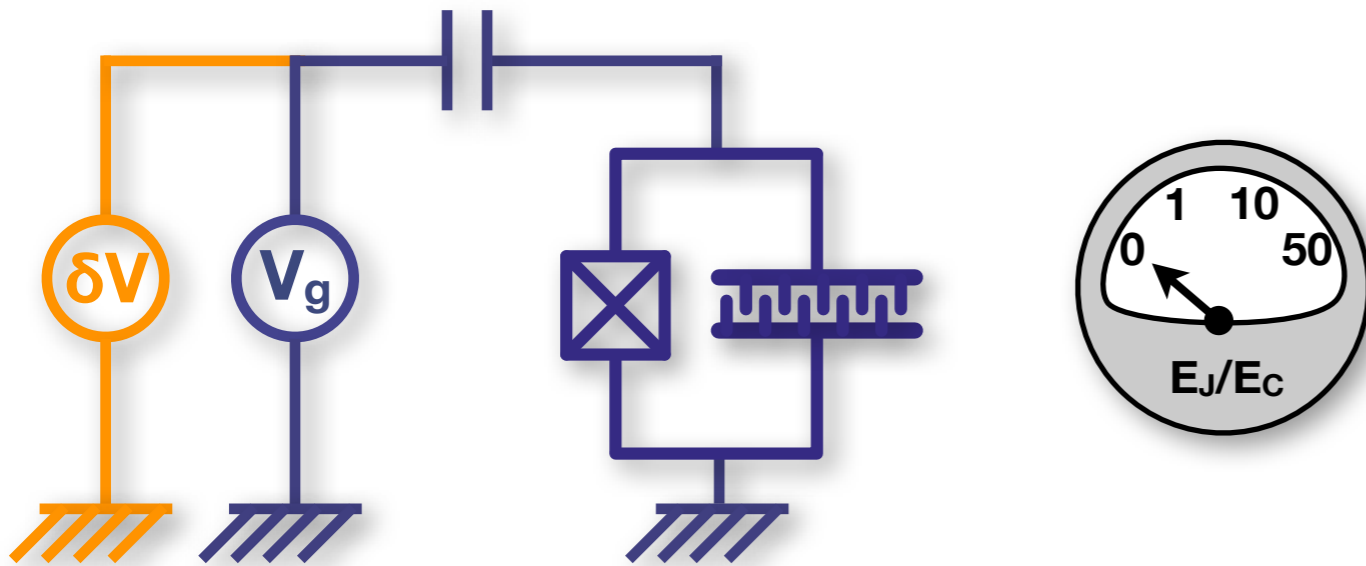
# Superconducting qubits: transmon regime



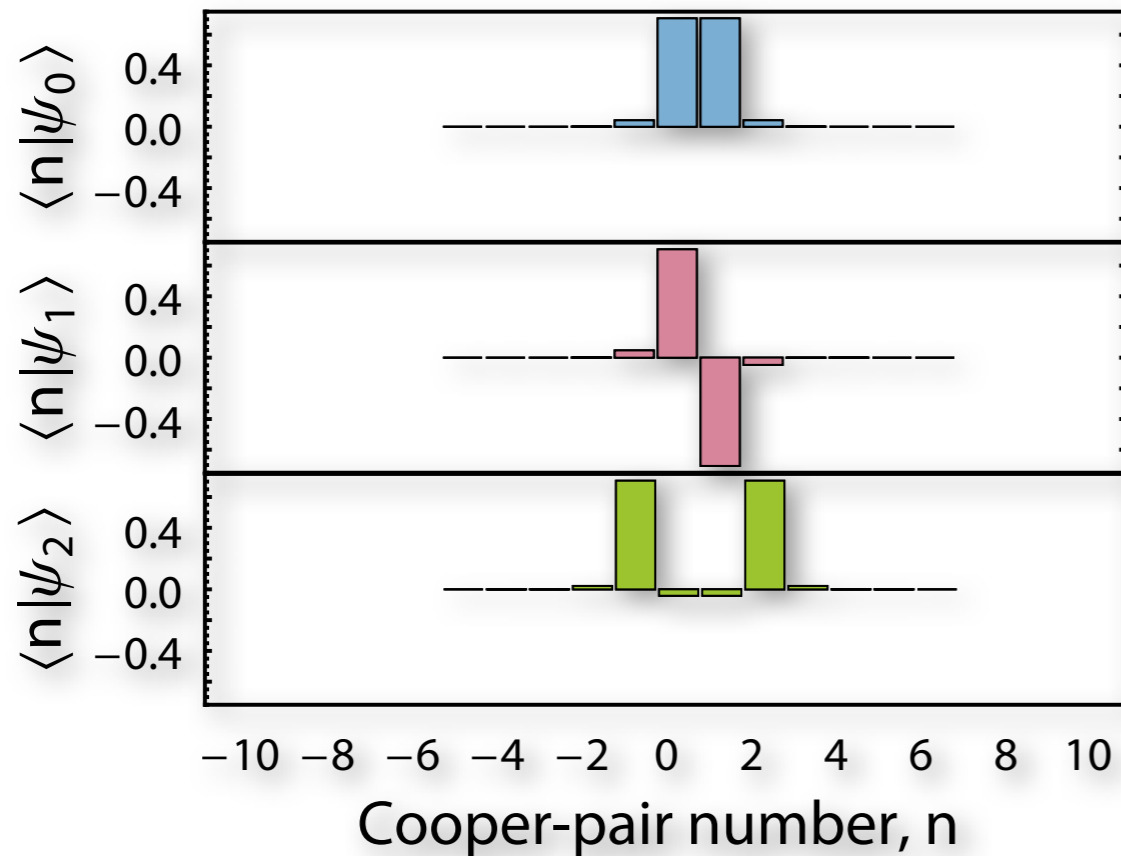
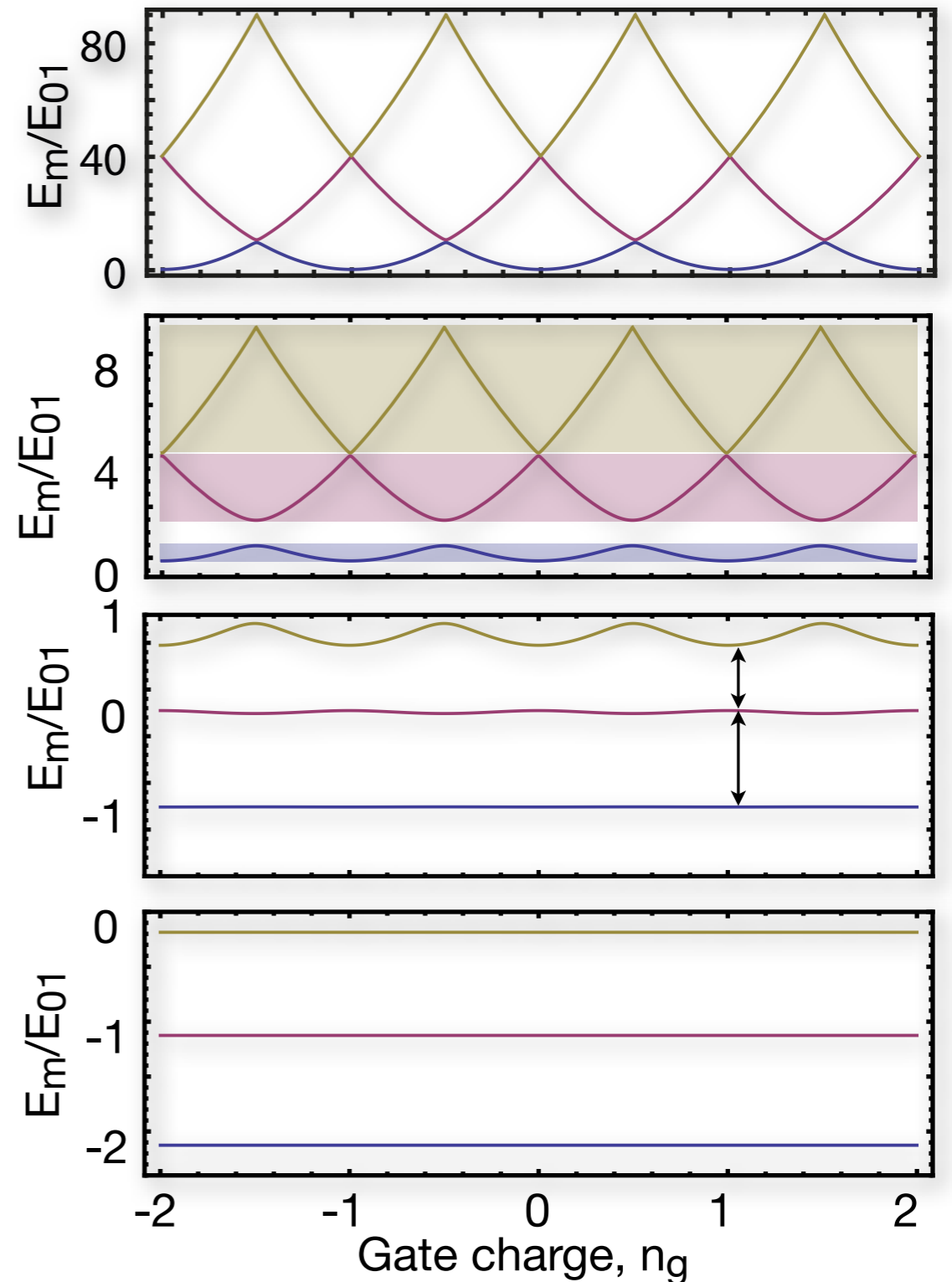
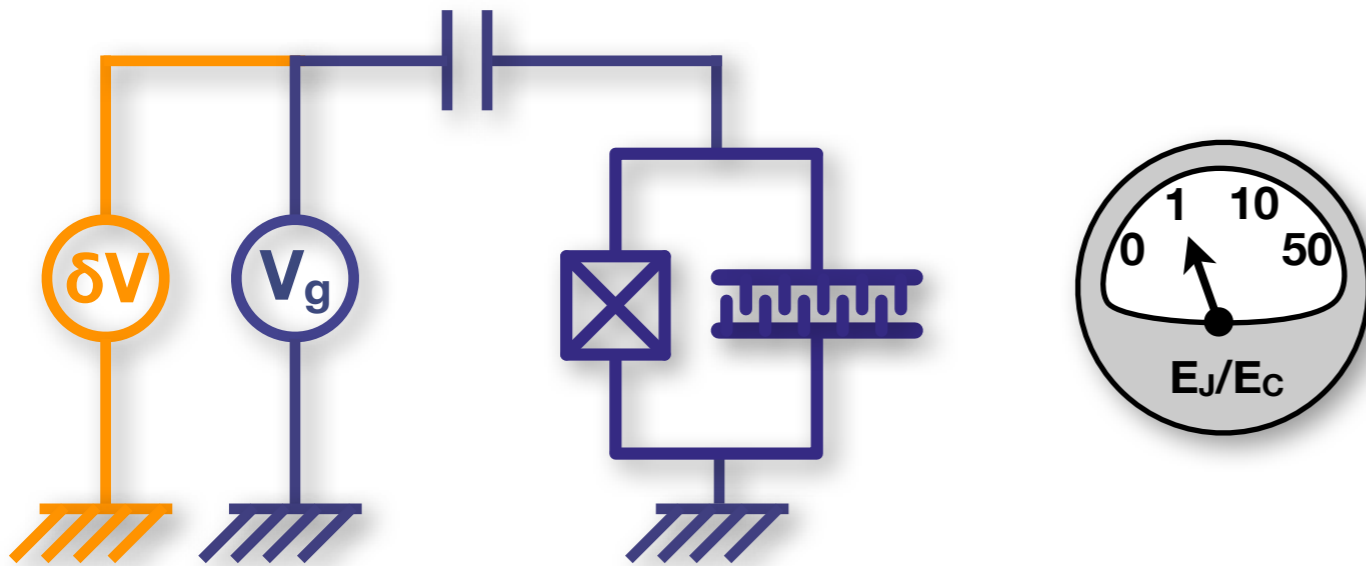
# Superconducting qubits: transmon regime



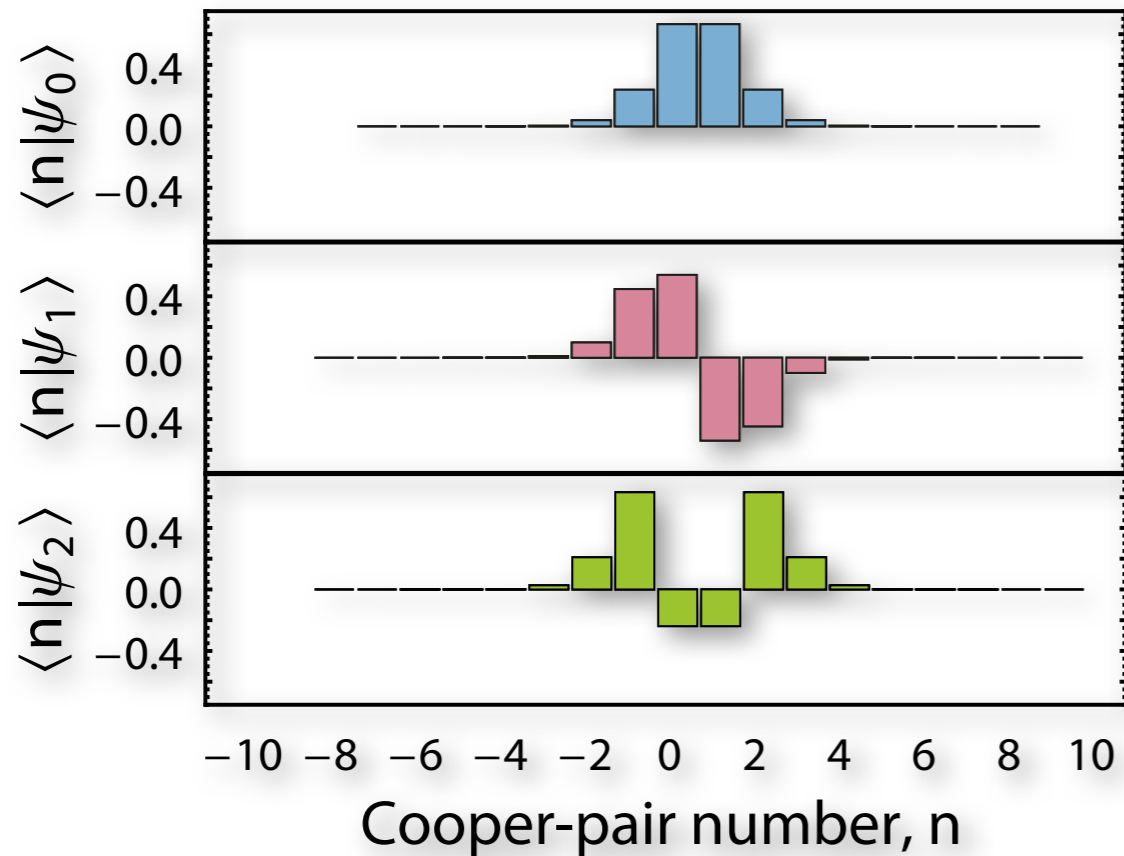
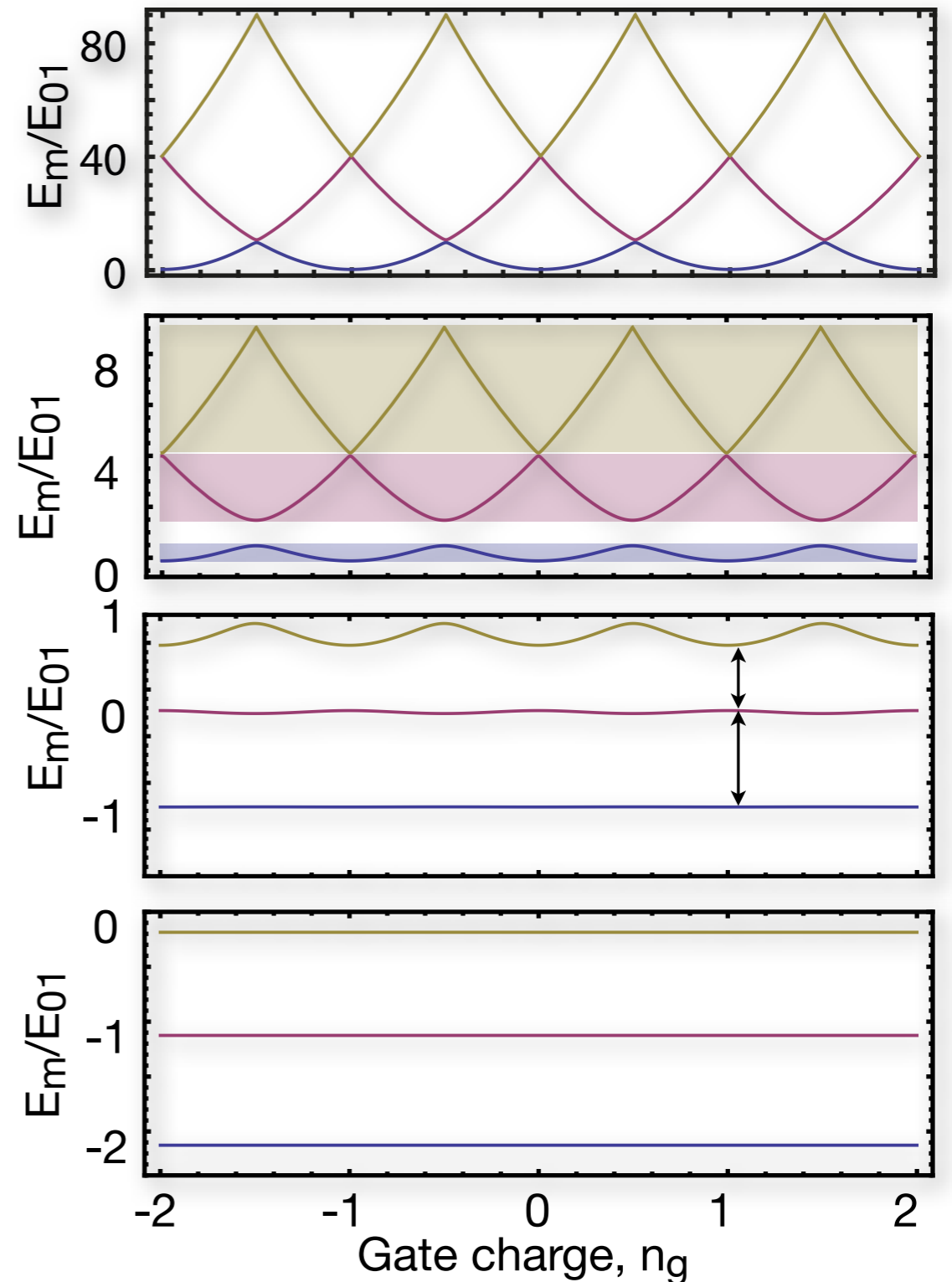
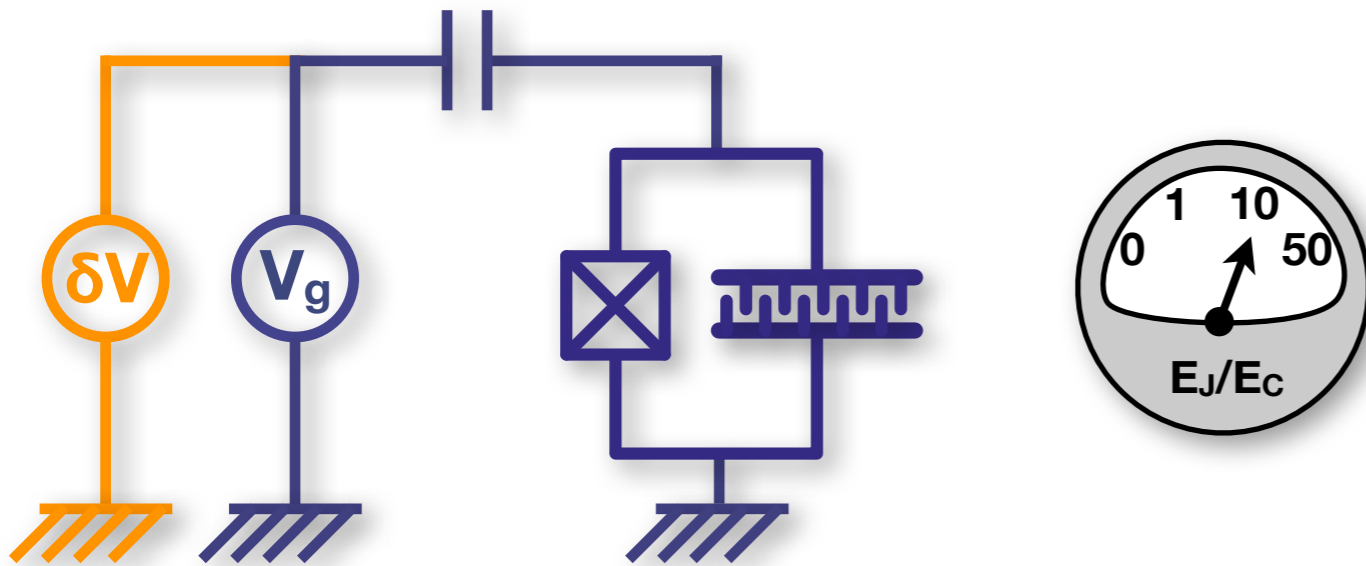
# Superconducting qubits: transmon regime



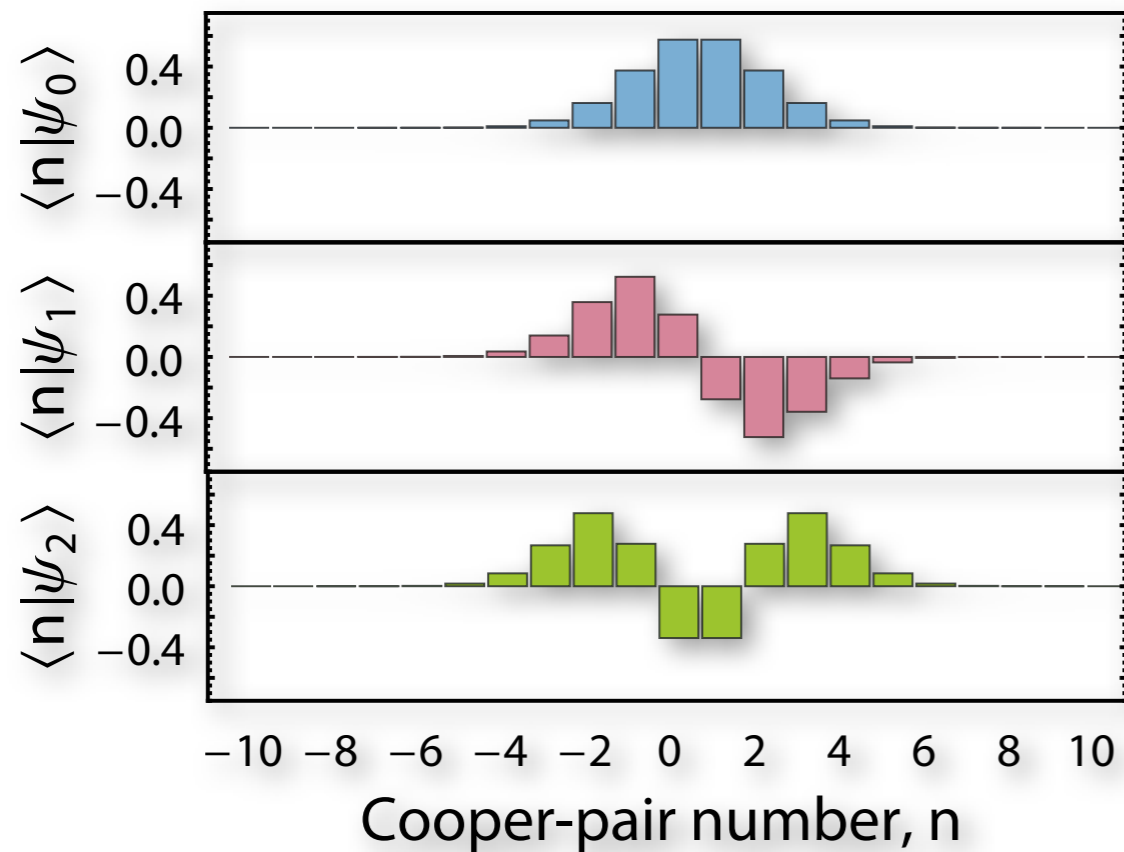
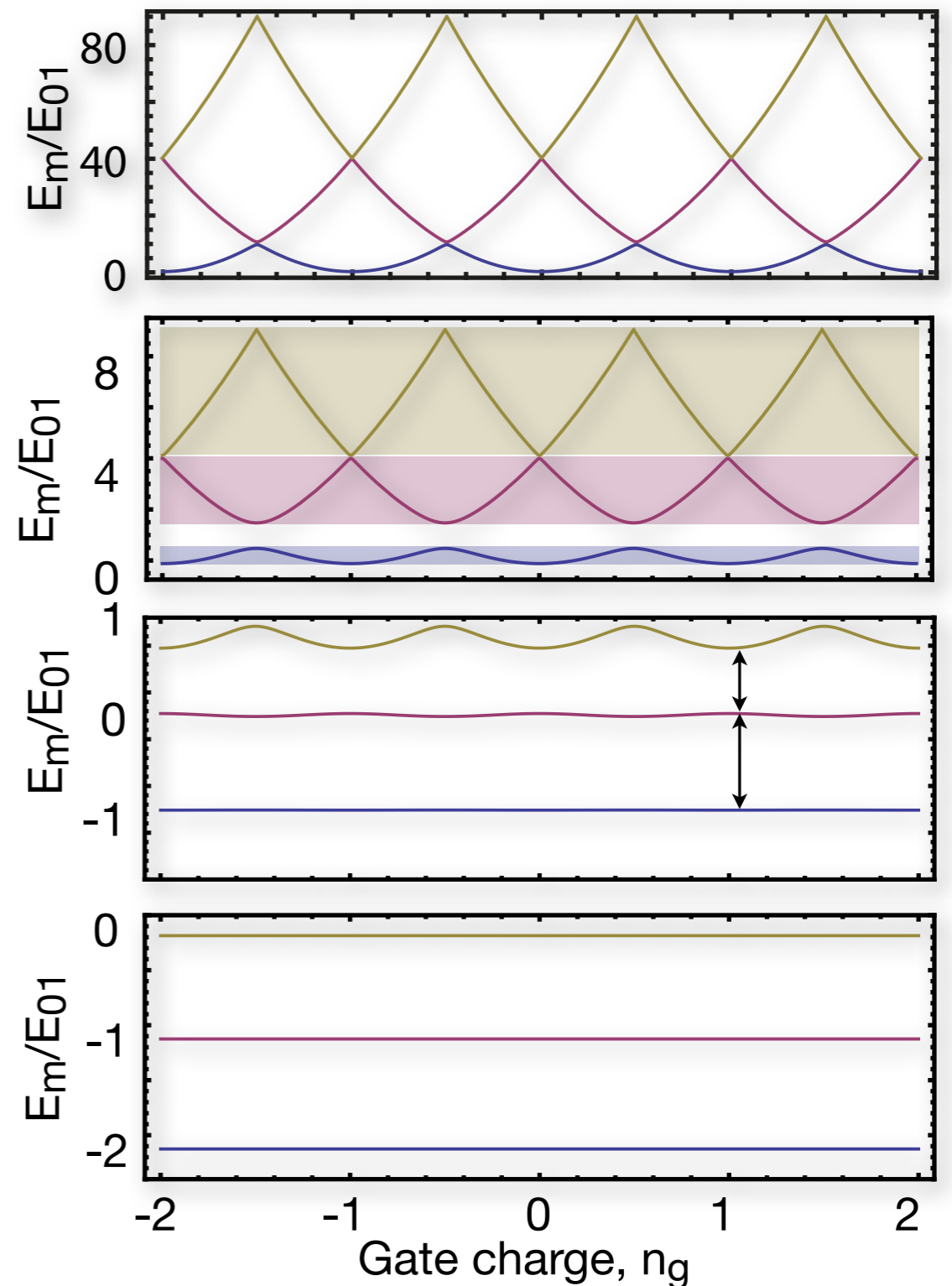
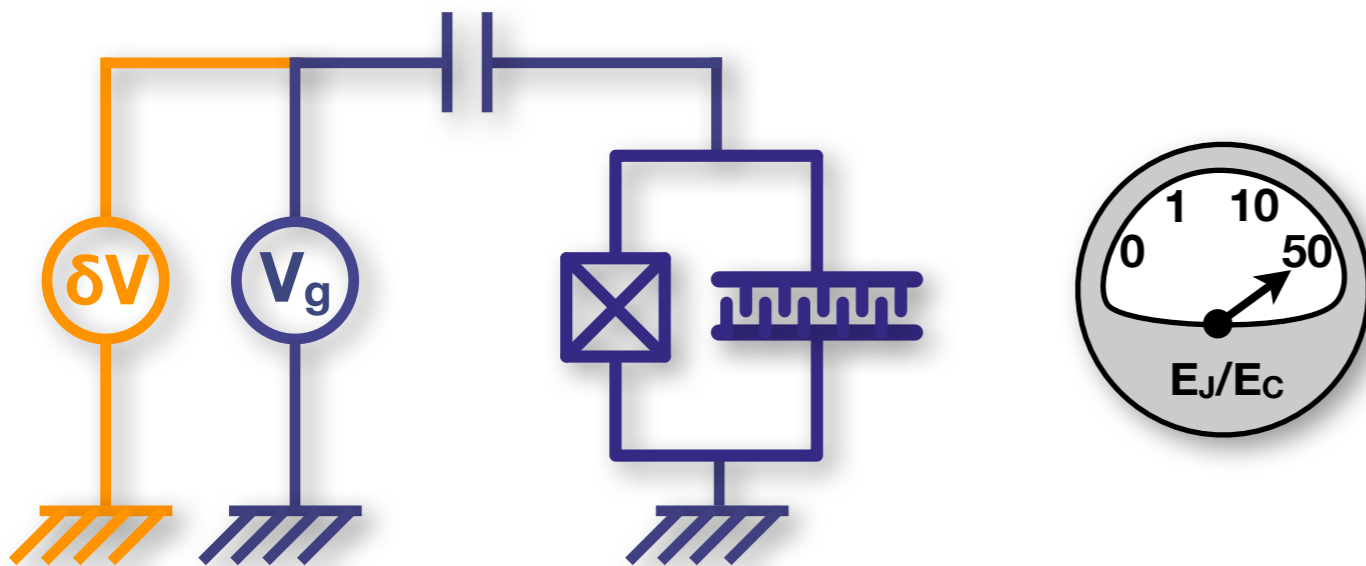
# Superconducting qubits: transmon regime



# Superconducting qubits: transmon regime

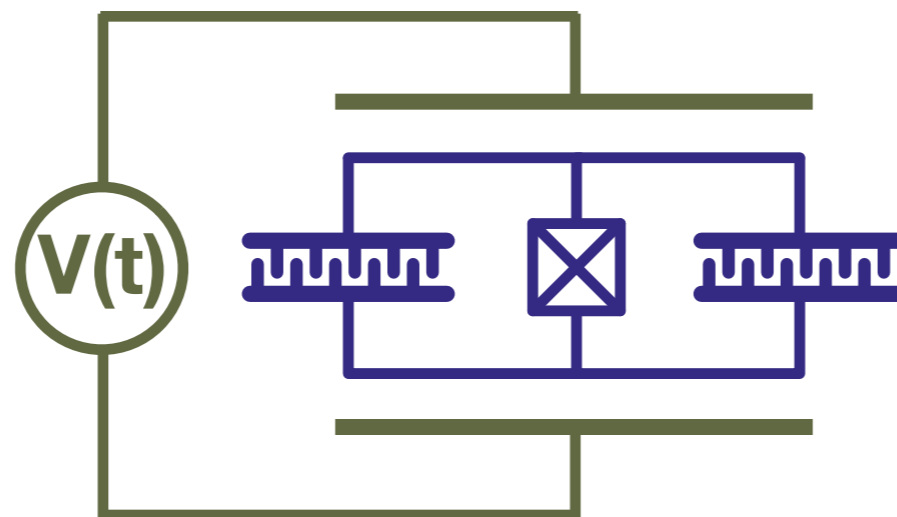
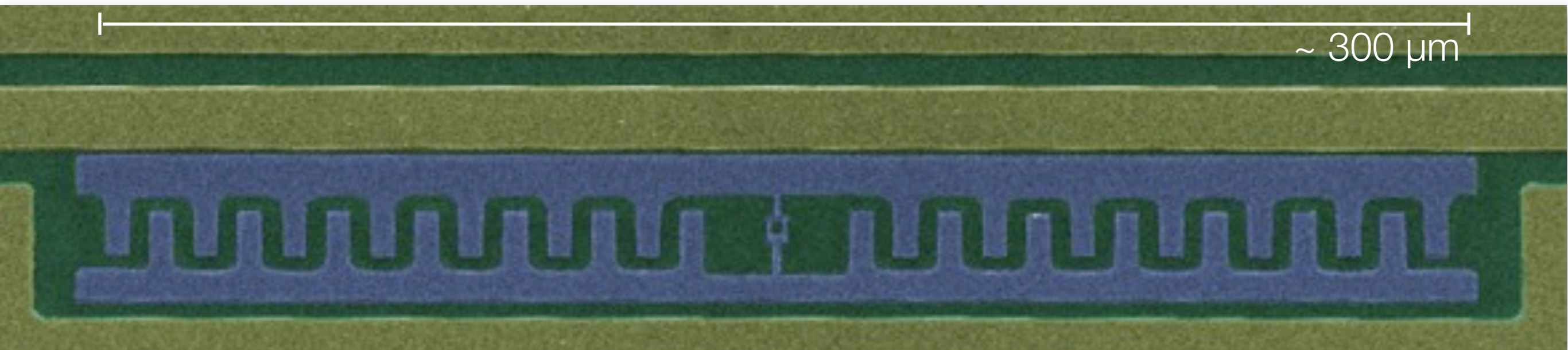


# Superconducting qubits: transmon regime



# Superconducting qubits: transmon regime

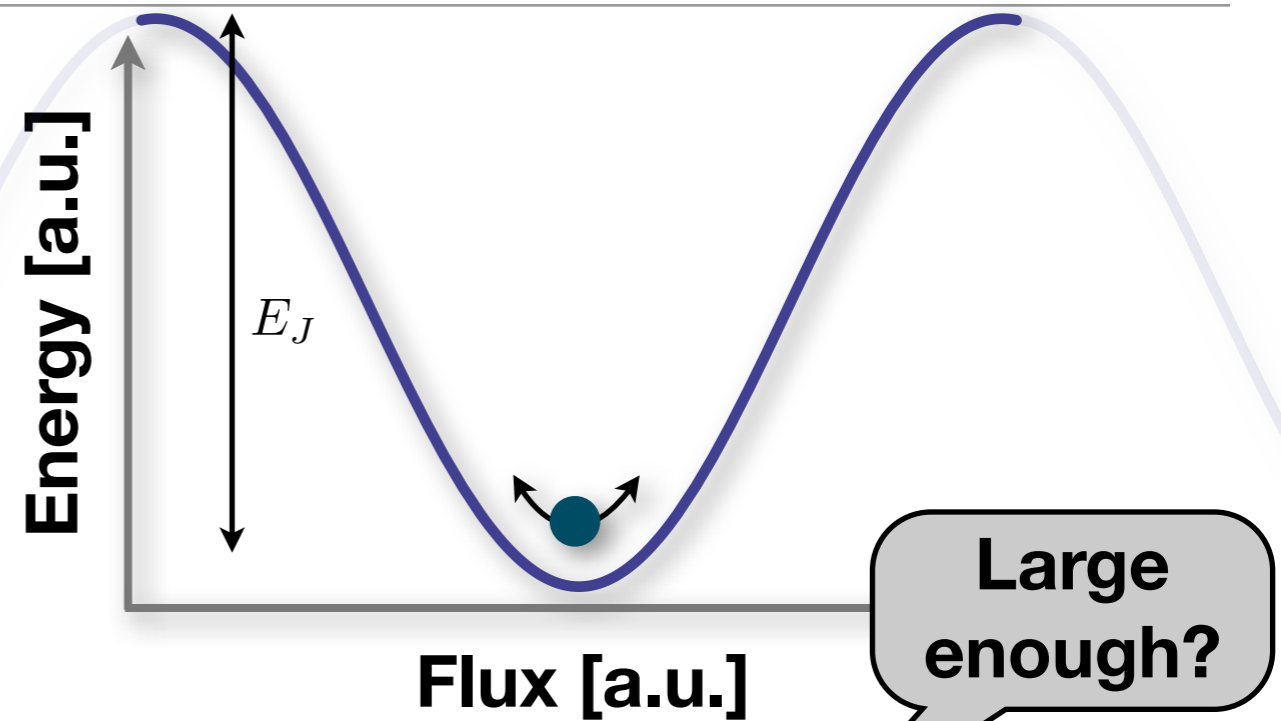
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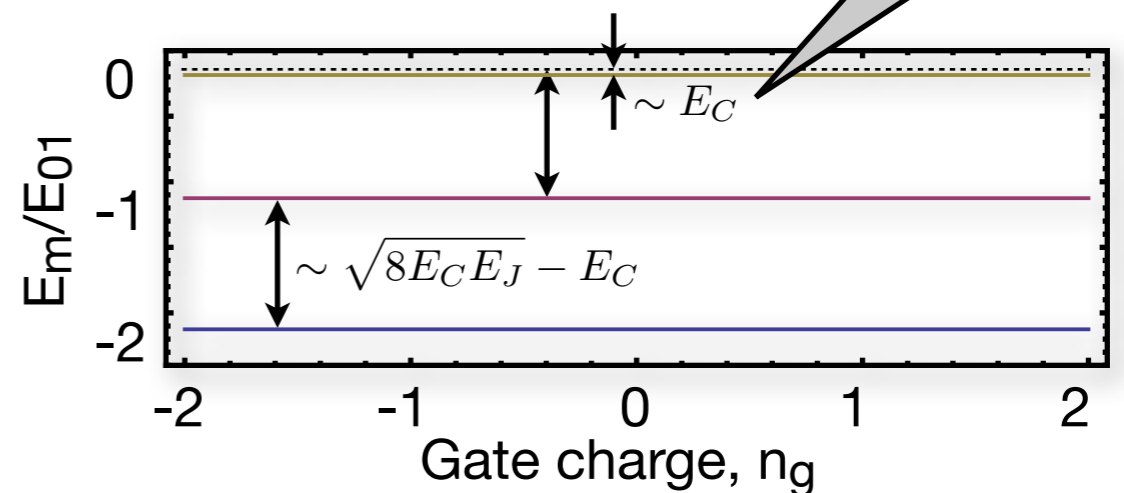
# Transmon regime: anharmonic oscillator

$$\begin{aligned}
 H &= 4E_C \hat{n}^2 - E_J \cos \hat{\phi} \\
 &\approx 4E_C \hat{n}^2 + \frac{1}{2} E_J \hat{\phi}^2 - \frac{1}{24} E_J \hat{\phi}^4 \\
 &\equiv \frac{\hat{n}^2}{2C'} + \frac{\hat{\phi}^2}{2L'} - \frac{1}{24} E_J \hat{\phi}^4
 \end{aligned}$$



$$\begin{aligned}
 \hat{n} &= i\sqrt{\hbar} \left( \frac{E_J}{32E_C} \right)^{1/4} (\hat{b}^\dagger - \hat{b}) \\
 \hat{\phi} &= \sqrt{\hbar} \left( \frac{2E_C}{E_J} \right)^{1/4} (\hat{b}^\dagger + \hat{b})
 \end{aligned}$$

$$\begin{aligned}
 H &\approx \sqrt{8E_C E_J} \hat{b}^\dagger \hat{b} - \frac{E_C}{2} (\hat{b}^\dagger \hat{b}^\dagger \hat{b} \hat{b} + 2\hat{b}^\dagger \hat{b}) \\
 &= (\sqrt{8E_C E_J} - E_C) \hat{b}^\dagger \hat{b} - \frac{E_C}{2} \hat{b}^\dagger \hat{b}^\dagger \hat{b} \hat{b}
 \end{aligned}$$

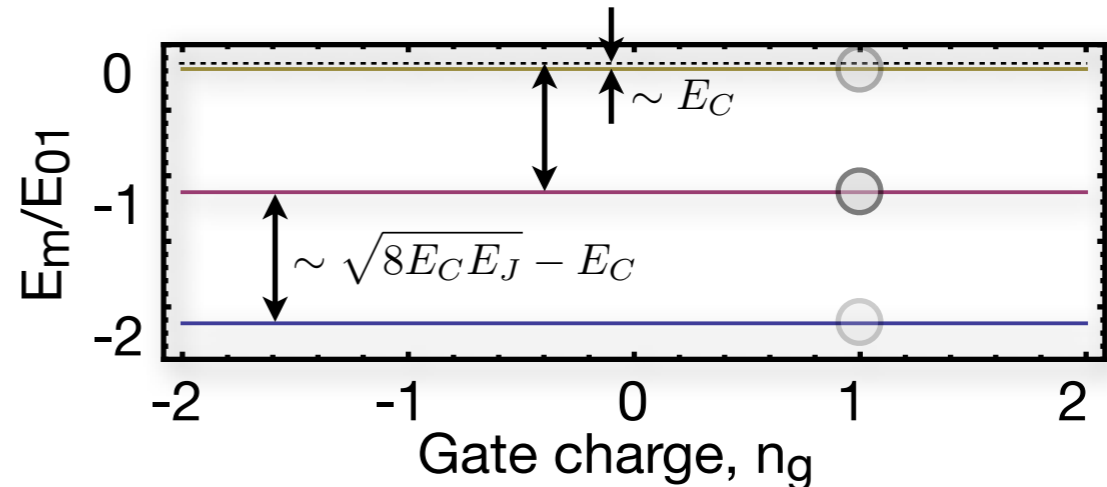
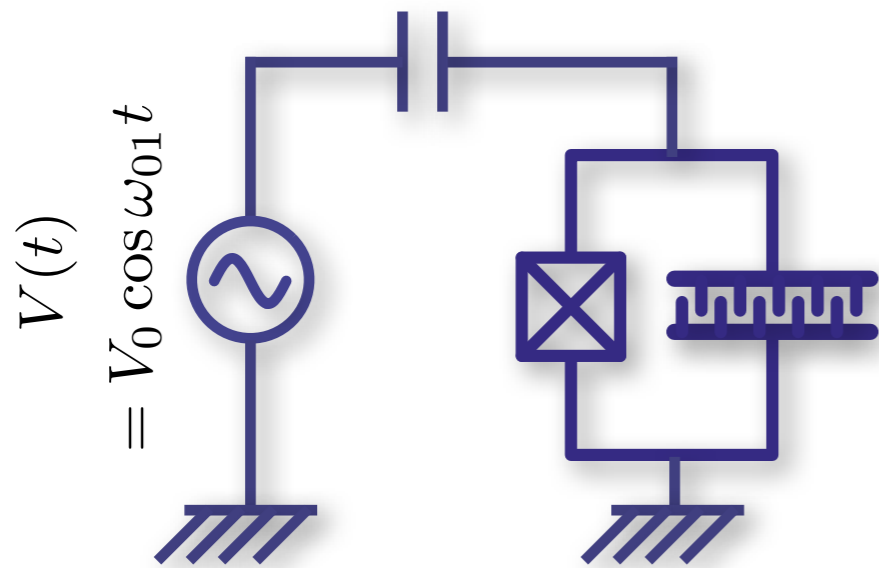


Typical values:

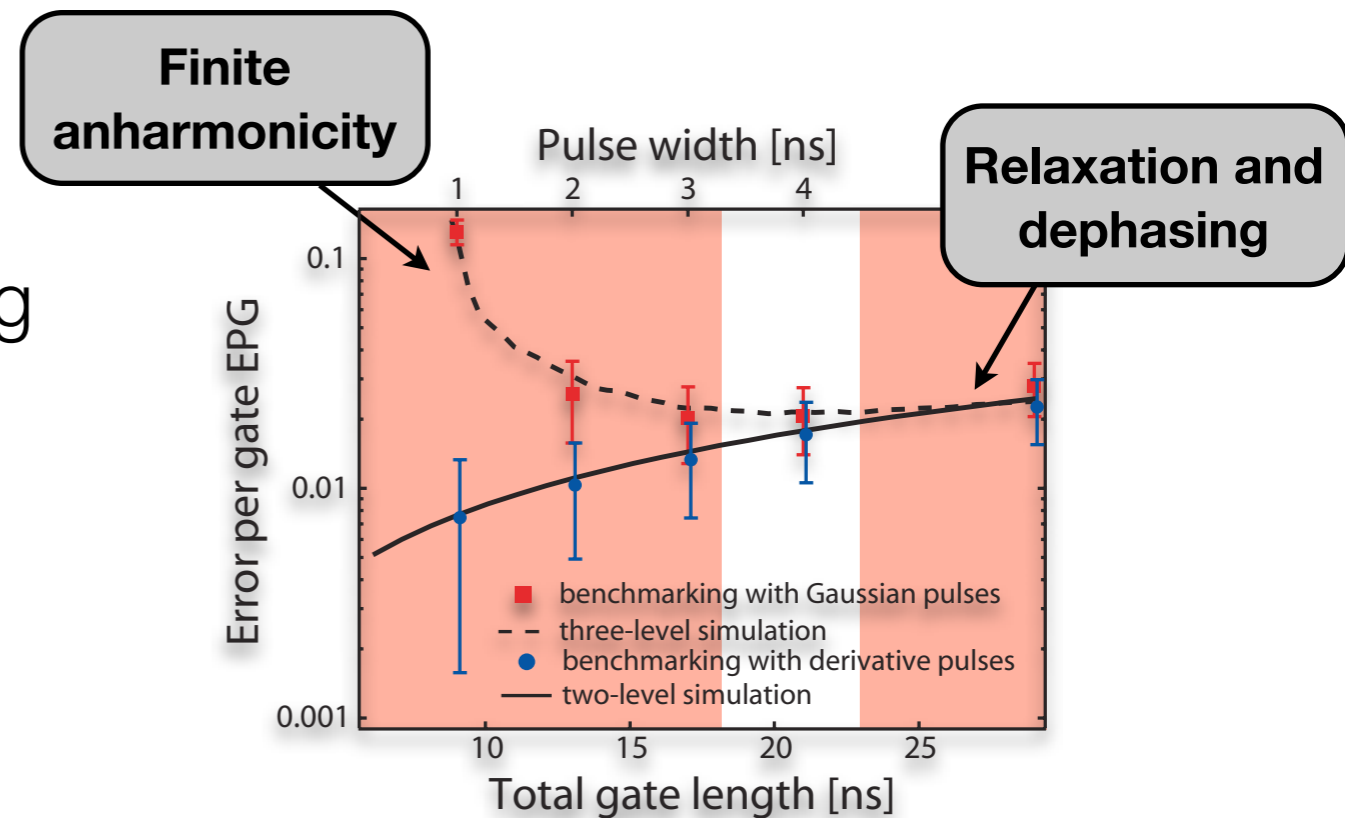
$$E_J \sim 20 \text{ GHz}$$

$$E_C \sim 400 \text{ MHz}$$

# Coherent control



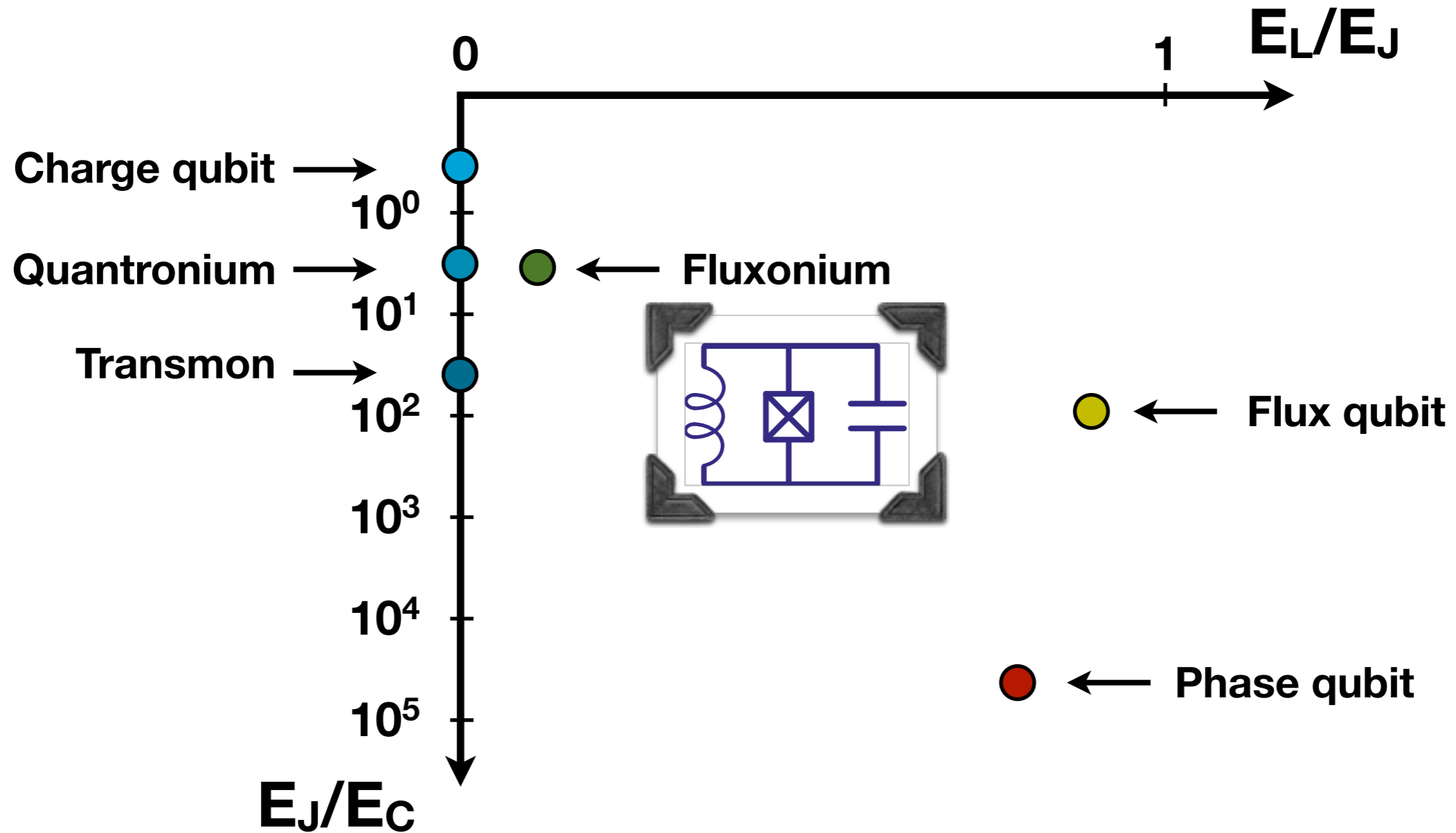
- Leakage reduction by pulse shaping
- Error per gates of 0.2%, similar to trapped ions results



Pulse shaping theory: F. Motzoi *et al.* Phys. Rev. Lett. **103**, 110501 (2009)

Exp: J. M. Chow *et al.* Phys. Rev. A **82**, 040305 (2010); E. Magesan *et al.*, Phys. Rev. Lett. **109**, 080505 (2012)

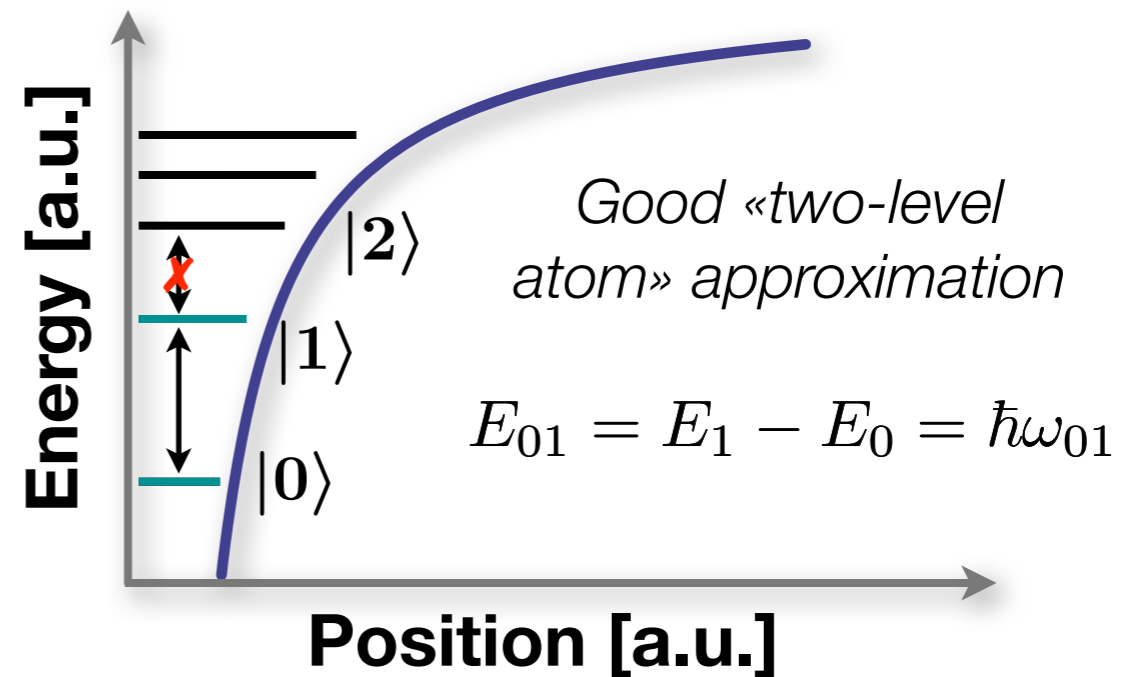
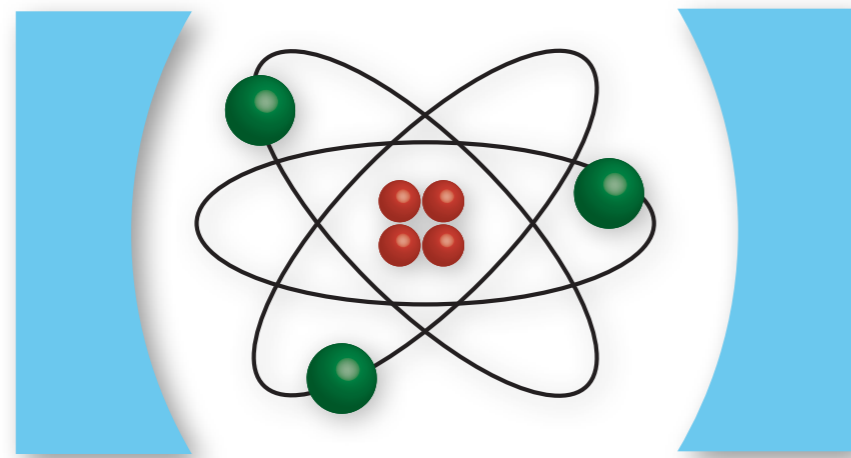
# Superconducting qubits, a family tree





# Circuit QED

# From atomic physics to quantum optics



- Control internal state by shining laser at the transition frequency

$$H = -\underbrace{\vec{d}}_g \cdot \vec{E}(t) \quad \text{with} \quad E(t) = E_0 \cos \omega_{01} t$$

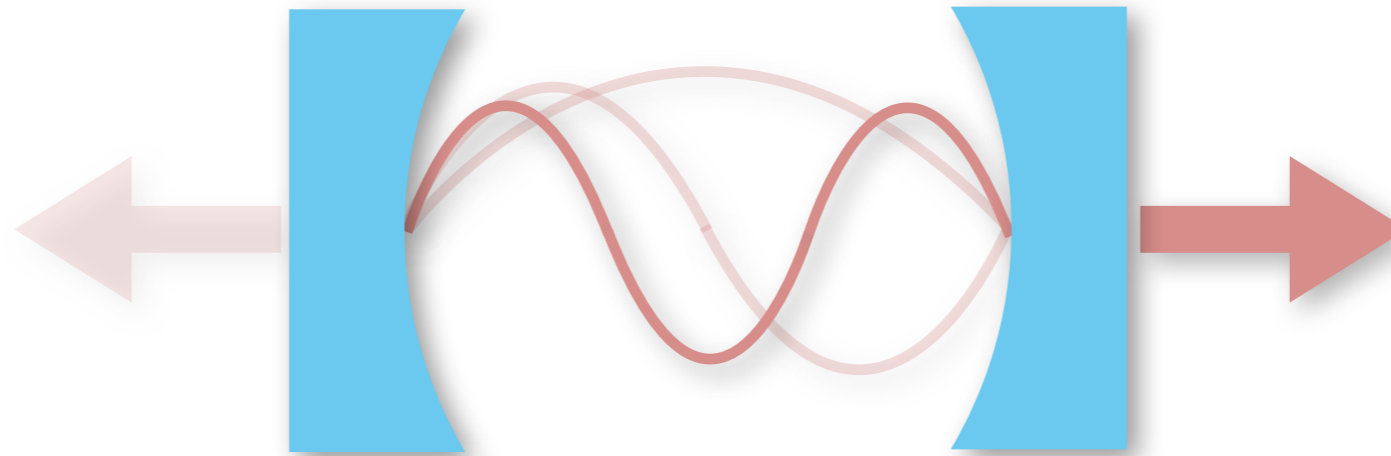
- Can the field of a single photon, or even *vacuum fluctuations*, have a large effect?

## Cavity QED

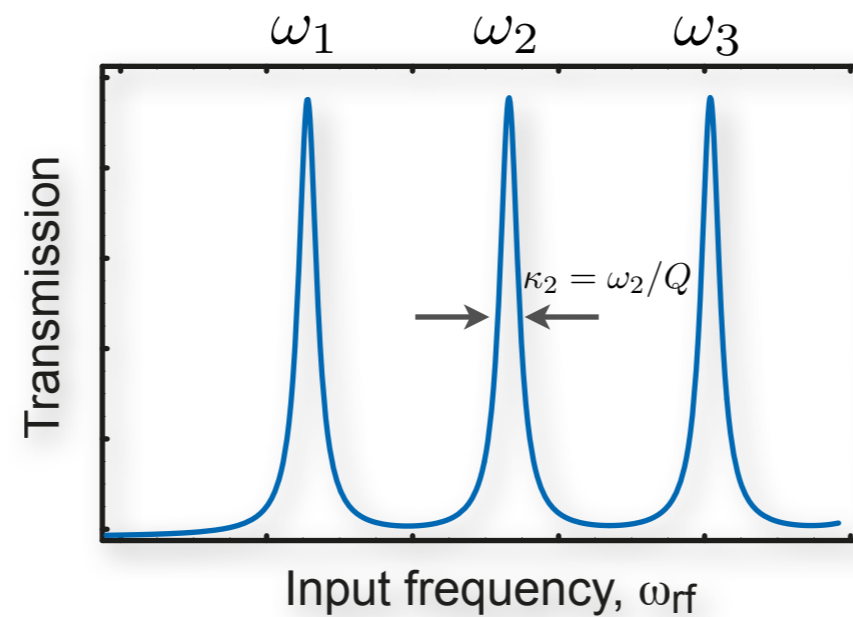
- 1) Work with large atoms ( $d$ )
- 2) Confine the field ( $E$ )

# Fabry–Pérot resonator

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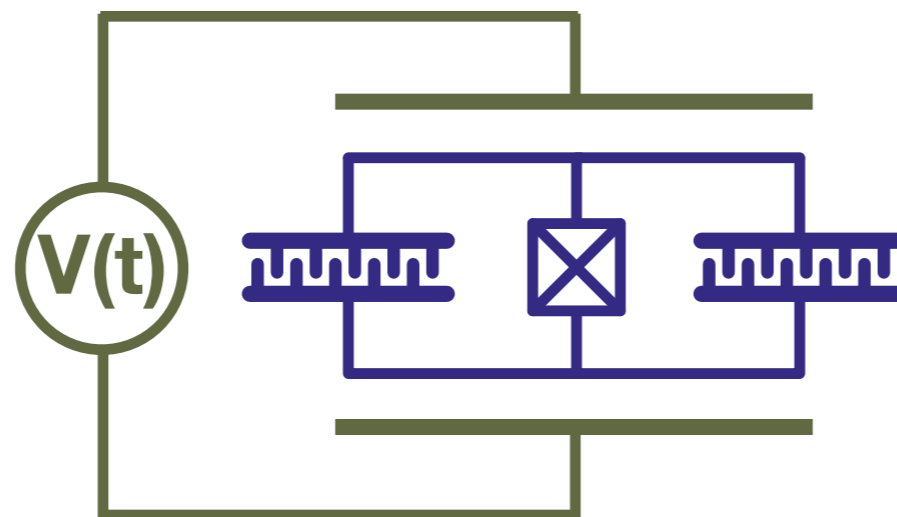
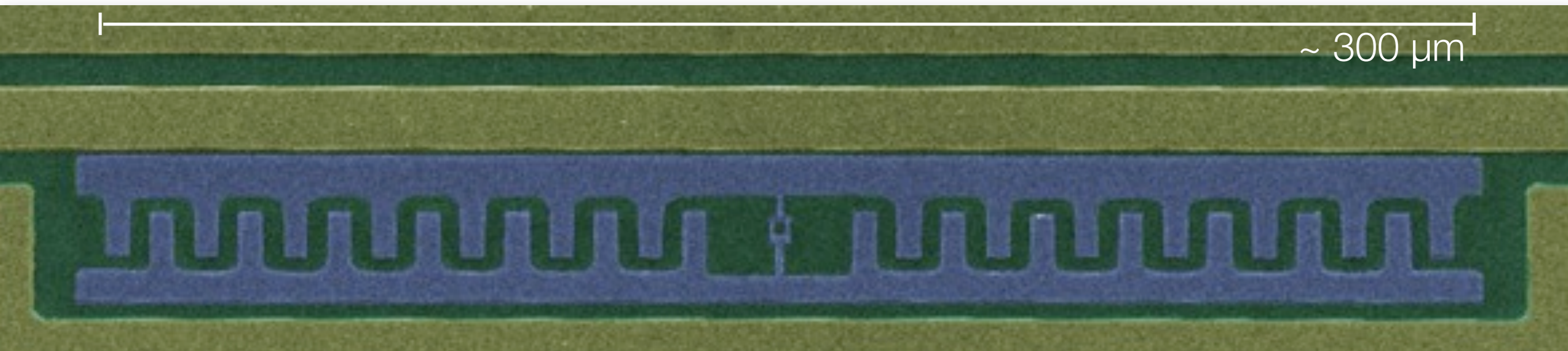


- Transmission vs input frequency:

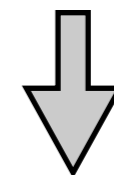


# From cavity to *circuit* QED

- Artificial atoms are **large**



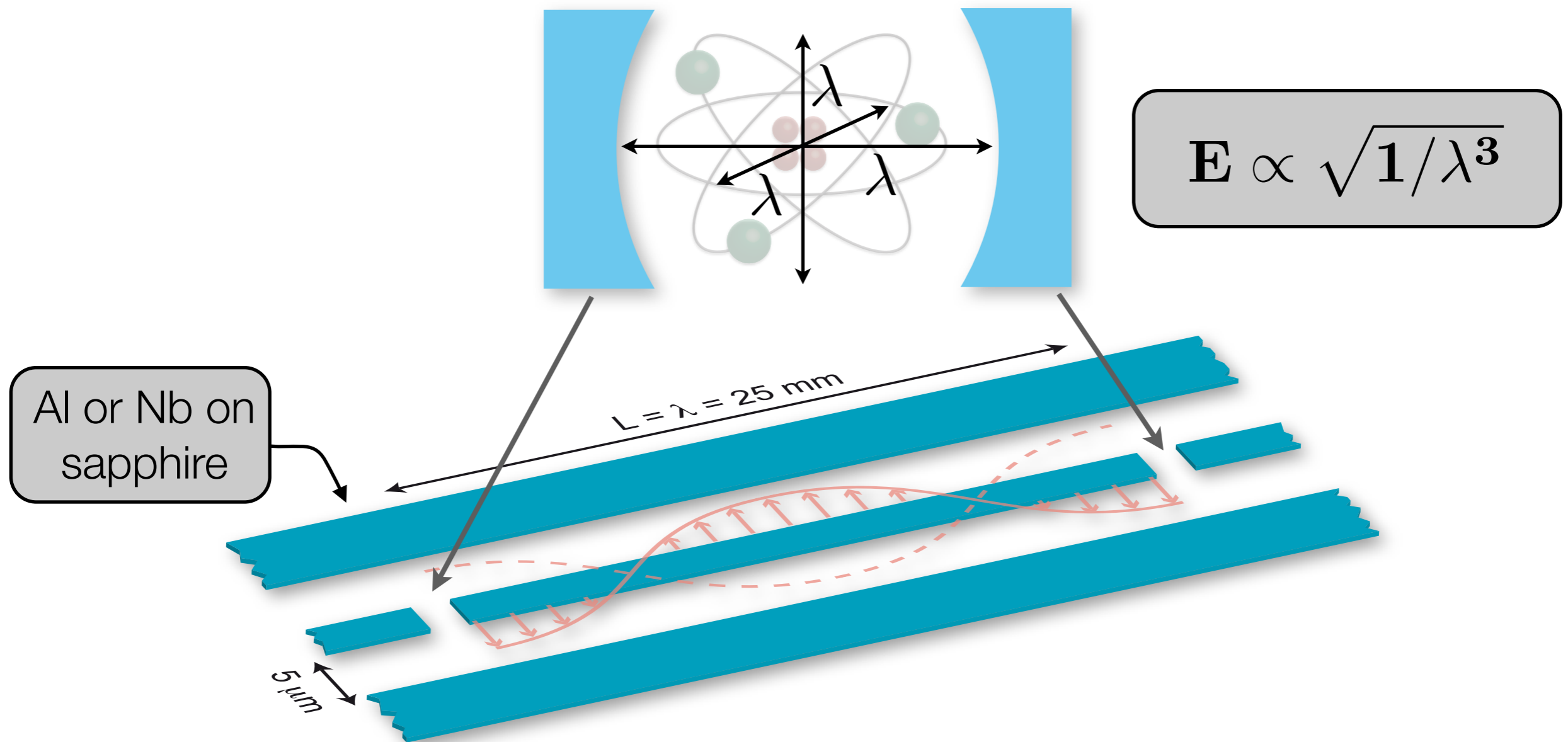
Artificial atoms have **large** dipole moments



Fast gates

# From cavity to *circuit* QED

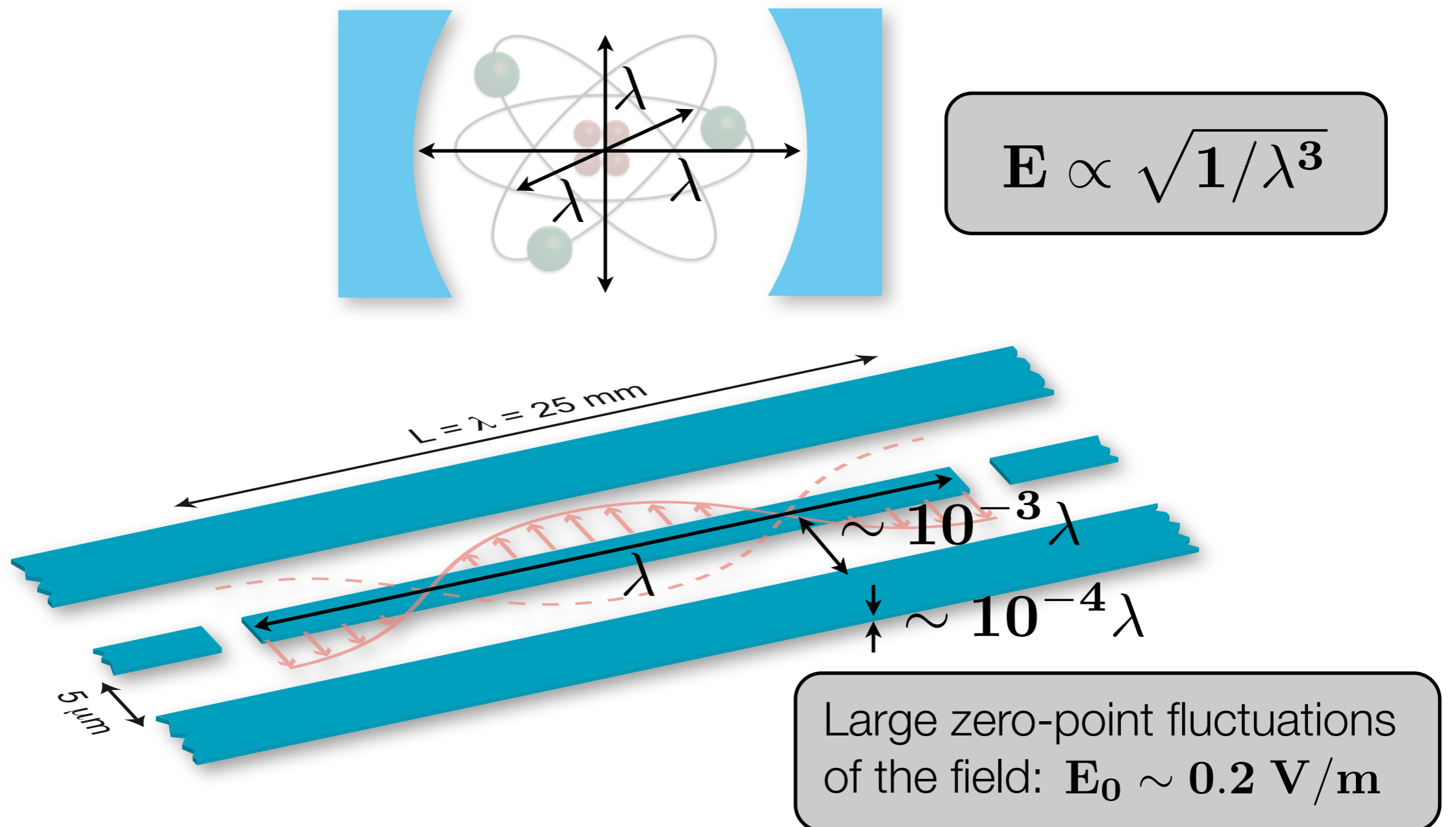
- E-field can be tightly confined



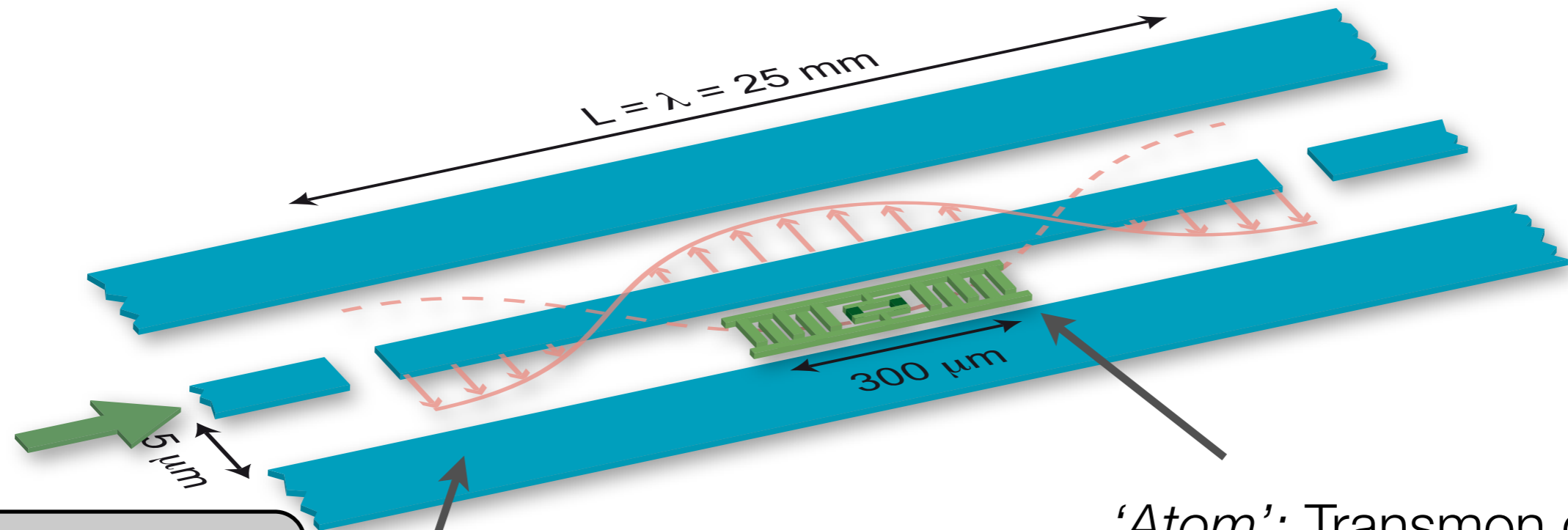


# From cavity to *circuit* QED

- E-field can be tightly confined



# From cavity to *circuit* QED



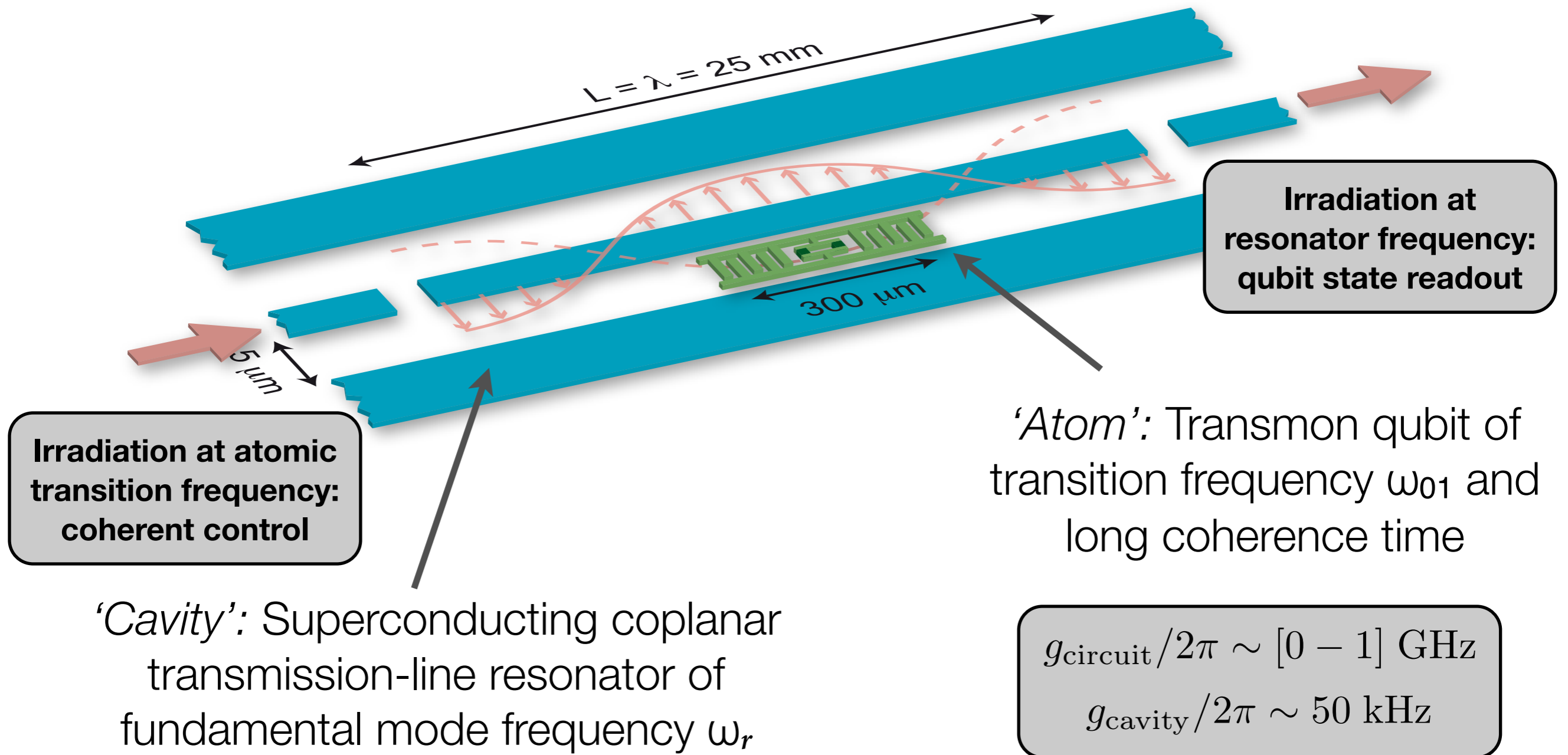
Irradiation at atomic transition frequency: coherent control

'Atom': Transmon qubit of transition frequency  $\omega_{01}$  and long coherence time

'Cavity': Superconducting coplanar transmission-line resonator of fundamental mode frequency  $\omega_r$

$g_{\text{circuit}}/2\pi \sim [0 - 1] \text{ GHz}$   
 $g_{\text{cavity}}/2\pi \sim 50 \text{ kHz}$

# From cavity to *circuit* QED

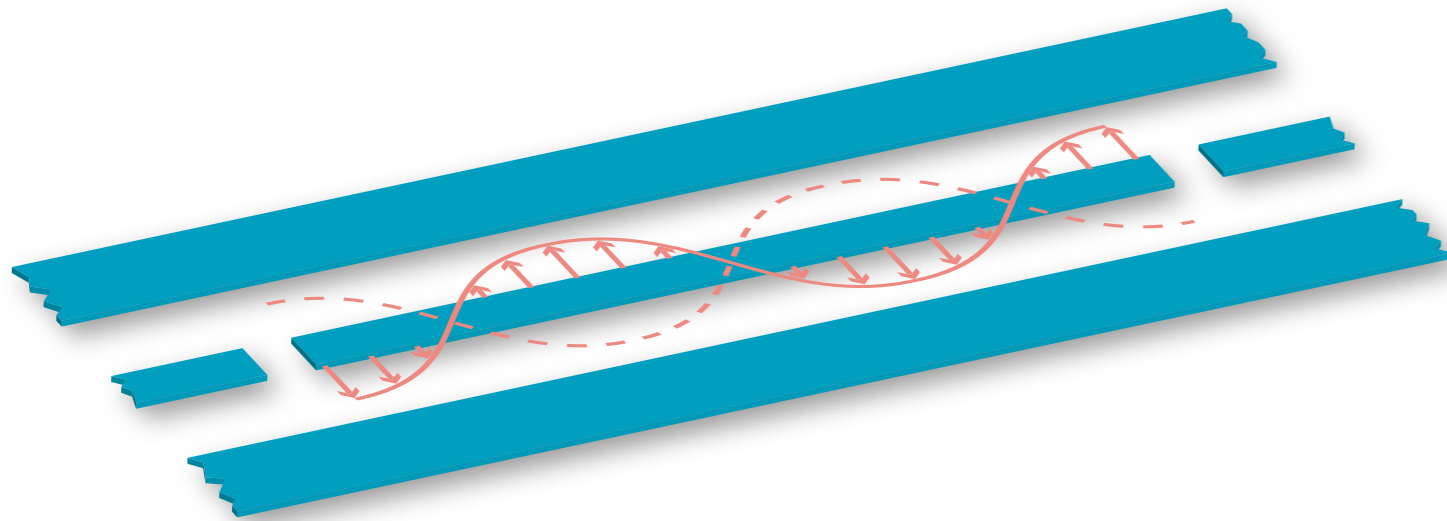


Proposal: Blais, Huang, Wallraff, Girvin & Schoelkopf, Phys. Rev. A **69**, 062320 (2004)

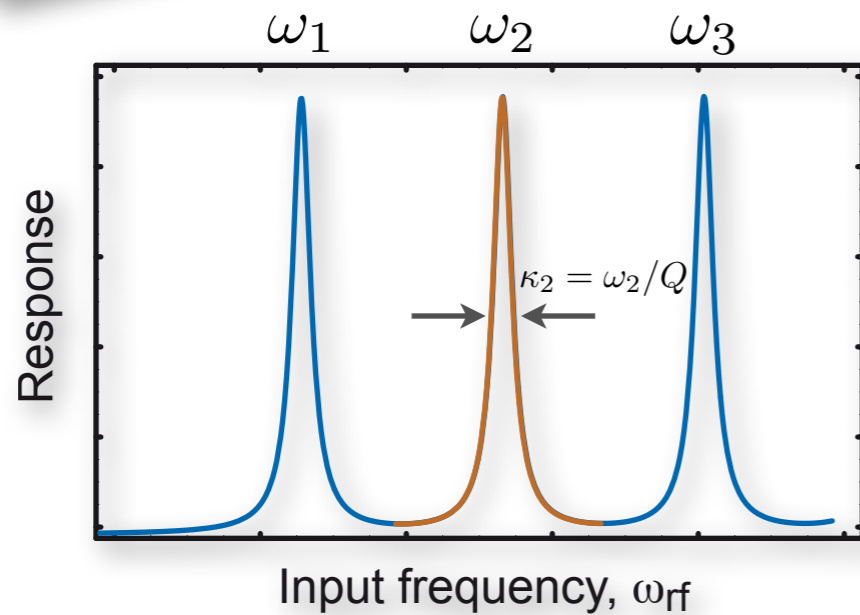
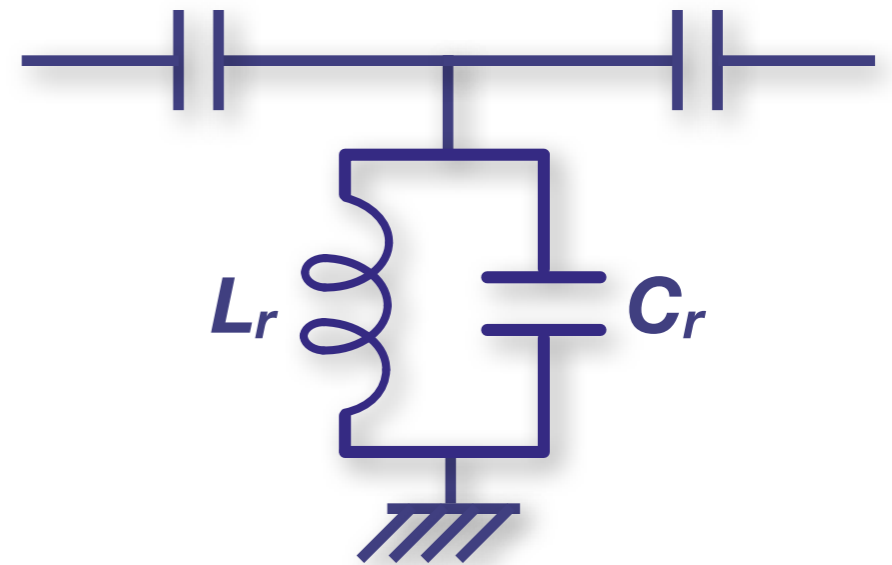
First realization: Wallraff, Schuster, Blais, Frunzio, Huang, Majer, Kumar, Girvin & Schoelkopf. Nature **431**, 162 (2004)

Ultrastrong coupling: Bourassa, Gambetta, Abdumalikov, Astafiev, Nakamura, Blais. Phys. Rev. A **80**, 032109 (2009)

# Resonator: One-mode approximation



$\approx$



$$\omega_r = \sqrt{1/L_r C_r}$$

$$\kappa = \omega_r / Q$$

Voltage across the resonator:

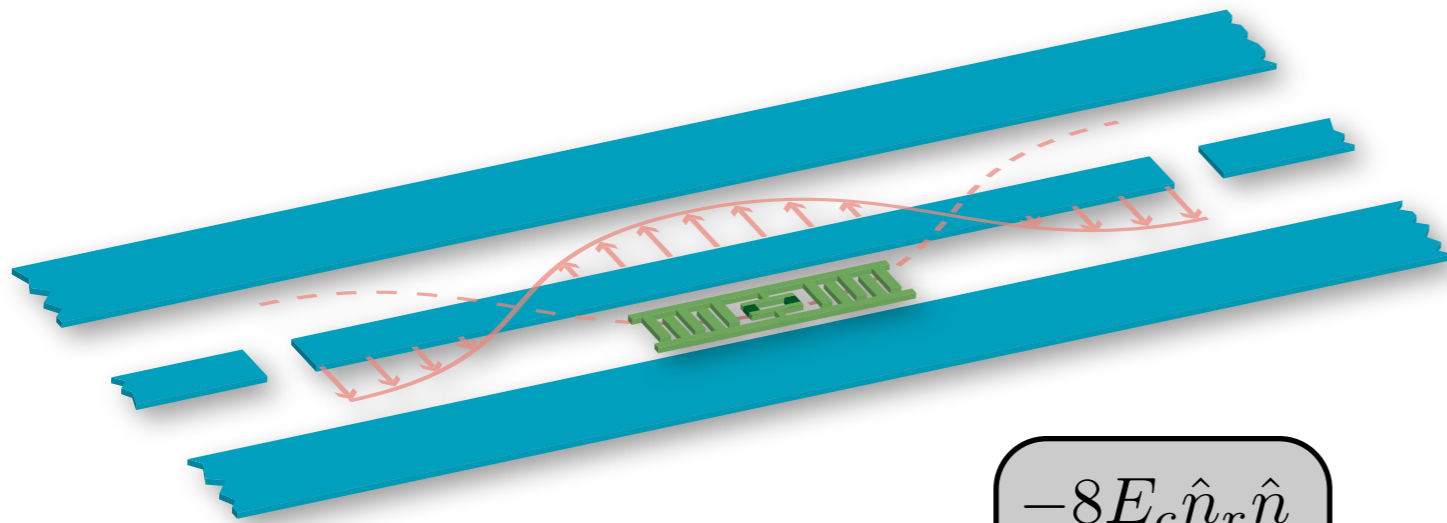
$$\hat{V}_r = \hat{Q}_r / C_r \quad \langle 0 | \hat{V}_r | 0 \rangle = 0$$

$$\sqrt{\langle 0 | \hat{V}_r^2 | 0 \rangle} = \sqrt{\frac{\hbar \omega_r}{2 C_r}} \sim 1 \mu\text{V}$$

Zero-point field fluctuations:

$$E_0 = \frac{\Delta \hat{V}_r}{d} \sim \frac{1 \mu\text{V}}{5 \mu\text{m}} \sim 0.2 \text{ V/m}$$

# Bringing it all together: Jaynes-Cummings model



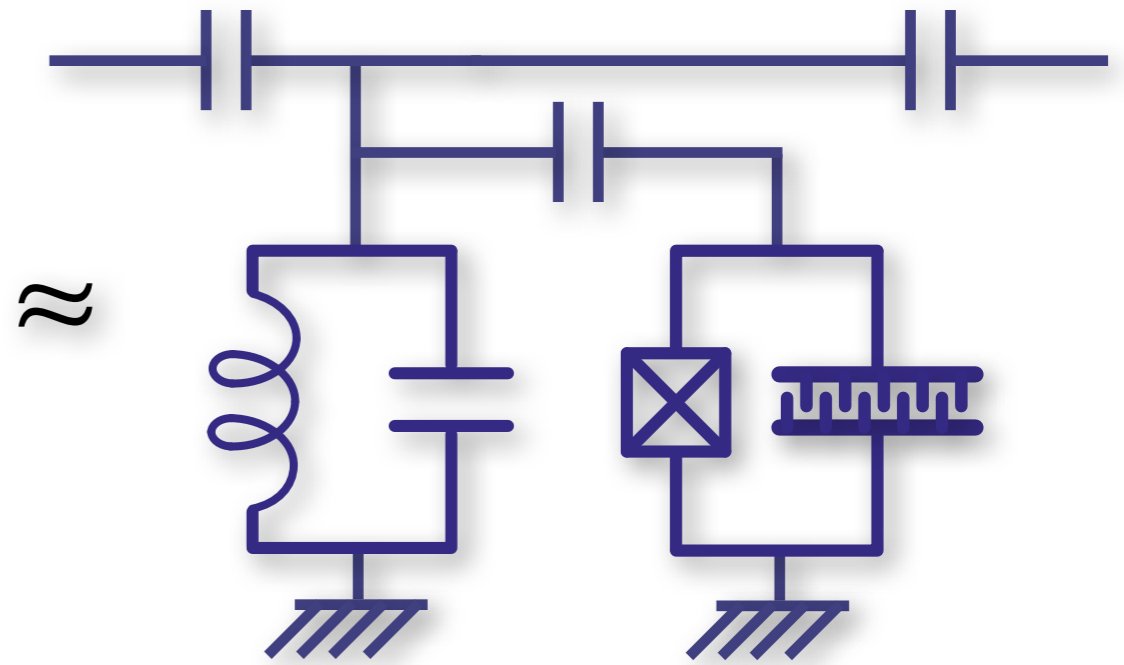
$$-8E_c \hat{n}_r \hat{n}$$

$$\hat{H} = \left( \frac{\hat{Q}_r^2}{2C_r} + \frac{\hat{\Phi}_r^2}{2L_r} \right) + \left( 4E_C [\hat{n} - \hat{n}_r]^2 - E_J \cos \hat{\phi} \right)$$

$$= \omega_r \hat{a}^\dagger \hat{a} + \left( \omega_{01} \hat{b}^\dagger \hat{b} - \frac{E_c}{2} \hat{b}^\dagger \hat{b}^\dagger \hat{b} \hat{b} \right) - g(\hat{a}^\dagger - \hat{a})(\hat{b}^\dagger - \hat{b})$$

$$= \omega_r \hat{a}^\dagger \hat{a} + \frac{\omega_{01}}{2} \hat{\sigma}_z - g(\hat{a}^\dagger - \hat{a})(\hat{\sigma}_+ - \hat{\sigma}_-) \quad \leftarrow \text{TLS approx.}$$

$$\approx \omega_r \hat{a}^\dagger \hat{a} + \frac{\omega_{01}}{2} \hat{\sigma}_z + g(\hat{a}^\dagger \hat{\sigma}_- + \hat{a} \hat{\sigma}_+) \quad \leftarrow \text{RWA}$$

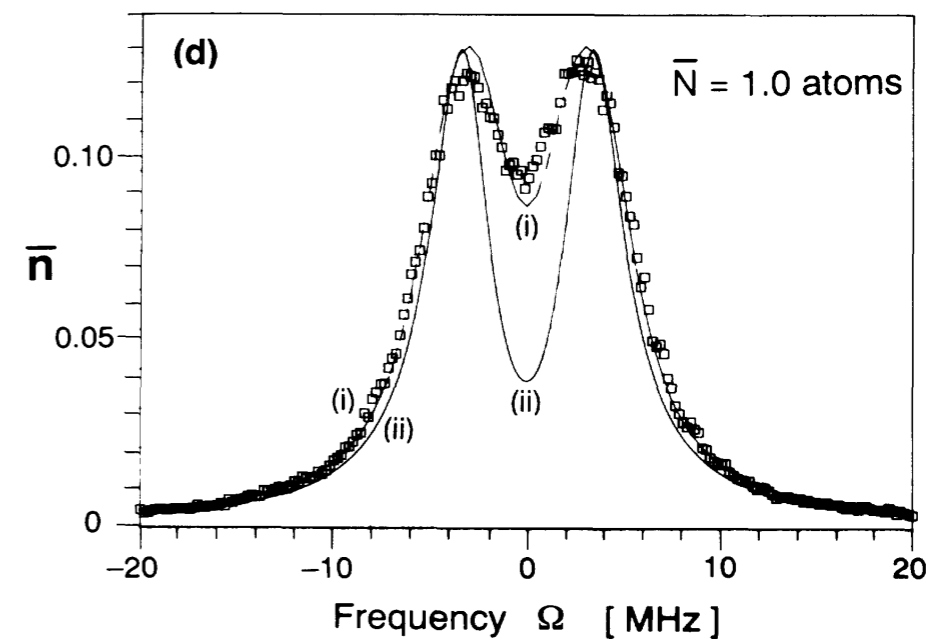
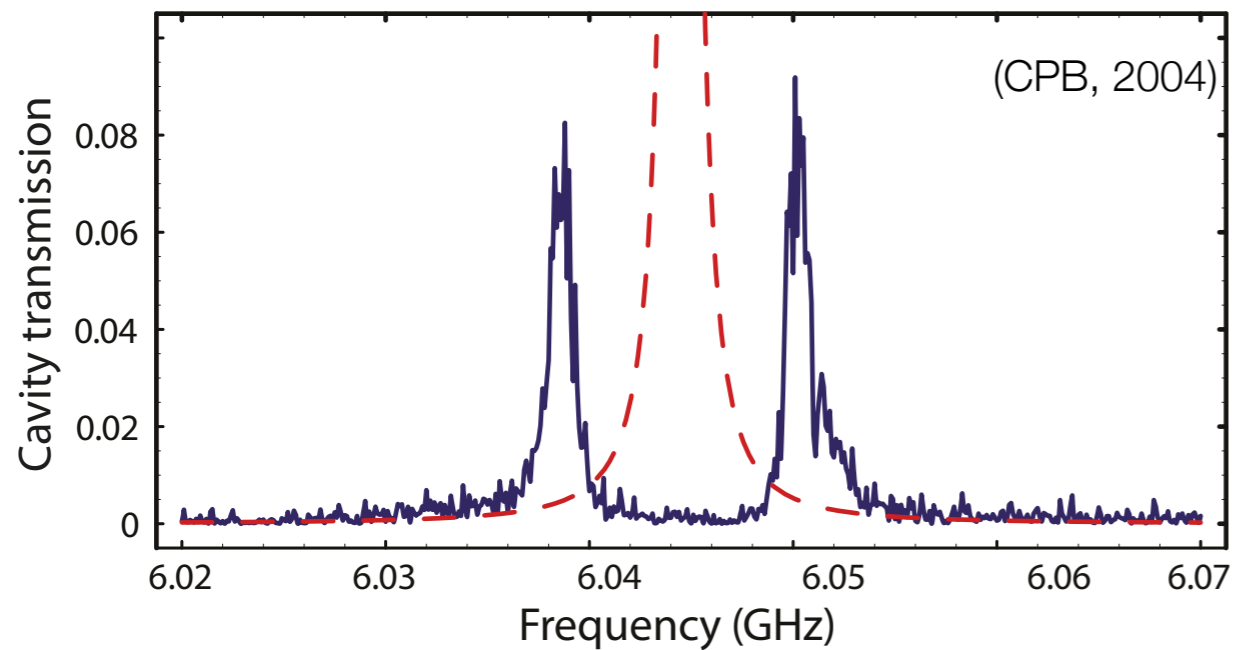
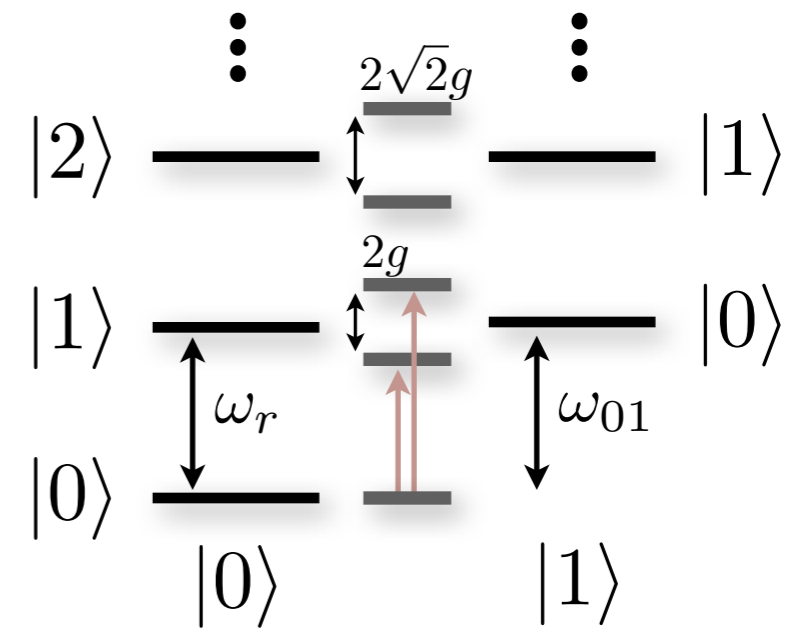
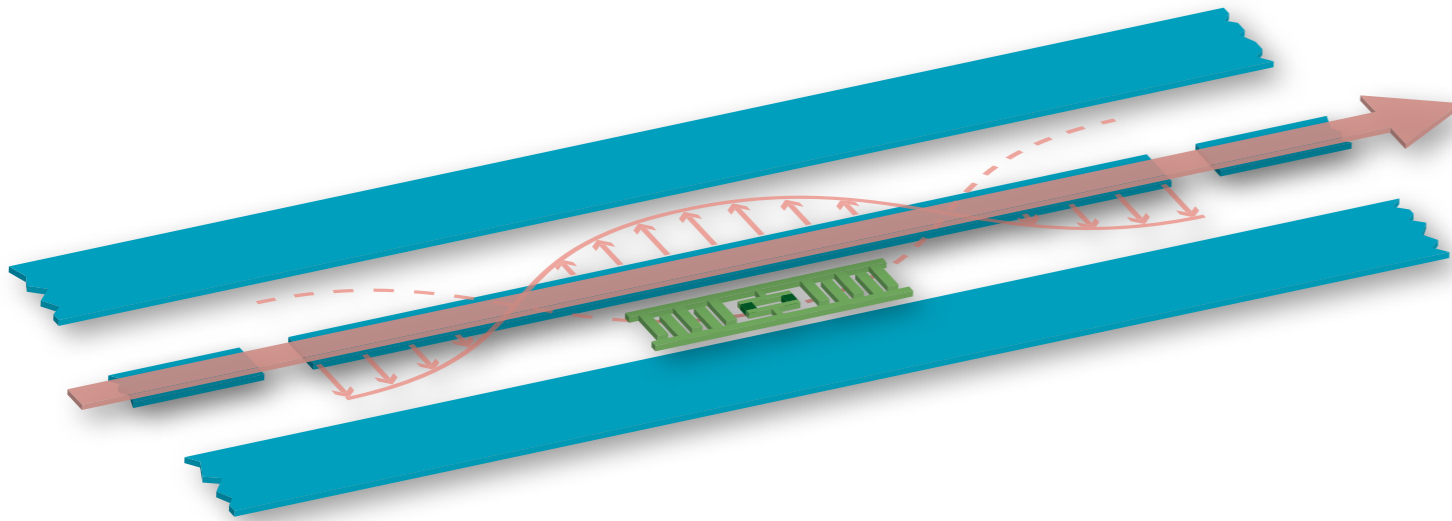


Energy exchange between 'light' and 'matter' at rate  $2g$

$$g_{\text{circuit}}/2\pi \sim [0 - 1] \text{ GHz}$$

$$g_{\text{cavity}}/2\pi \sim 50 \text{ kHz}$$

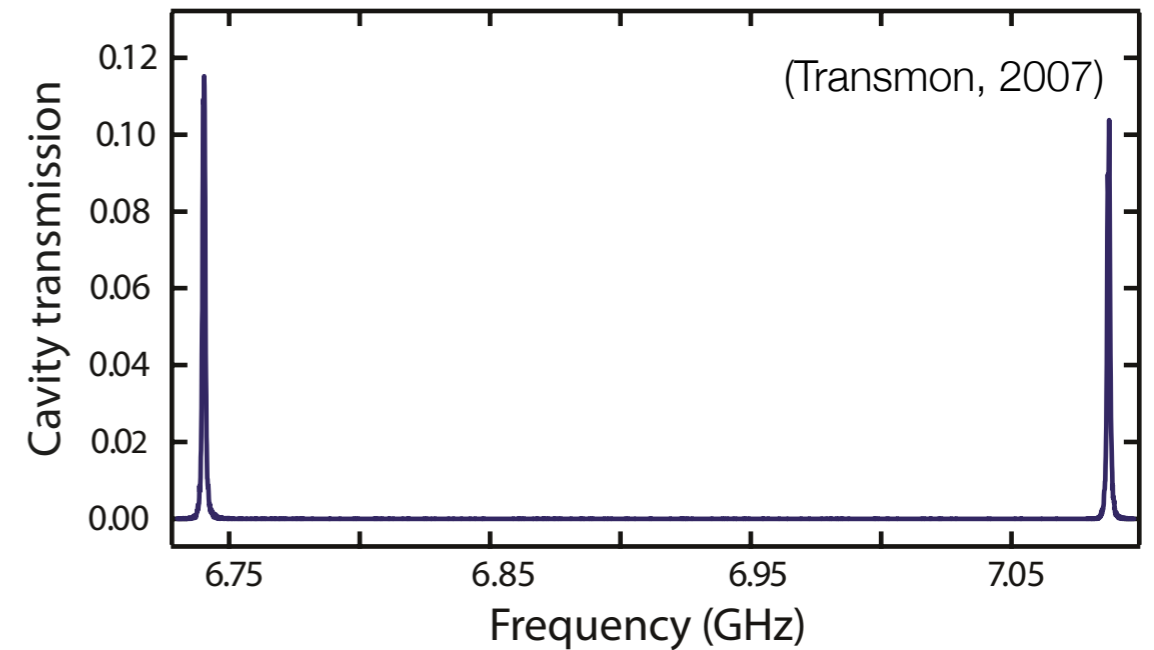
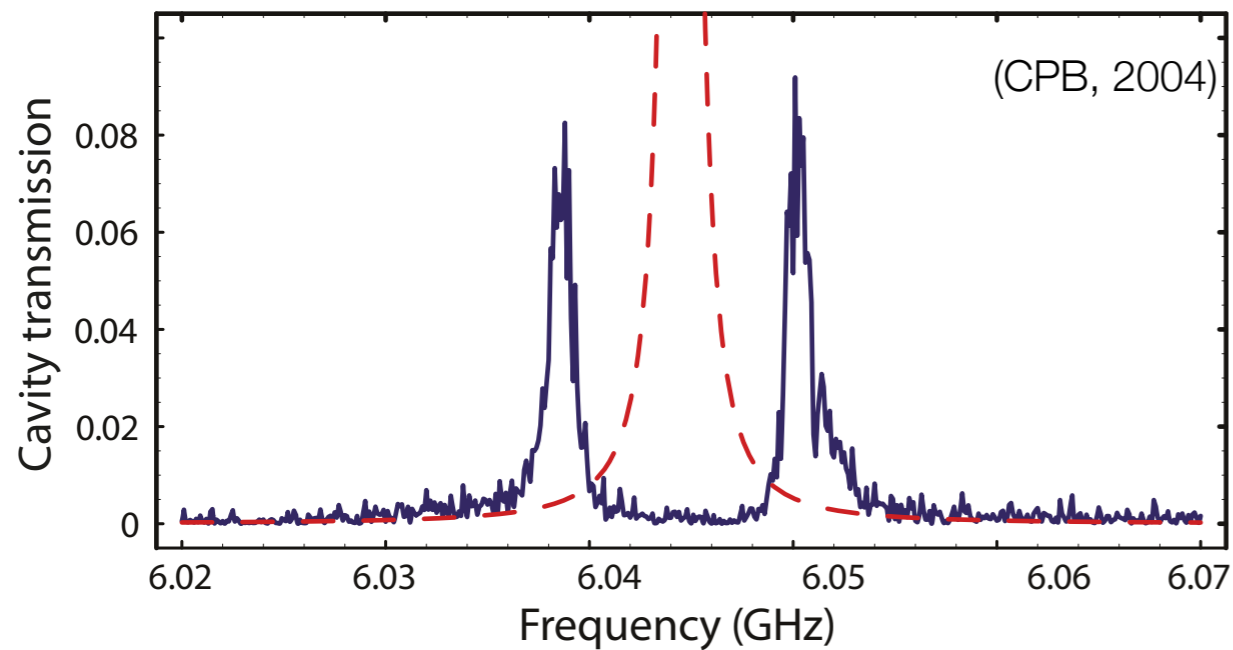
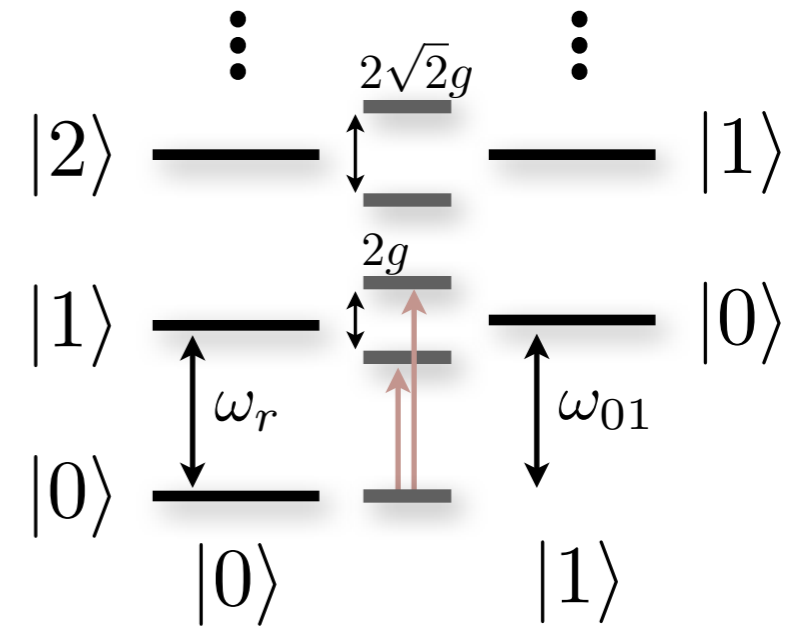
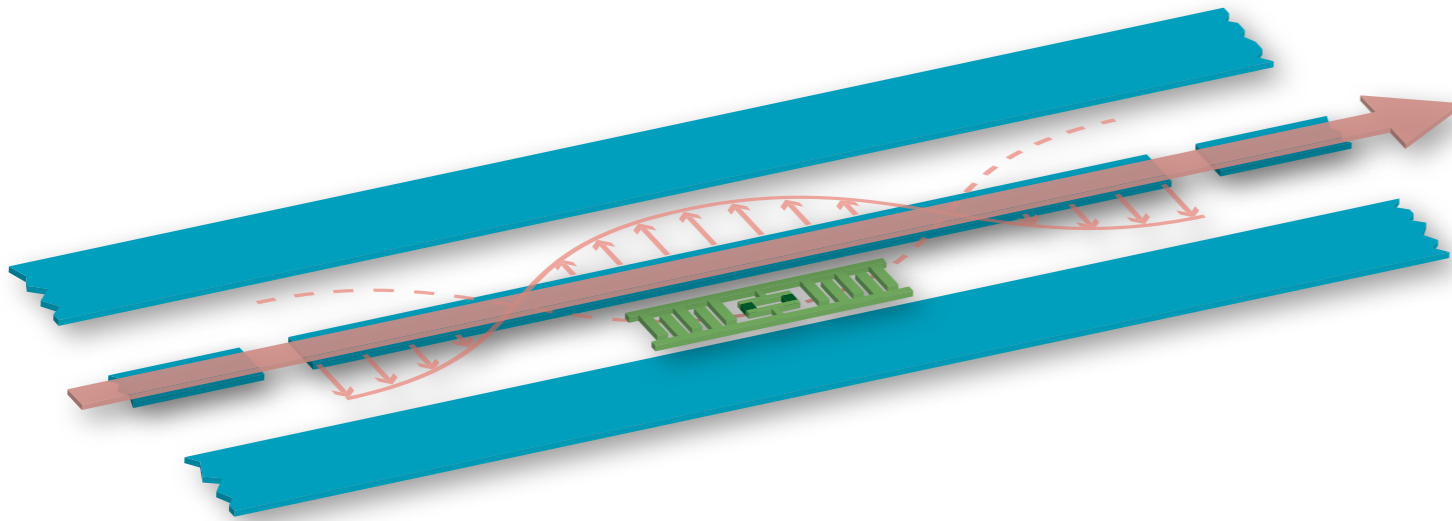
# Jaynes-Cummings Hamiltonian



Circuit QED: Wallraff, Schuster, Blais, Frunzio, Huang, Majer, Kumar, Girvin & Schoelkopf. Nature 431, 162 (2004)

Cavity QED: R. J. Thompson, G. Rempe and H. J. Kimble, PRL **68**, 1132 (1992)

# Jaynes-Cummings Hamiltonian



Circuit QED: Wallraff, Schuster, Blais, Frunzio, Huang, Majer, Kumar, Girvin & Schoelkopf. Nature 431, 162 (2004)

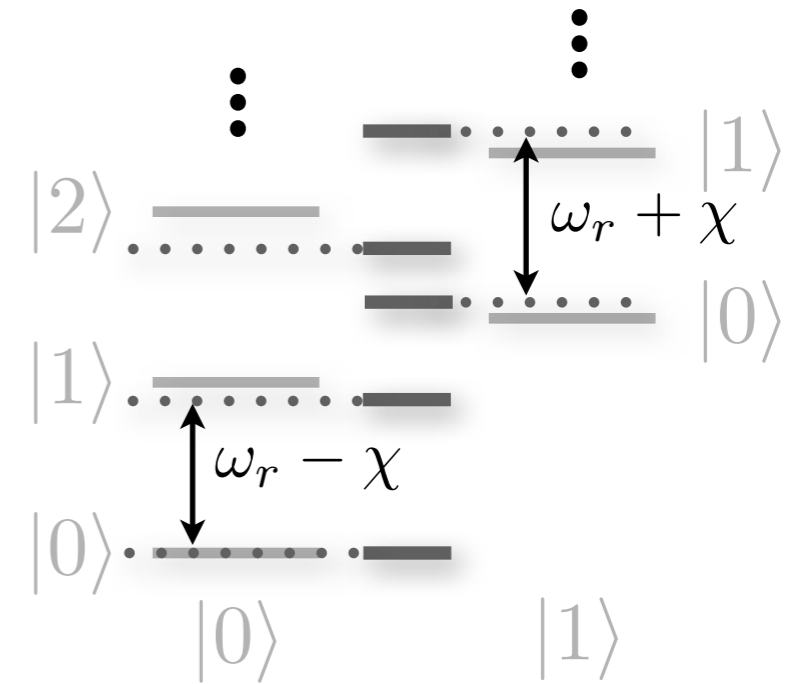
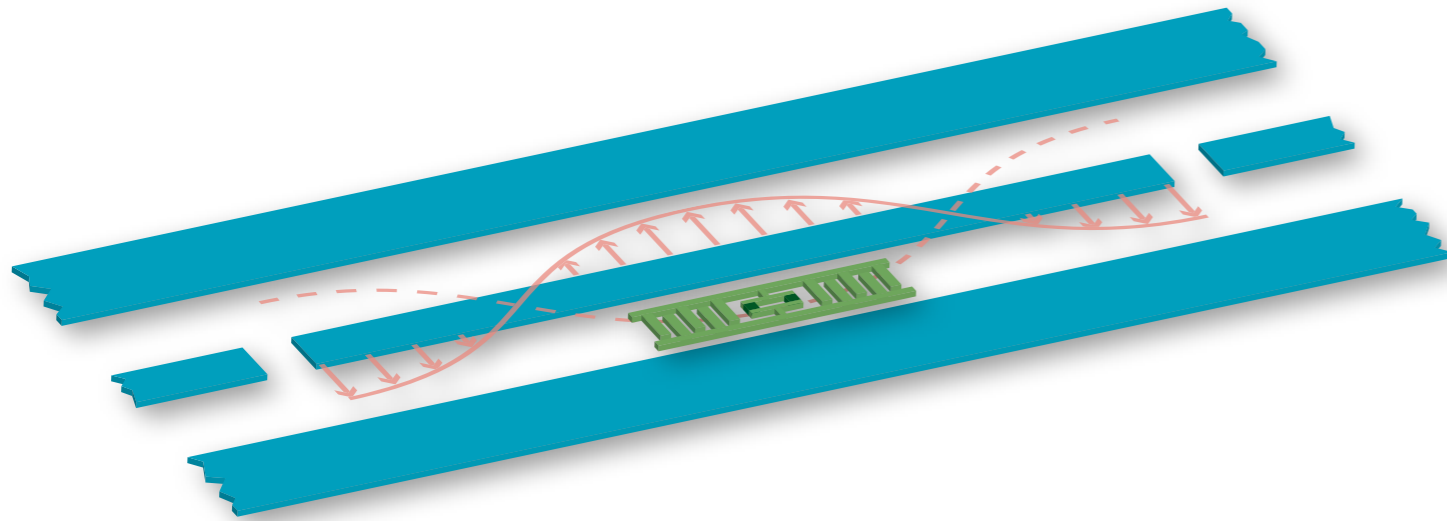
Cavity QED: R. J. Thompson, G. Rempe and H. J. Kimble, PRL **68**, 1132 (1992)

A grayscale image of an abacus, tilted slightly to the right. The abacus has two rows of beads on vertical rods. A central banner with a torn, paper-like edge is overlaid on the middle of the abacus. The banner contains the text "QIP with cQED" in a bold, black, sans-serif font.

**QIP with cQED**



# Dispersive regime of the JC Hamiltonian



Dispersive interaction:  $H \approx (\omega_r + \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + \frac{\tilde{\omega}_{01}}{2} \hat{\sigma}_z$   
 ( $|\Delta| = |\omega_{01} - \omega_r| \gg g$ )

Resonator  
frequency

Dispersive  
interaction strength  
 $\chi = g^2 / \Delta$

Qubit transition  
frequency

# Dispersive regime of the JC Hamiltonian

---

Dispersive interaction:  $H \approx (\omega_r + \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + \frac{\tilde{\omega}_{01}}{2} \hat{\sigma}_z$   
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$\swarrow$                        $\nwarrow$                        $\swarrow$   
*Resonator*                      *Dispersive*                      *Qubit transition*  
*frequency*                      *interaction strength*                      *frequency*  
 $\chi = g^2 / \Delta$

Operator perturbation theory:

$$H = \omega_r a^\dagger a + \frac{\omega_a}{2} \sigma_z + g(a^\dagger \sigma_- + a \sigma_+)$$

$$H_{\text{eff}} = e^{-\lambda X} H e^{\lambda X} = H + \lambda [H, X] + \frac{\lambda^2}{2!} [[H, X], X] + \frac{\lambda^3}{3!} [[[H, X], X], X] + \dots$$

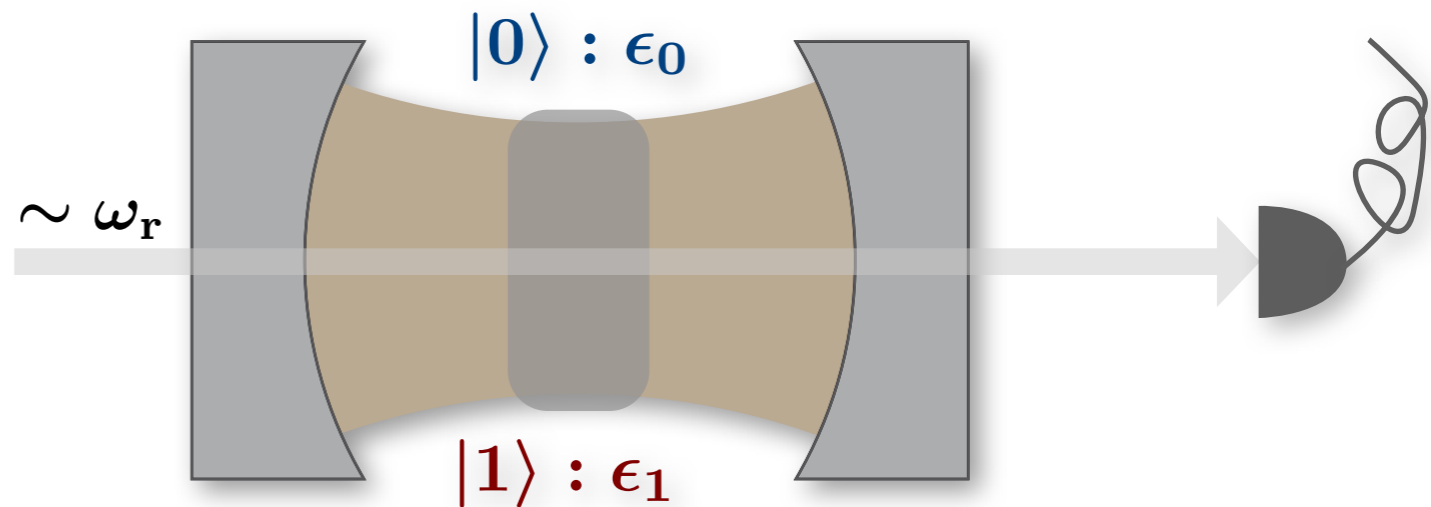
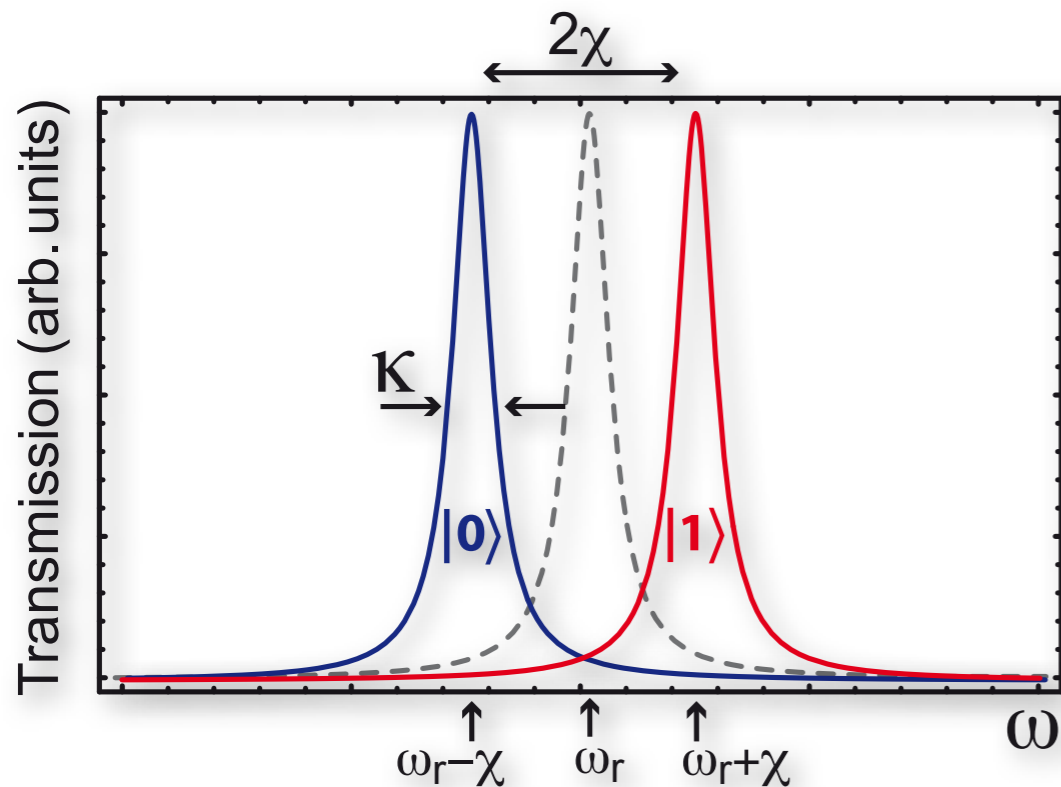
Choose  $X$  and  $\lambda$  such as to cancel non-diagonal term to second order:

$$X = a^\dagger \sigma_- - a \sigma_+ \qquad \lambda = -g / \Delta$$

# Dispersive regime: Qubit readout

Dispersive interaction:  $H \approx (\omega_r + \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + \frac{\tilde{\omega}_{01}}{2} \hat{\sigma}_z$   
 ( $|\Delta| = |\omega_{01} - \omega_r| \gg g$ )

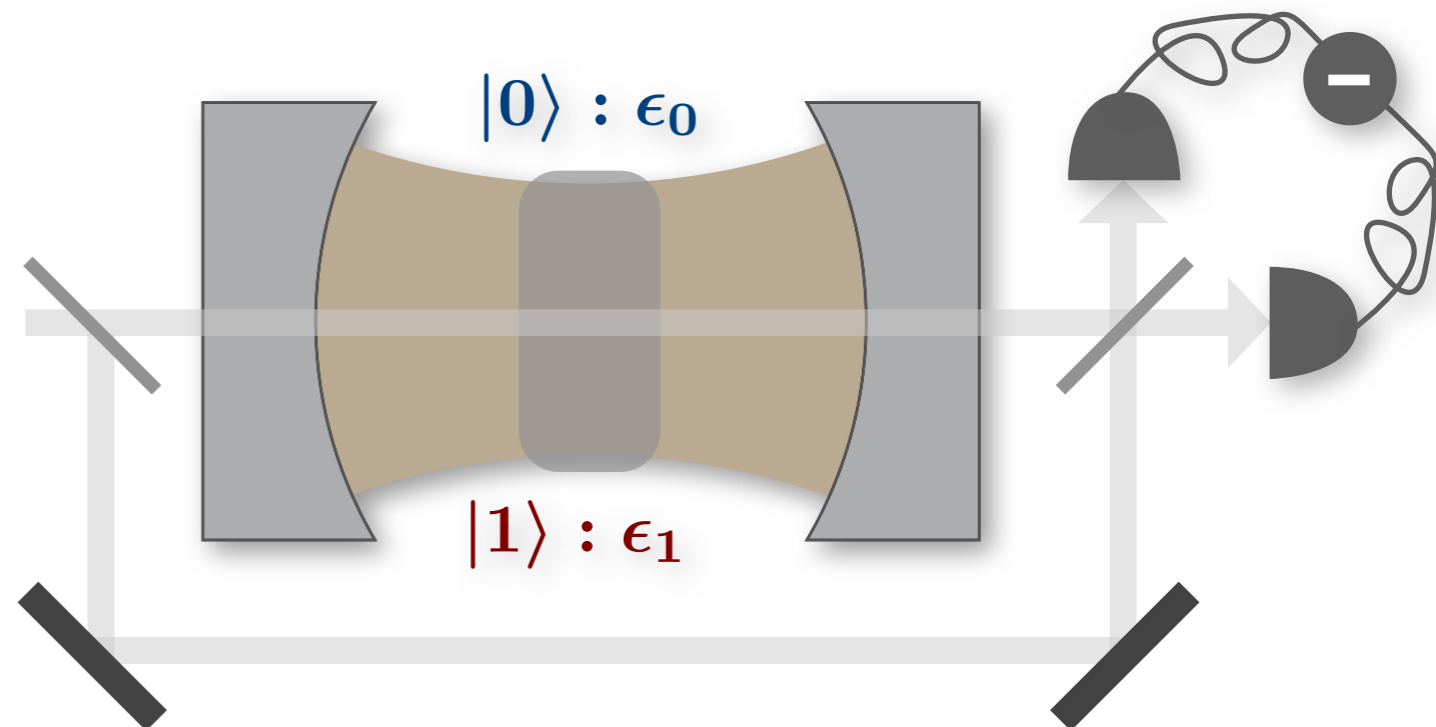
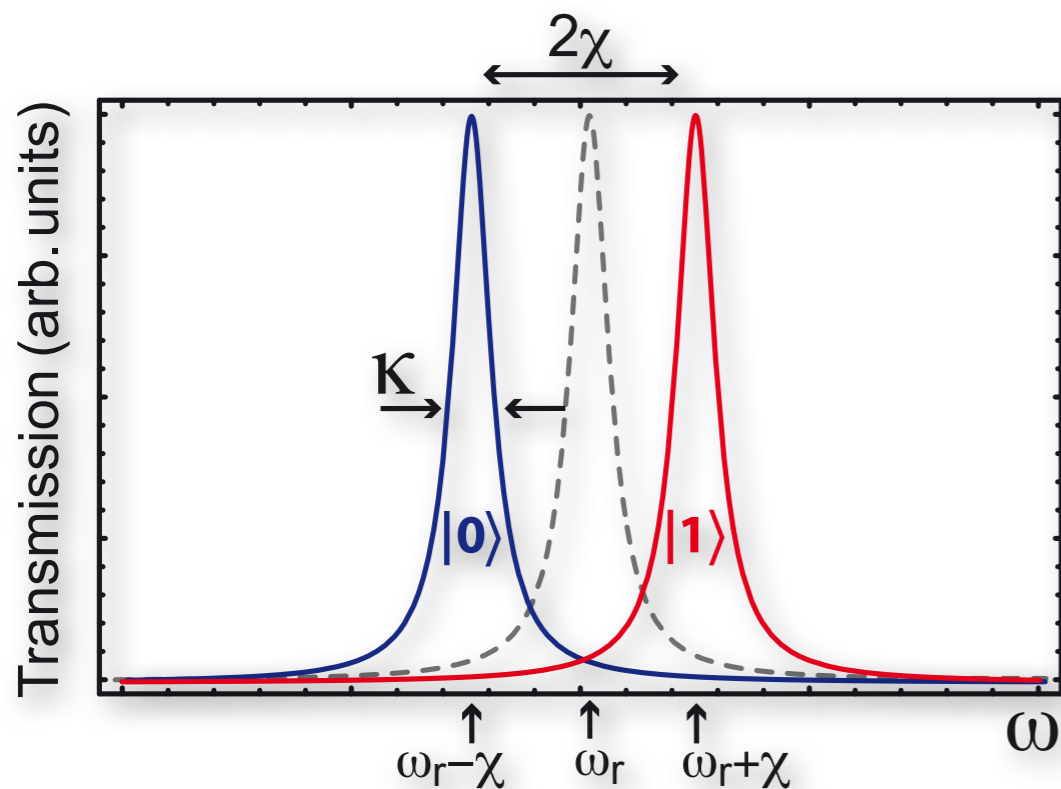
$\omega_r$ : Resonator frequency  
 $\chi$ : Dispersive interaction strength  
 $\chi = g^2 / \Delta$   
 $\tilde{\omega}_{01}$ : Qubit transition frequency



# Dispersive regime: Qubit readout

Dispersive interaction:  $H \approx (\omega_r + \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + \frac{\tilde{\omega}_{01}}{2} \hat{\sigma}_z$   
 ( $|\Delta| = |\omega_{01} - \omega_r| \gg g$ )

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 $\tilde{\omega}_{01}$ : Qubit transition frequency

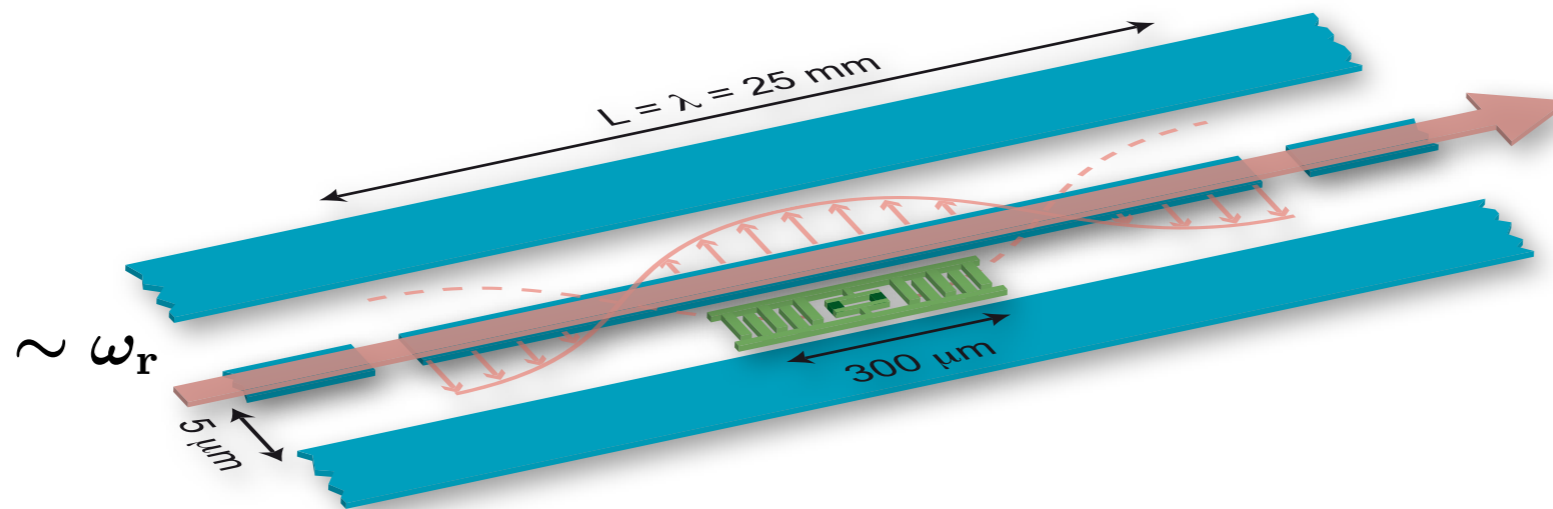


# Dispersive regime: Single-qubit gates

Dispersive interaction:  $H \approx (\omega_r + \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + \frac{\tilde{\omega}_{01}}{2} \hat{\sigma}_z$   
( $|\Delta| = |\omega_{01} - \omega_r| \gg g$ )

Resonator frequency      Dispersive interaction strength      Qubit transition frequency

$\chi = g^2 / \Delta$

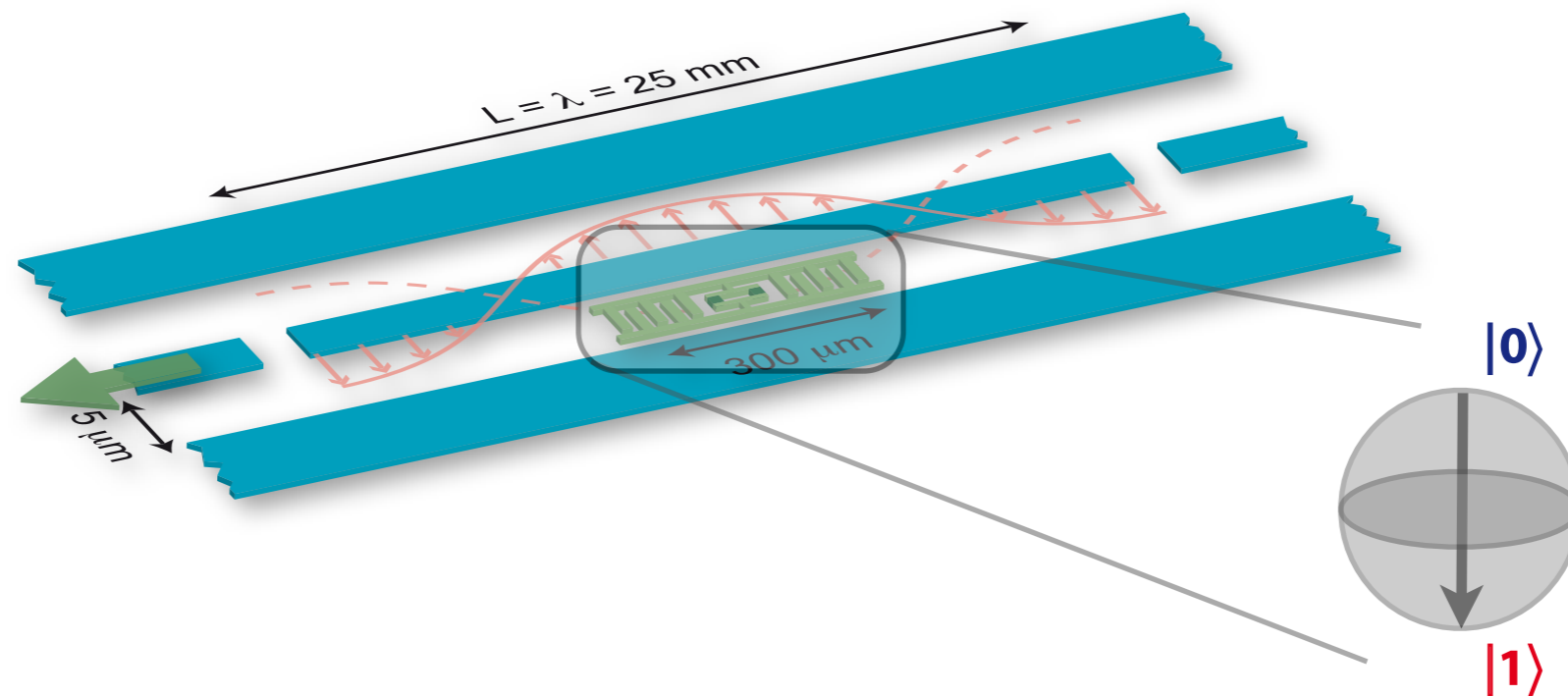


# Dispersive regime: Single-qubit gates

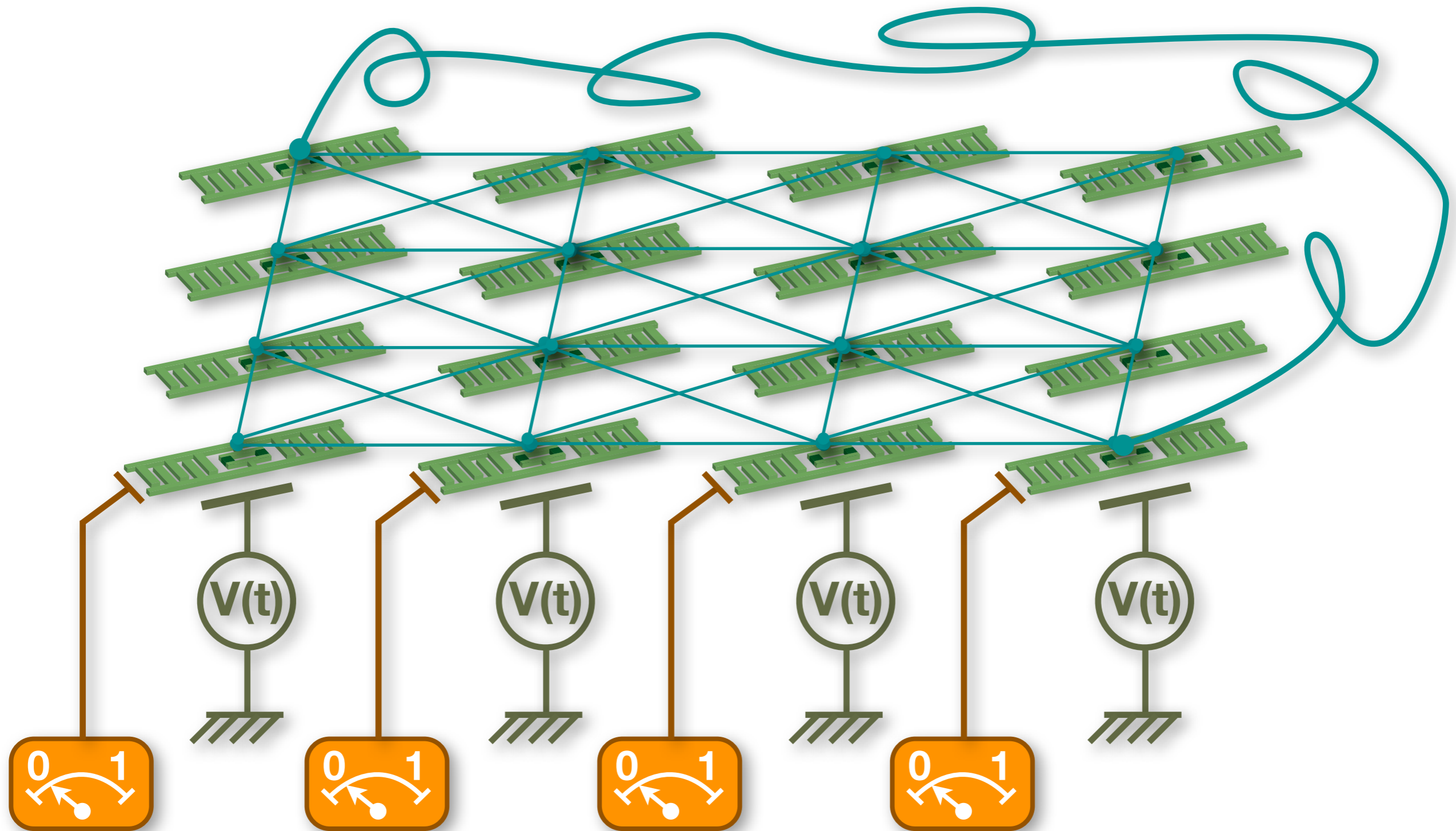
Dispersive interaction:  $H \approx (\omega_r + \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + \frac{\tilde{\omega}_{01}}{2} \hat{\sigma}_z$   
( $|\Delta| = |\omega_{01} - \omega_r| \gg g$ )

Resonator frequency      Dispersive interaction strength      Qubit transition frequency

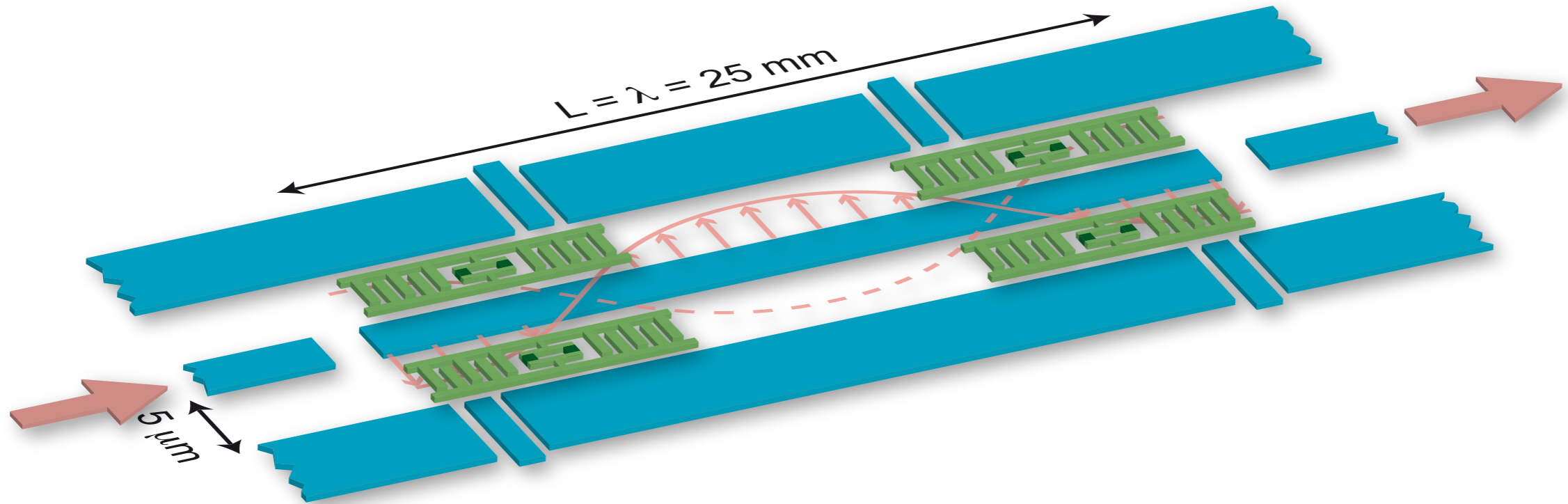
$\chi = g^2 / \Delta$



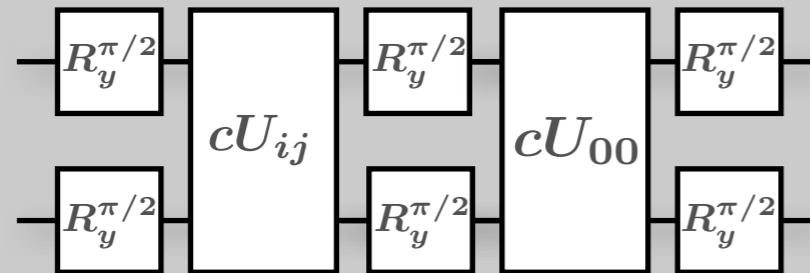
# Quantum information processing ~~with~~ *without* circuit QED



# Quantum information processing with circuit QED



**First two-qubit quantum algorithms implemented in a solid-state quantum information processor**



L. DiCarlo *et al*, Nature **460**, 240 (2009)

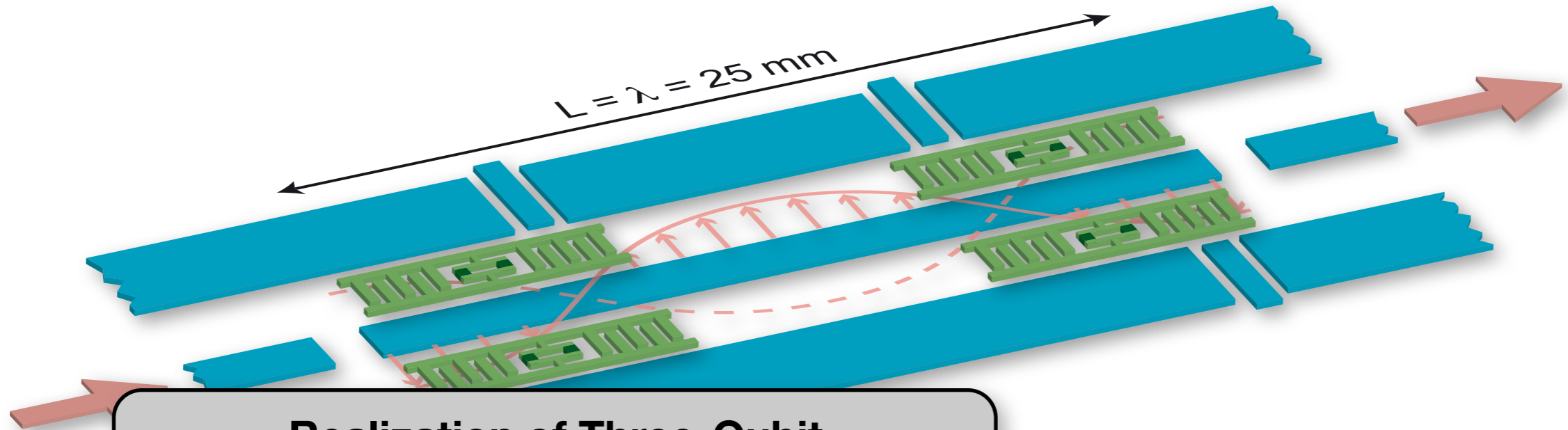
**3-qubit entanglement generated with 88% fidelity**

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)$$

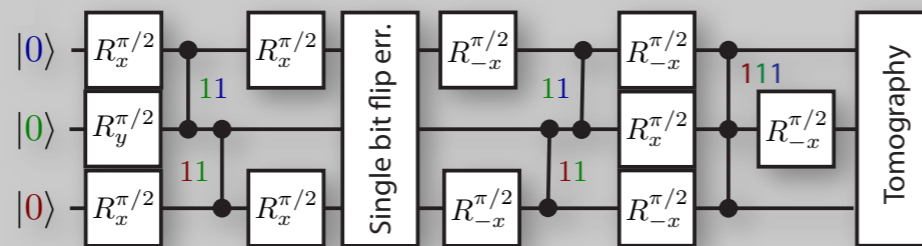
L. DiCarlo *et al*, Nature **467**, 574 (2010)



# Quantum information processing with circuit QED

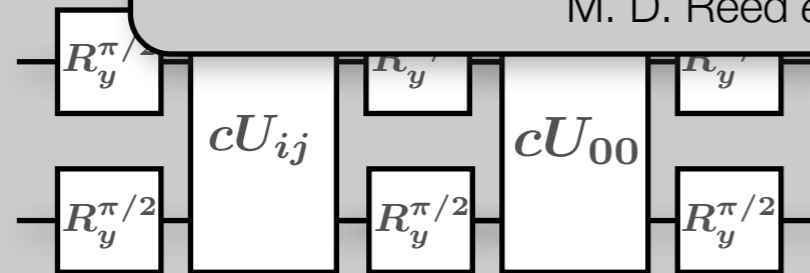


## Realization of Three-Qubit Quantum Error Correction



M. D. Reed *et al*, Nature **482**, 382 (2011)

First two  
impleme  
i



L. DiCarlo *et al*, Nature **460**, 240 (2009)

GHZ bit entanglement generated  
with 88% fidelity

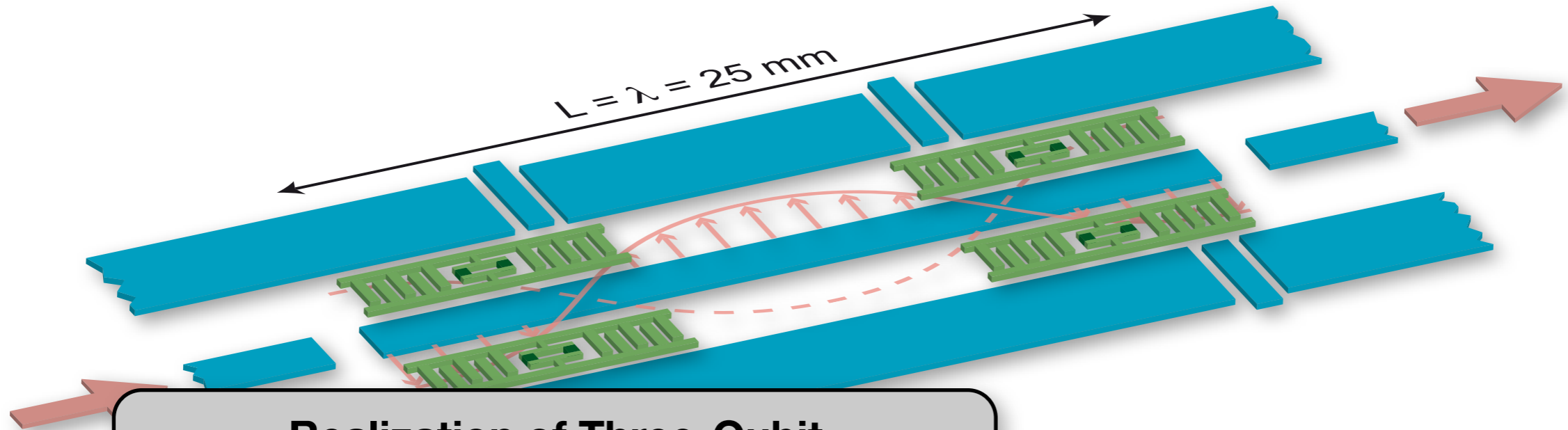
$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)$$

L. DiCarlo *et al*, Nature **467**, 574 (2010)

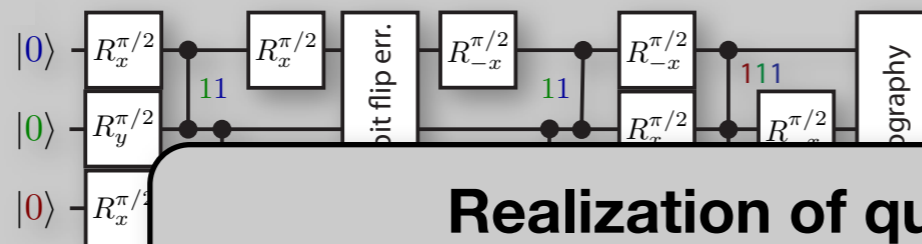
Quantum gates: A. Blais *et al*, Phys. Rev. A **75**, 032329 (2007)

Multi-qubit readout: S. Filipp *et al*, Phys. Rev. Lett. **102**, 200402 (2009)

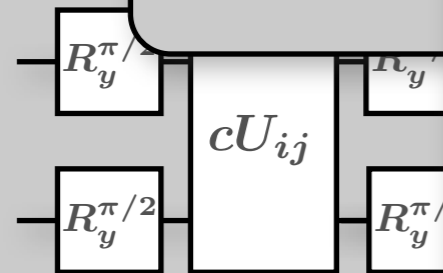
# Quantum information processing with circuit QED



## Realization of Three-Qubit Quantum Error Correction

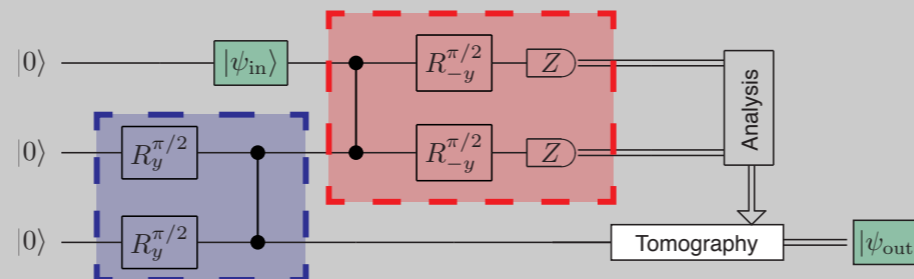


First two  
impleme  
i



L. DiCarlo

## Realization of quantum teleportation



L. Steffen *et al*, ArXiv: arXiv:1302.5621

ment generated  
o fidelity

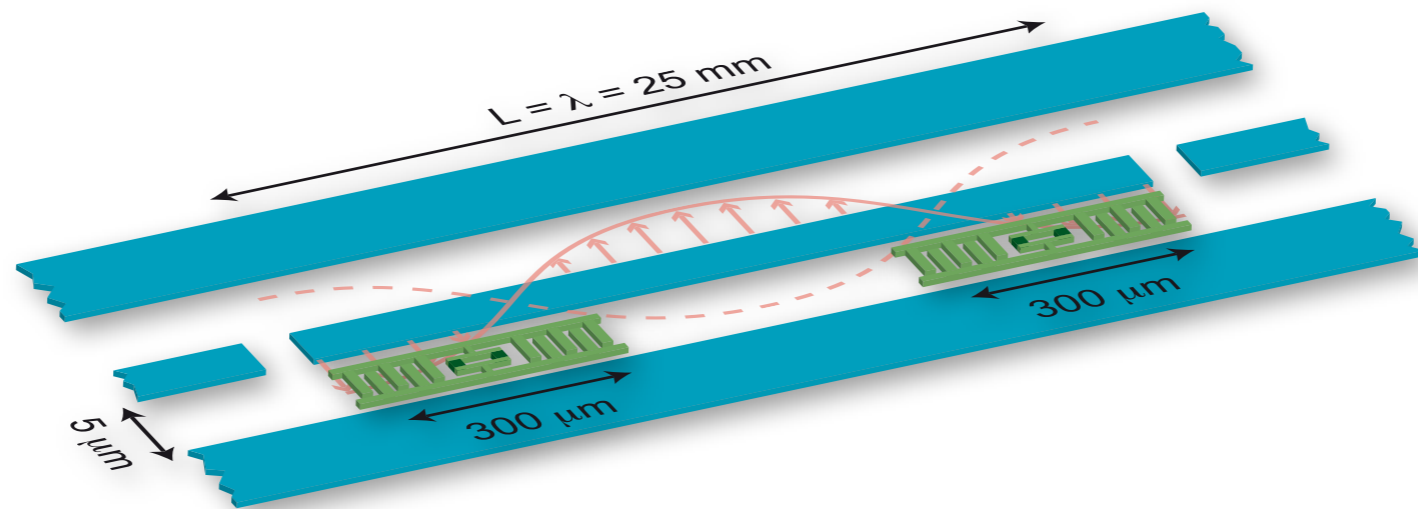
$$|000\rangle + |111\rangle$$

*et al*, Nature **467**, 574 (2010)

Quantum gates: A. Brats *et al*, Phys. Rev. A **75**, 032329 (2007)

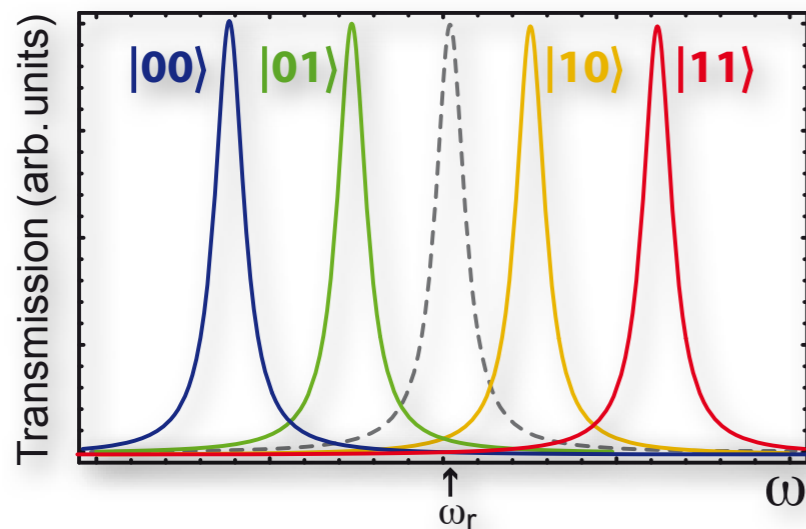
Multi-qubit readout: S. Filipp *et al*, Phys. Rev. Lett. **102**, 200402 (2009)

# Multi-qubit circuit QED



Dispersive interaction:  $H = (\omega_r + \hat{\chi}) a^\dagger a + \frac{\omega_{a1}}{2} \sigma_{z1} + \frac{\omega_{a2}}{2} \sigma_{z2}$

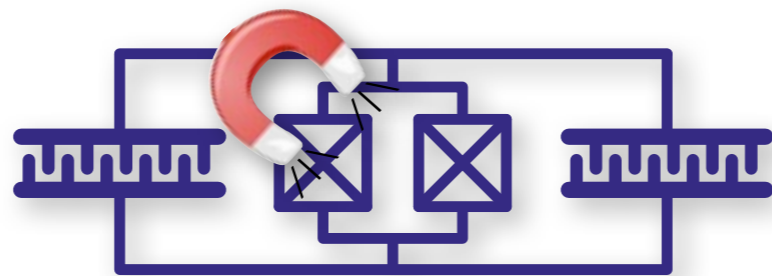
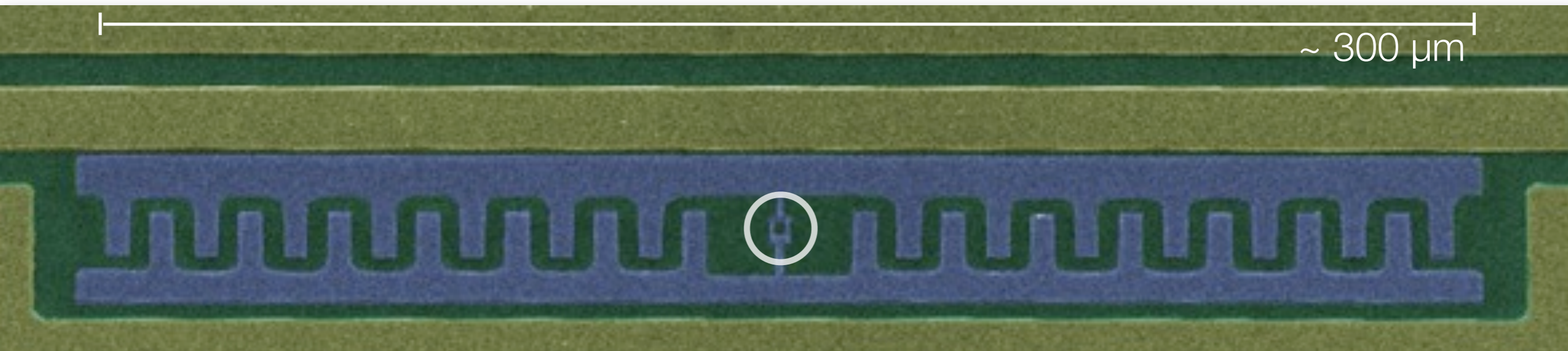
$$\hat{\chi} = \chi_1 \sigma_{z1} + \chi_2 \sigma_{z2}$$



- Simultaneous qubit readout
- Single-qubit gates in parallel
- Weak measurement

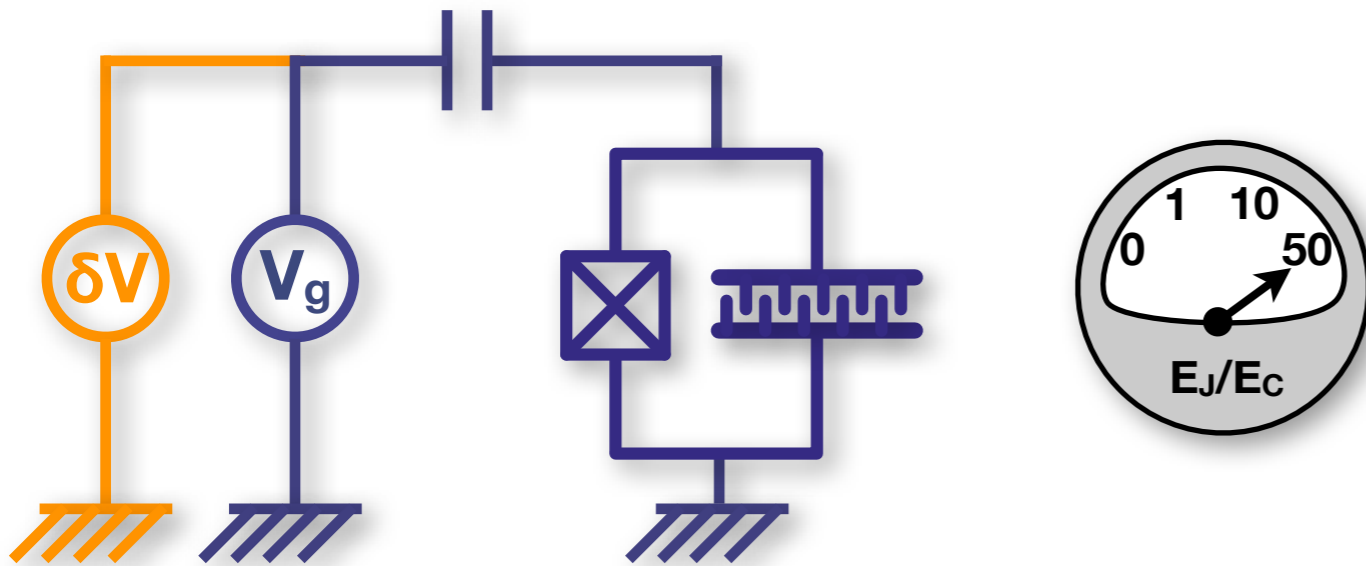
# Superconducting qubits: transmon regime

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Qubit transition frequency  
can be changed by  $\sim 1$  GHz  
in a few ns

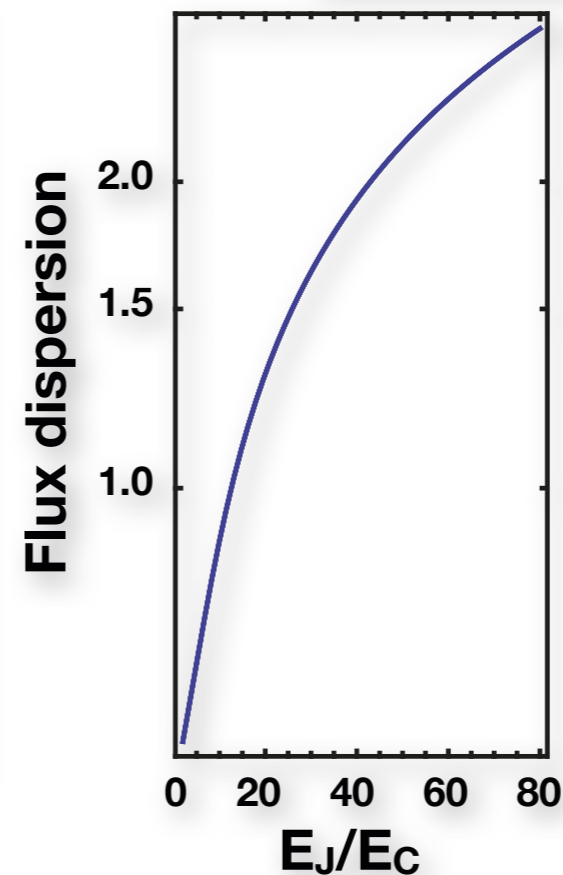
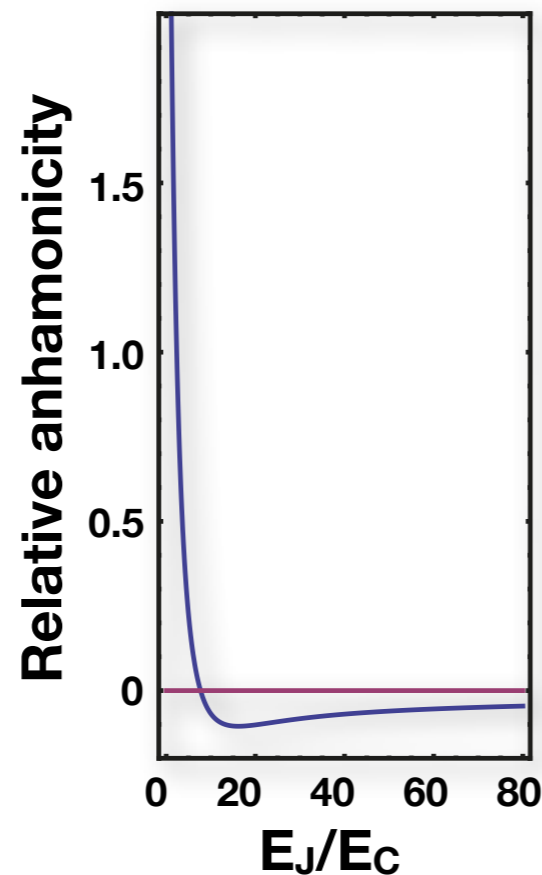
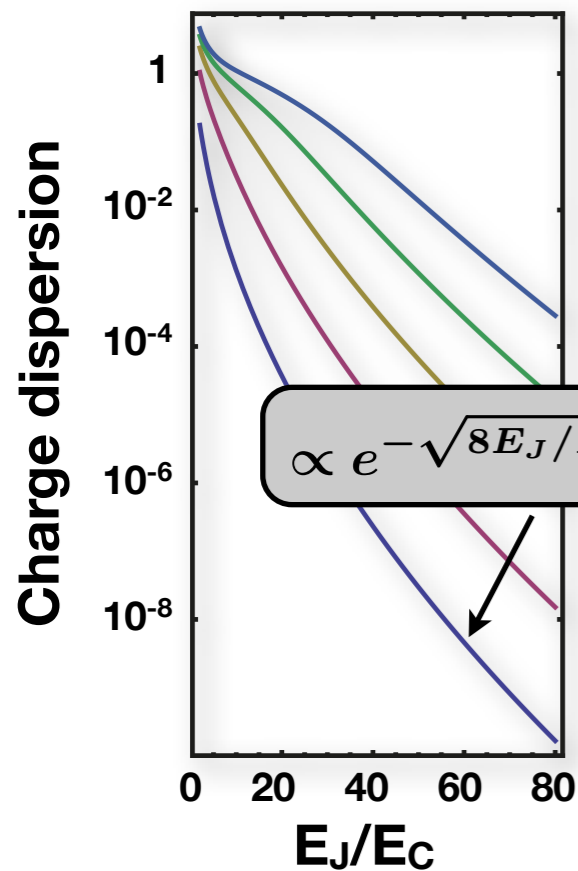
# Flux dispersion



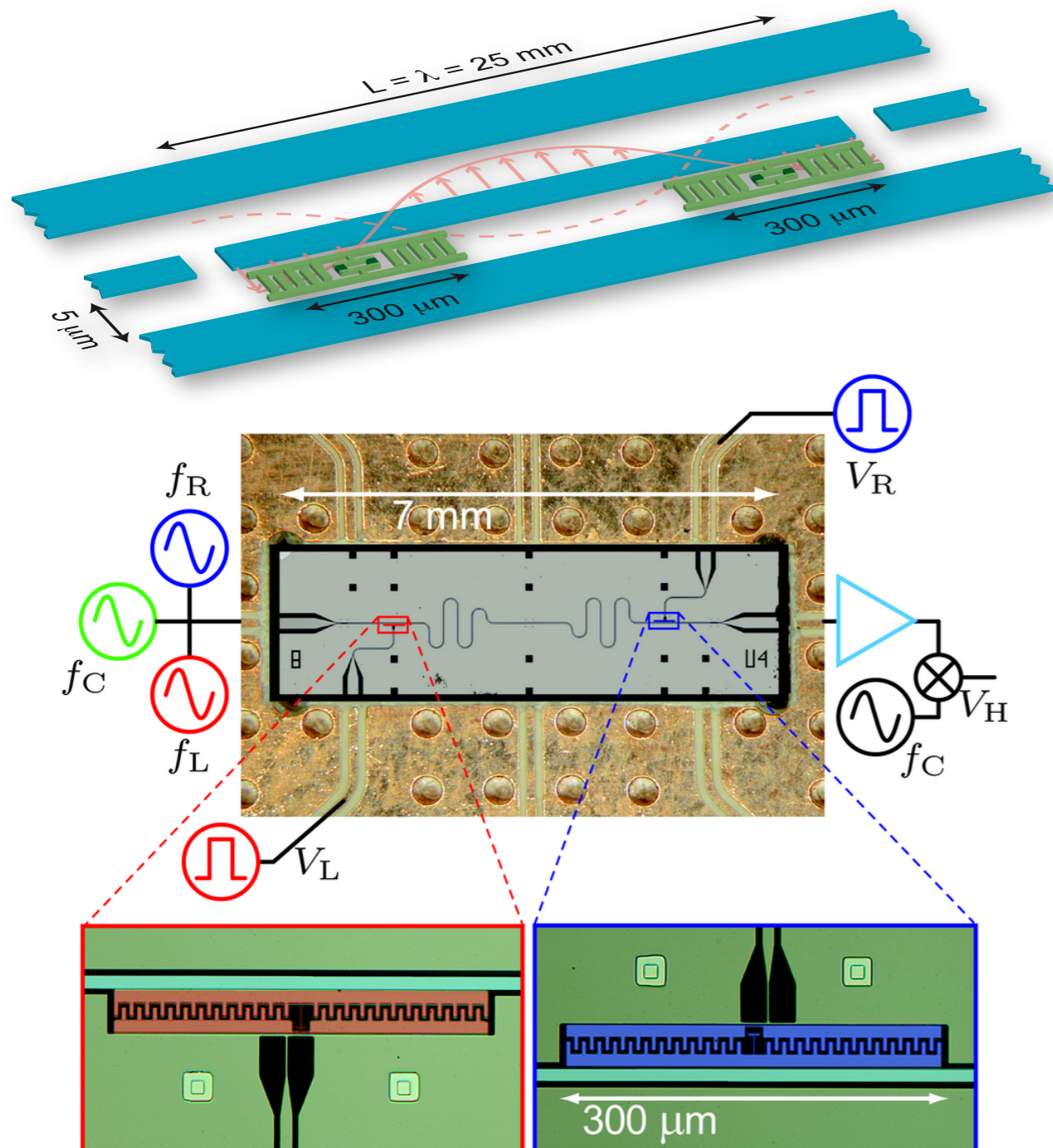
$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos \hat{\phi}$$

Flux in the SQUID loop:

$$E_J \rightarrow E_J \cos \left( \frac{\pi \Phi_x}{\Phi_0} \right)$$



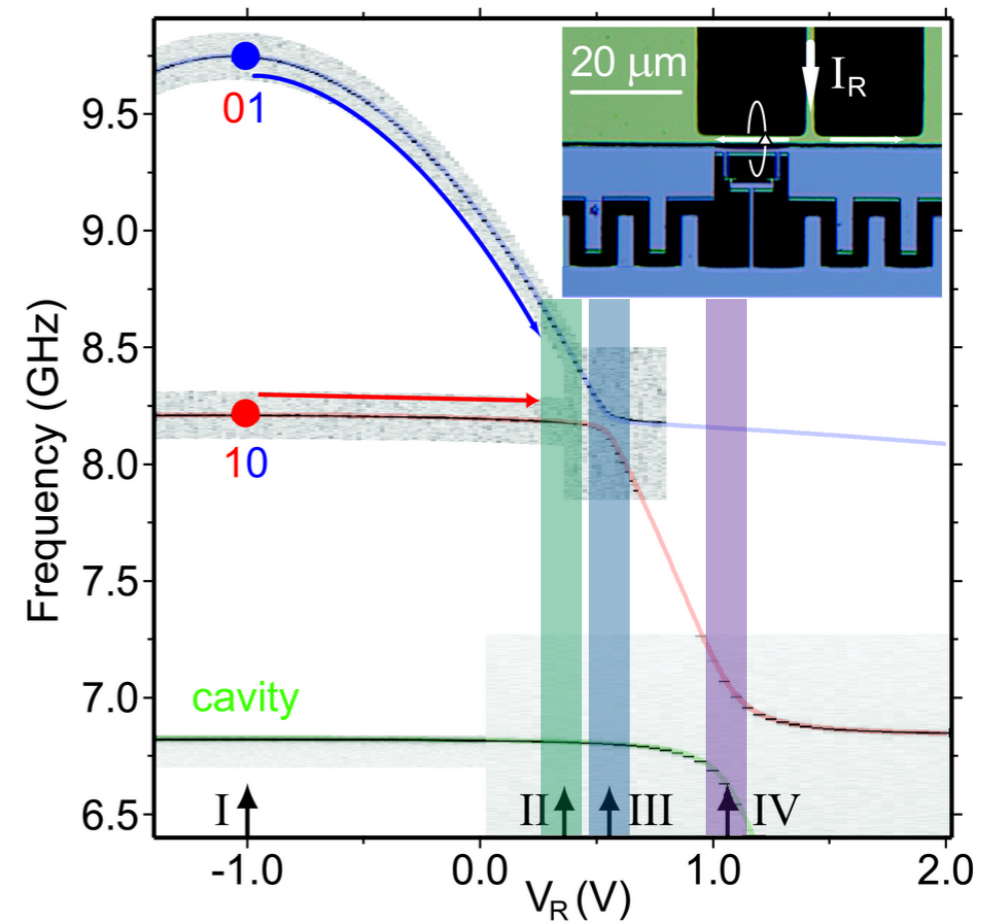
# Generating highly entangled states



$T_{1L} \sim 1.3 \mu\text{s}$   
 $T_{2L} \sim 1.8 \mu\text{s}$

$T_{1R} \sim 0.8 \mu\text{s}$   
 $T_{2R} \sim 1.2 \mu\text{s}$

Flux-tunable frequencies:

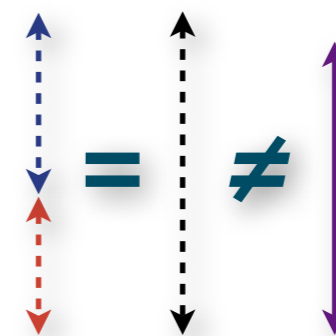
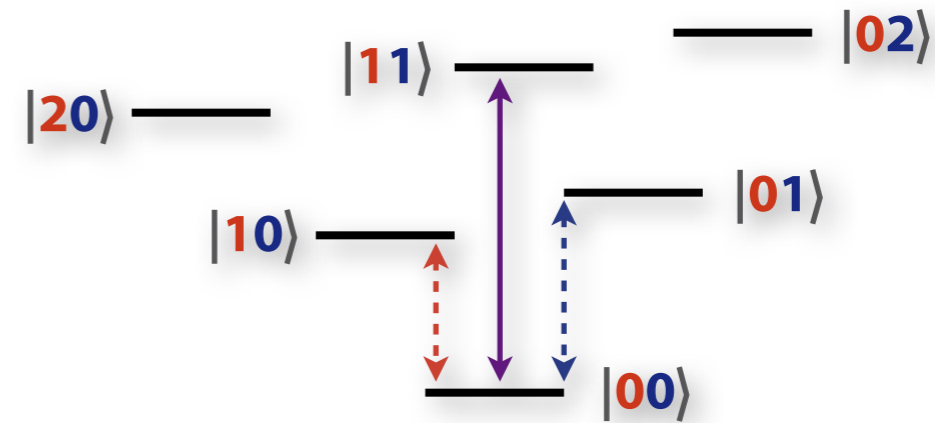
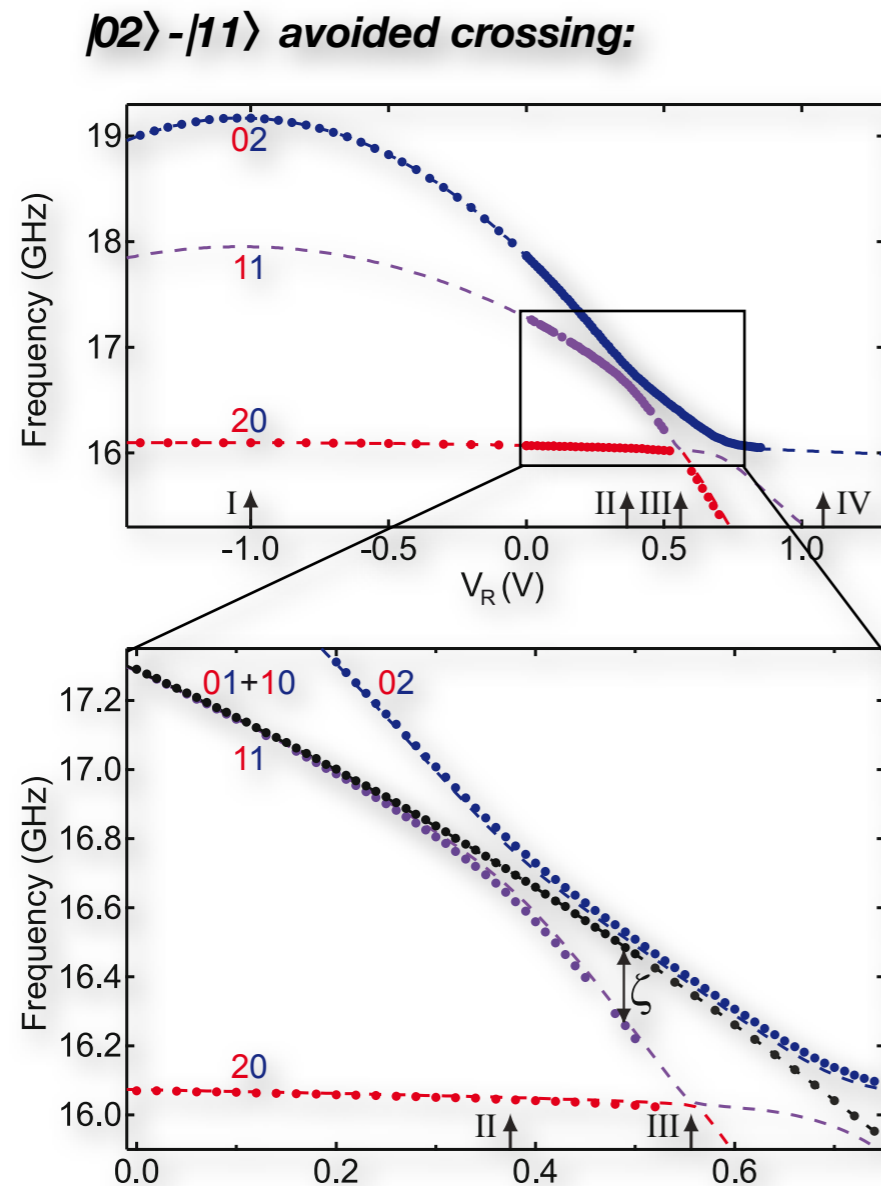
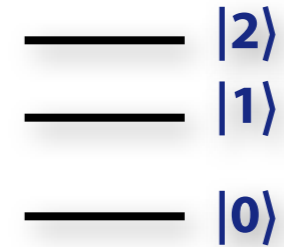


**Quantum bus**  
 J. Majer *et al.* Nature **449**,  
 443 (2007)

**Vacuum Rabi Splitting**  
 Wallraff *et al.* Nature **431**,  
 162 (2004)

# Generating highly entangled states

Transmon qubit: a slightly anharmonic oscillator

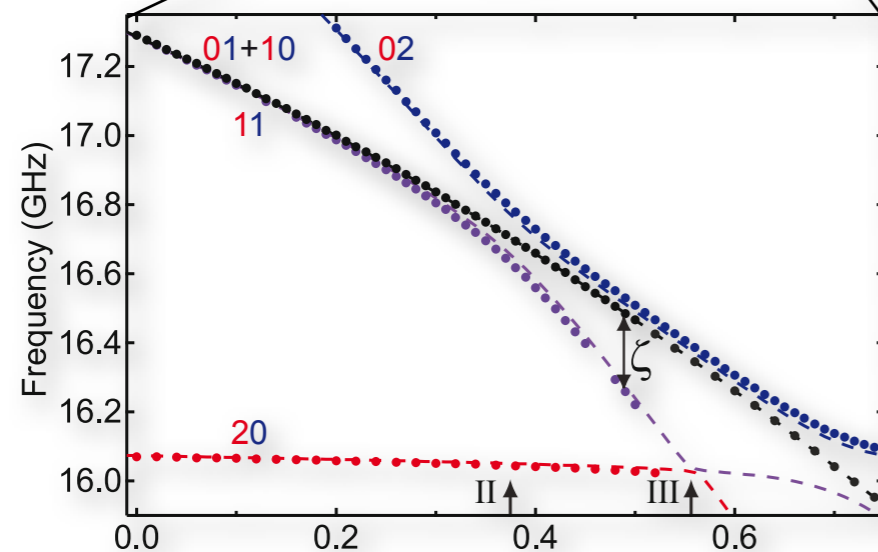
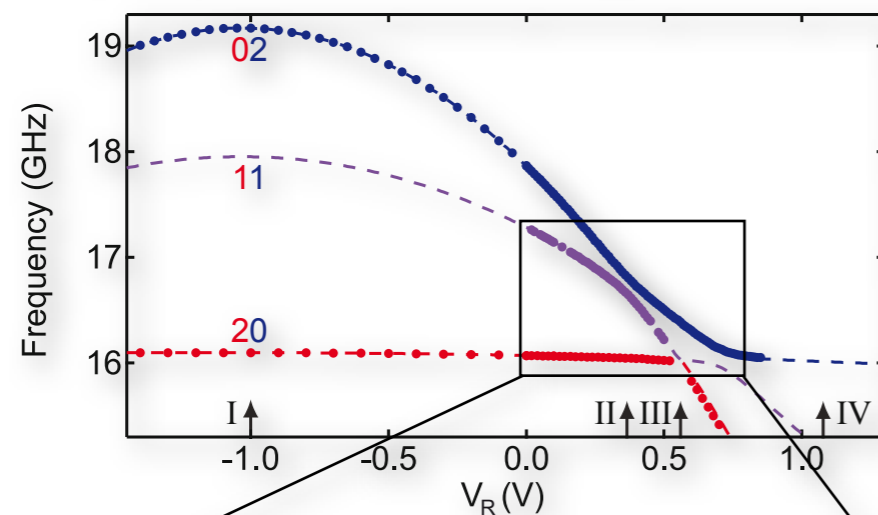


# Generating highly entangled states

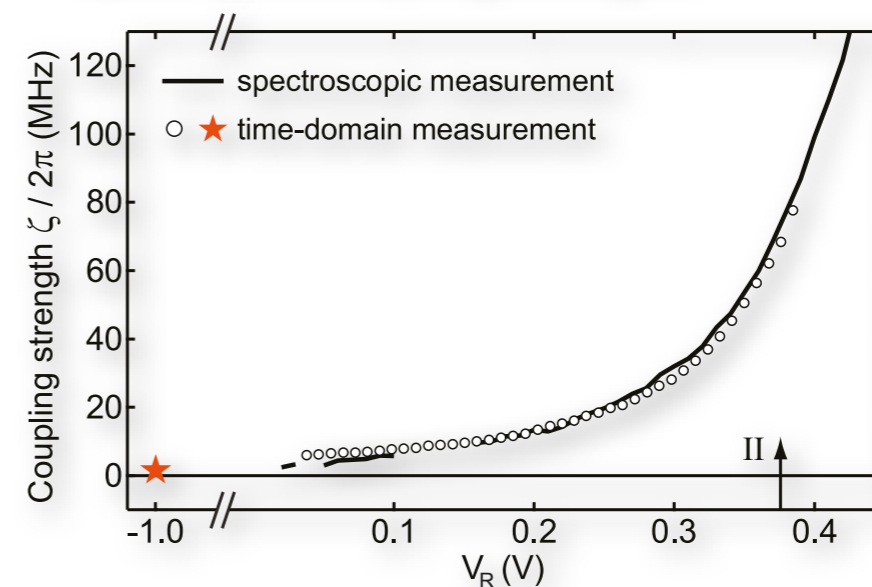
Transmon qubit: a slightly anharmonic oscillator

—  $|2\rangle$   
—  $|1\rangle$   
—  $|0\rangle$

$|02\rangle - |11\rangle$  avoided crossing:



Qubit-qubit coupling strength:



Conditional phase gate:

$$cU = \begin{pmatrix} e^{i\phi_{00}} & 0 & 0 & 0 \\ 0 & e^{i\phi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\phi_{10}} & 0 \\ 0 & 0 & 0 & e^{i\phi_{11}} \end{pmatrix}$$

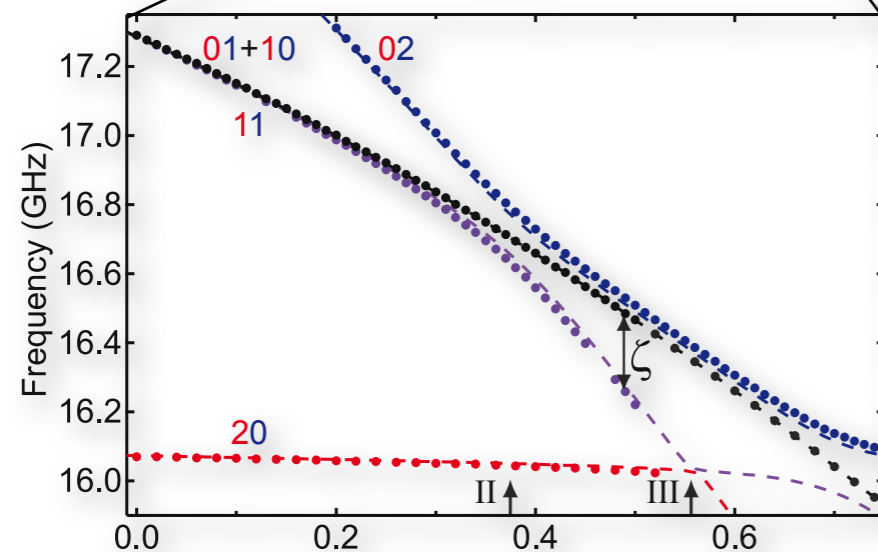
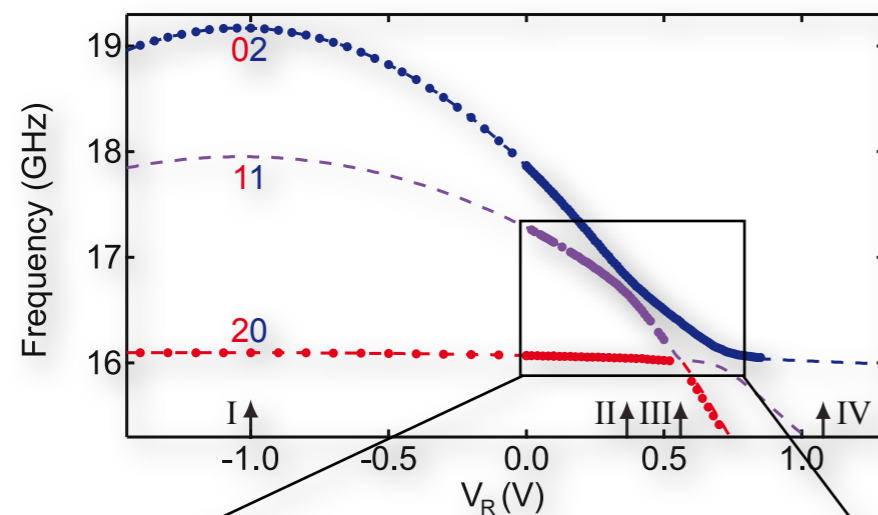


# Generating highly entangled states

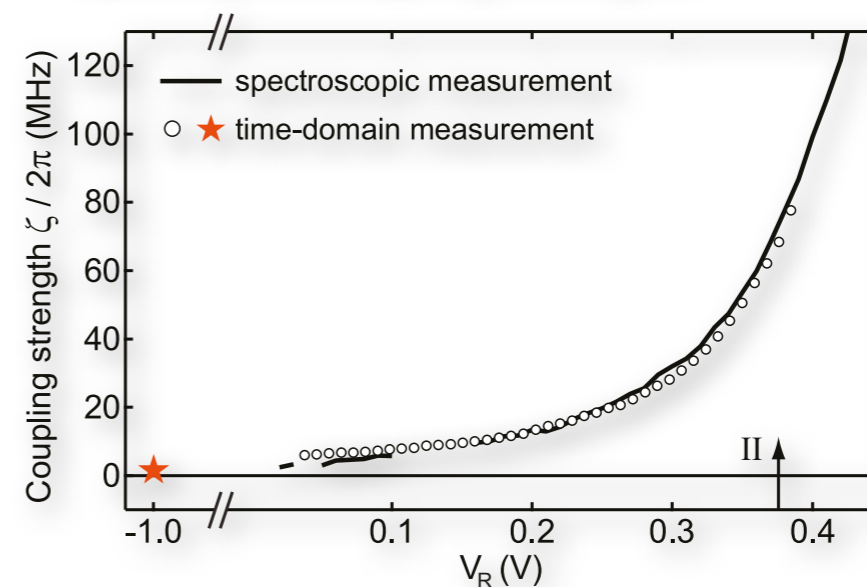
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Conditional phase gate:

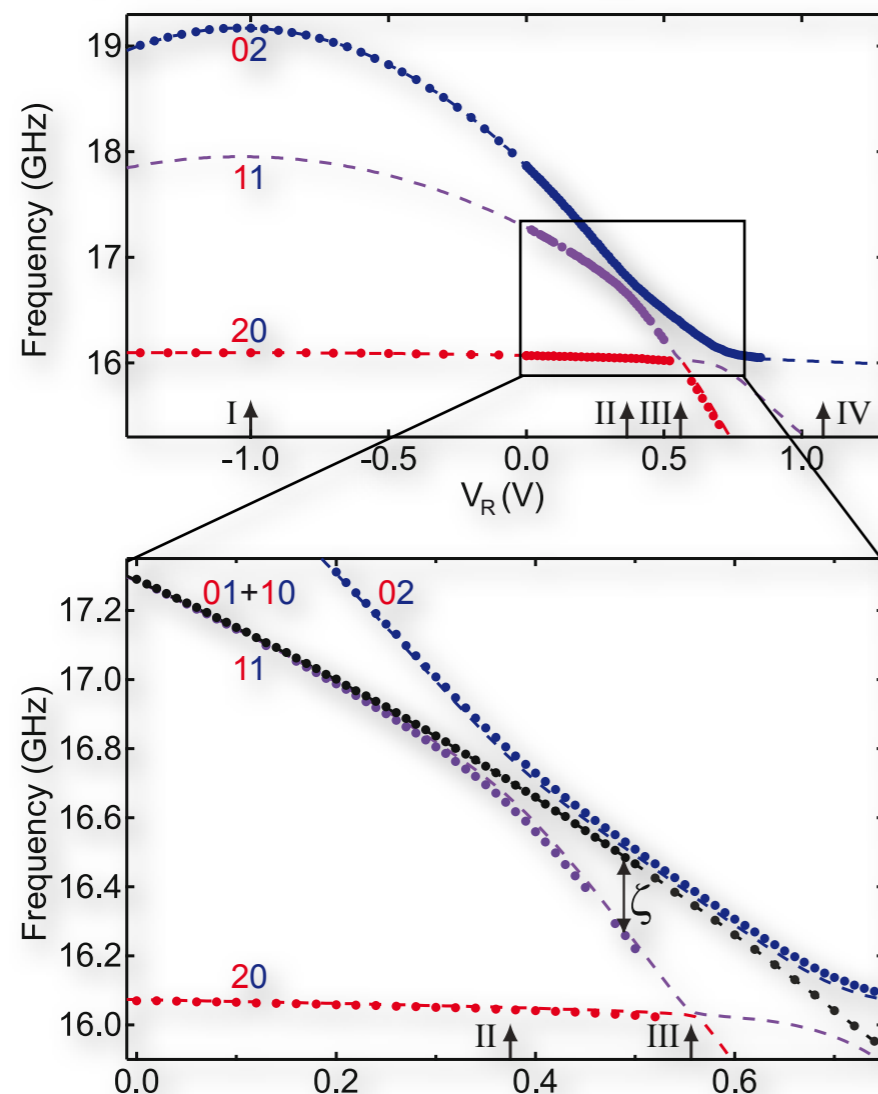
$$cU_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

# Generating highly entangled states

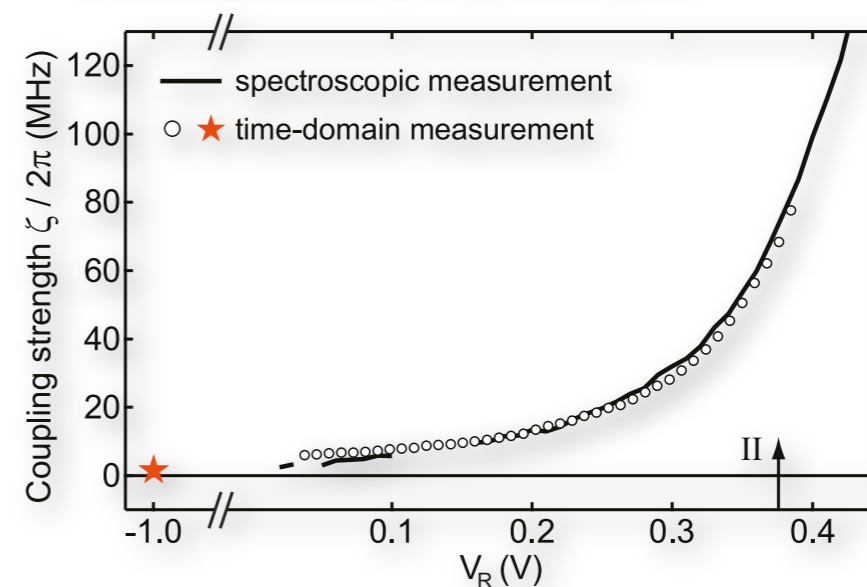
Transmon qubit: a slightly anharmonic oscillator

—  $|2\rangle$   
—  $|1\rangle$   
—  $|0\rangle$

$|02\rangle - |11\rangle$  avoided crossing:



Qubit-qubit coupling strength:



Conditional phase gate:

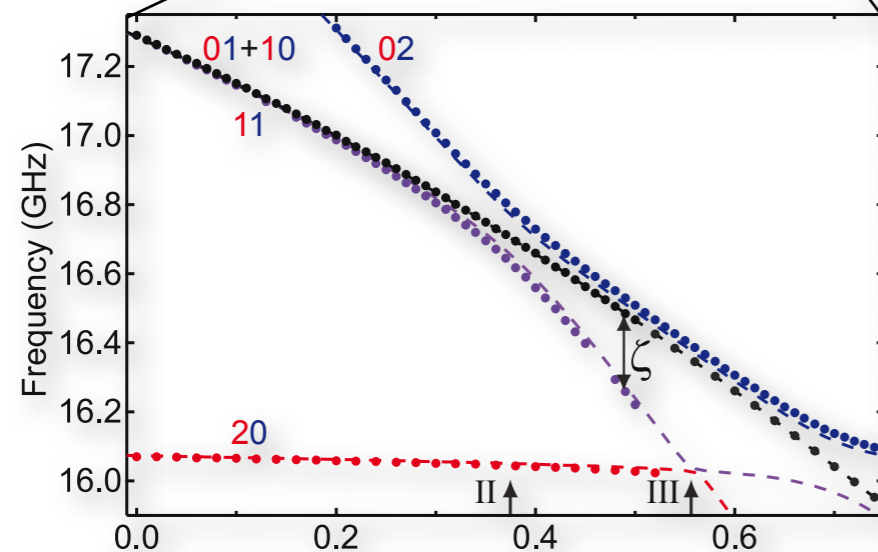
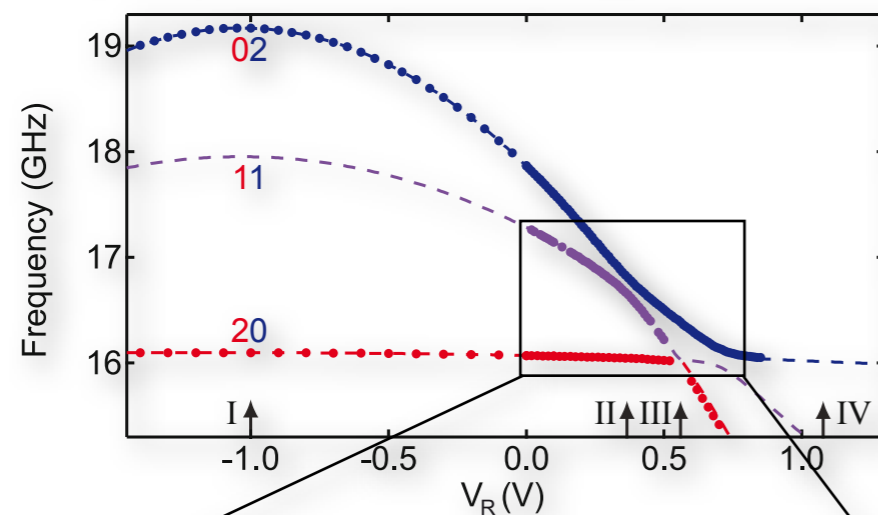
$$cU_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Generating highly entangled states

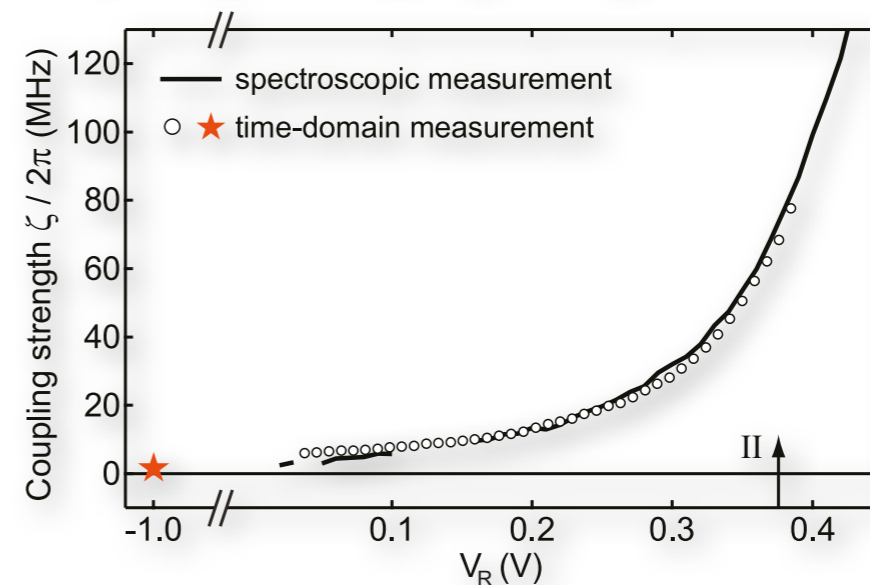
Transmon qubit: a slightly anharmonic oscillator

—  $|2\rangle$   
—  $|1\rangle$   
—  $|0\rangle$

$|02\rangle - |11\rangle$  avoided crossing:



Qubit-qubit coupling strength:



Conditional phase gate:

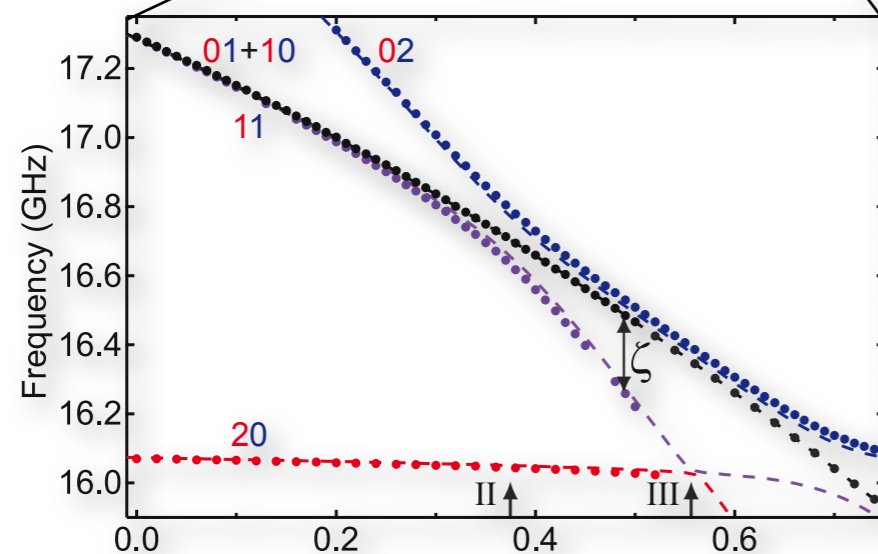
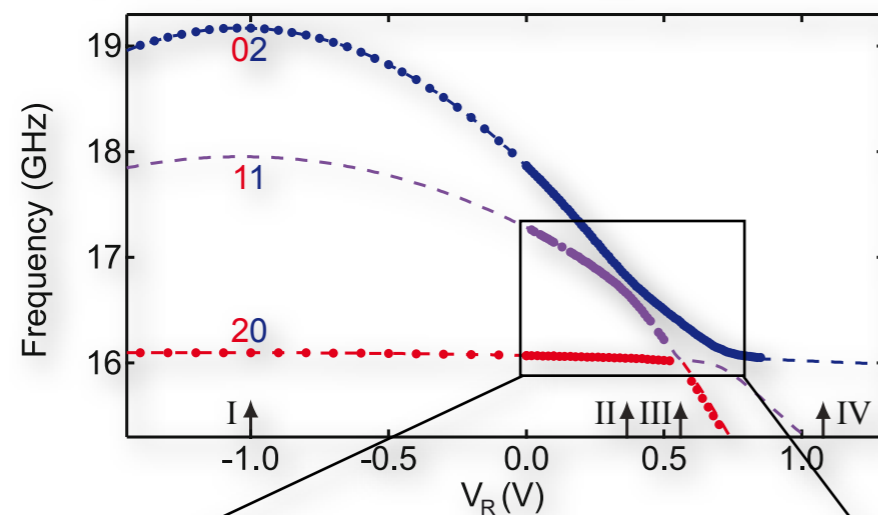
$$cU_{01} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Generating highly entangled states

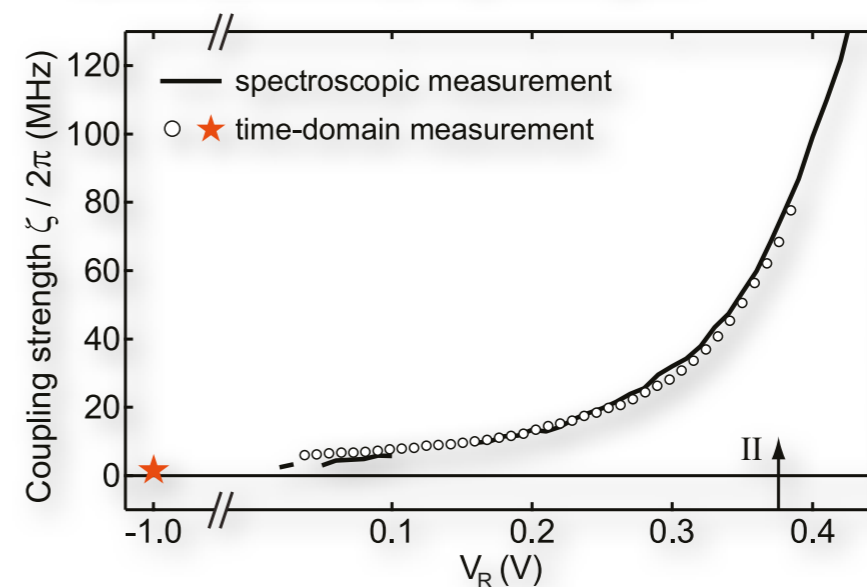
Transmon qubit: a slightly anharmonic oscillator

—  $|2\rangle$   
—  $|1\rangle$   
—  $|0\rangle$

$|02\rangle - |11\rangle$  avoided crossing:



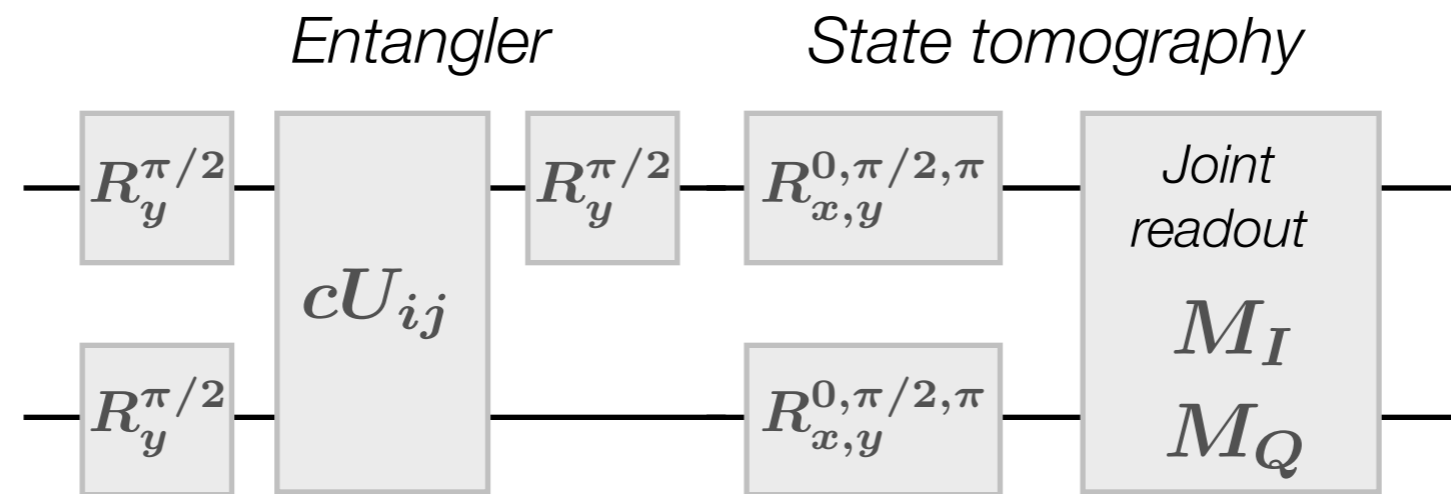
Qubit-qubit coupling strength:



Conditional phase gate:

$$cU_{00} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Generating highly entangled states

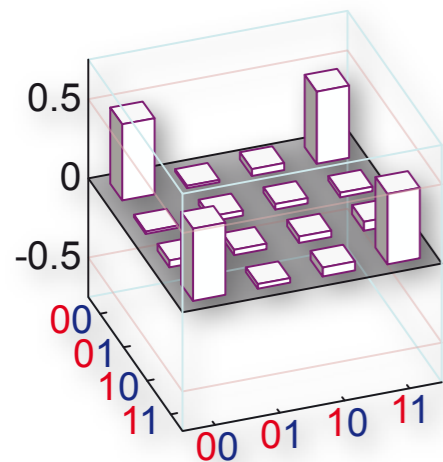


$$|\Psi^+\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle)$$

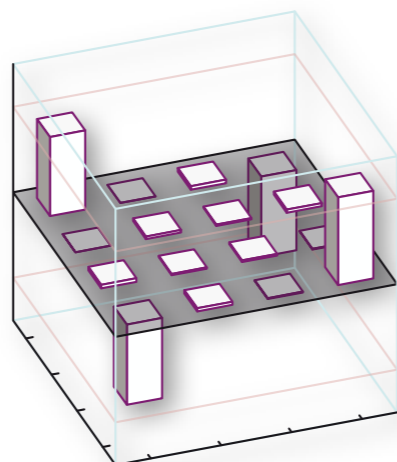
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)$$



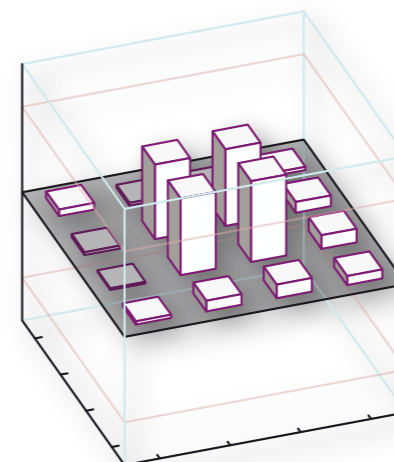
$$F = 0.91 \pm 0.01$$

$$C = 0.88 \pm 0.02$$



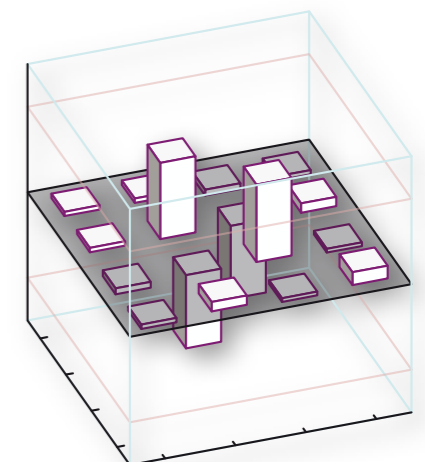
$$F = 0.94 \pm 0.01$$

$$C = 0.94 \pm 0.01$$



$$F = 0.90 \pm 0.01$$

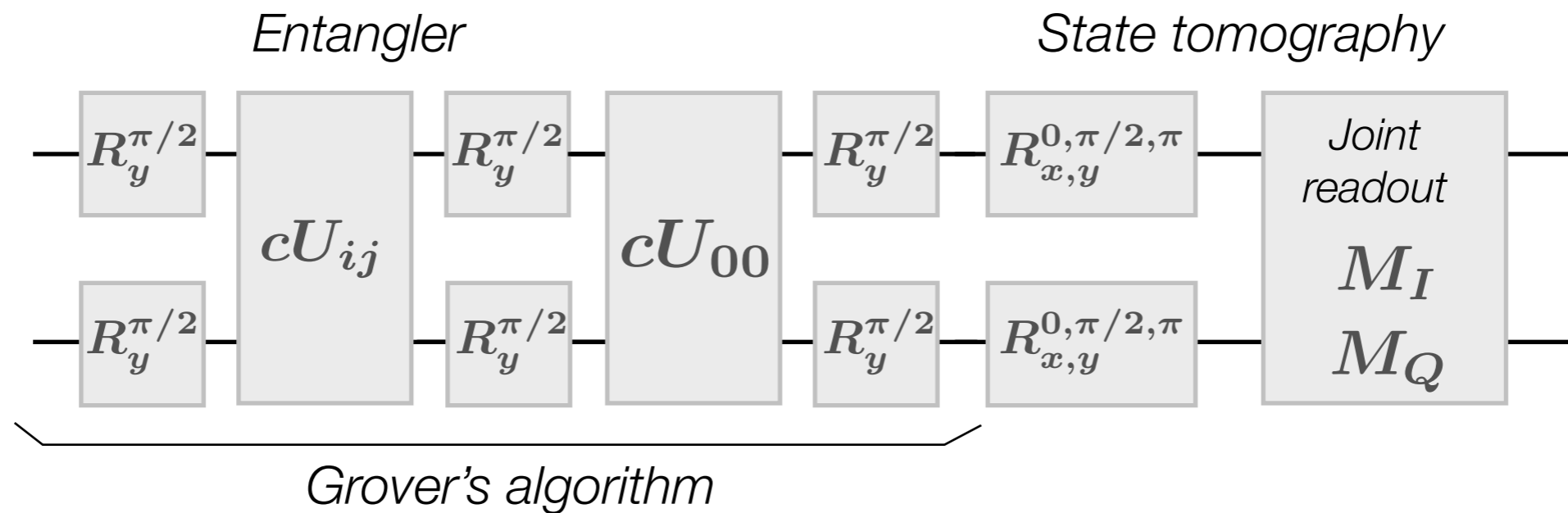
$$C = 0.86 \pm 0.02$$



$$F = 0.87 \pm 0.02$$

$$C = 0.81 \pm 0.04$$

# Quantum searching

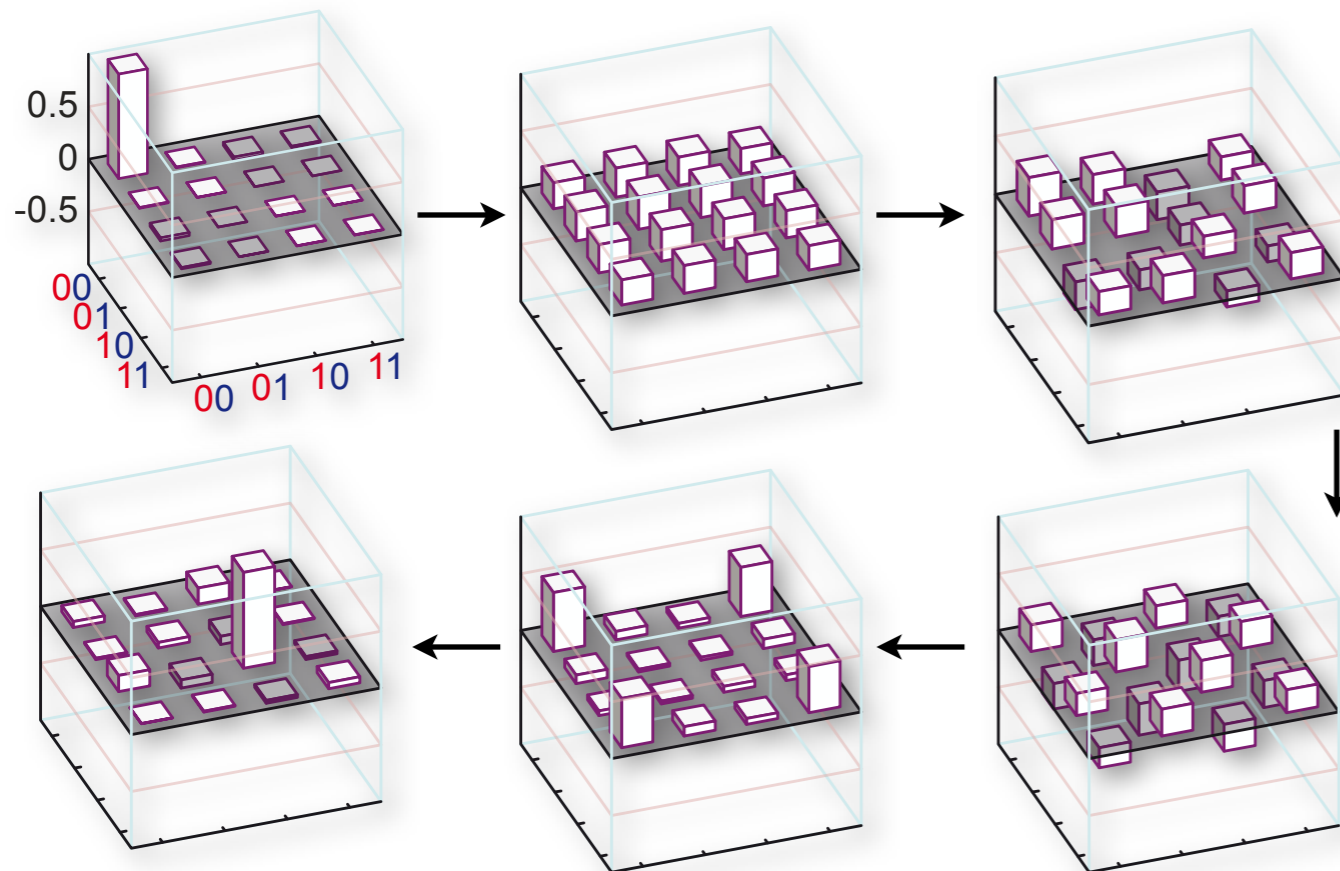
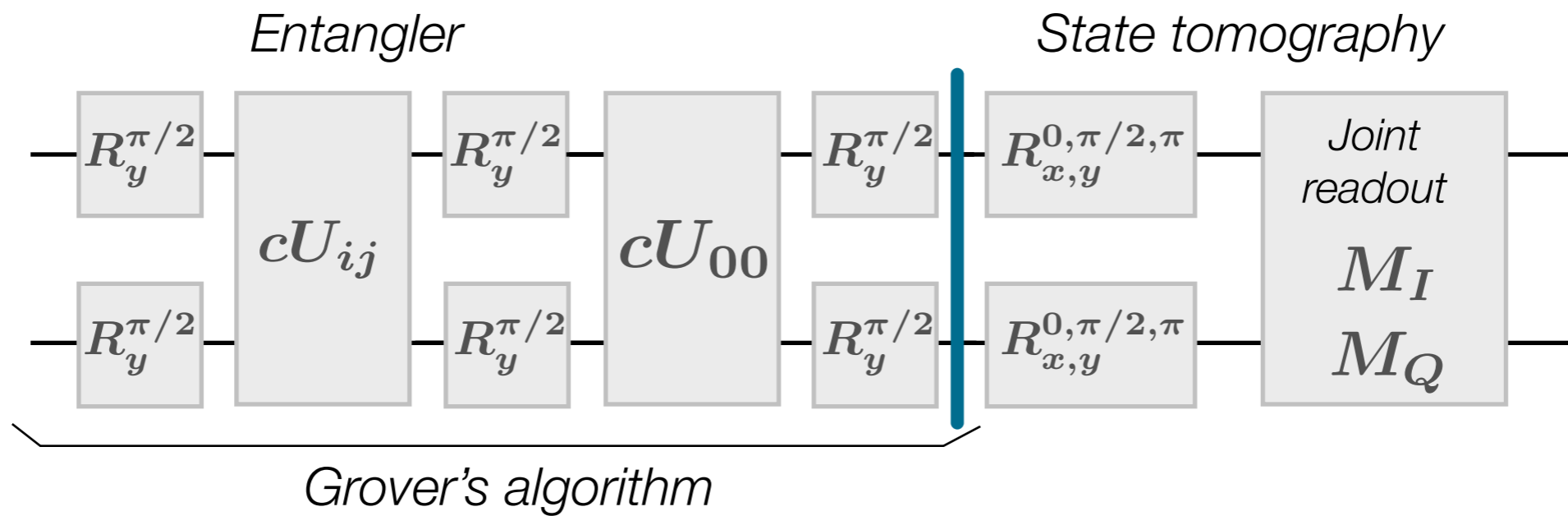


Determine  $x_0$  in the set  $x = \{00,01,10,11\}$  which is such that

$$f_{ij}(x) = \begin{cases} -1 & x = x_0 \\ +1 & \text{otherwise} \end{cases}$$

Classical:  $O(N)$   
Quantum:  $O(\sqrt{N})$

# Quantum searching



Fidelity = 0.85  
(Consistent with  $T_1$ )

*Deutsch–Jozsa algorithm also implemented*

A photograph of a circular electrical meter, possibly a voltmeter or ammeter, with a scale for AMPS and VOLTS. The meter is mounted on a light-colored surface, possibly a wall or panel, and is secured by four screws. The scale has markings for 10, 20, 40, and 80. The needle is positioned between 20 and 40. The text "Qubit readout in cQED" is overlaid on the meter in a bold, black font, with a white, torn paper effect behind it.

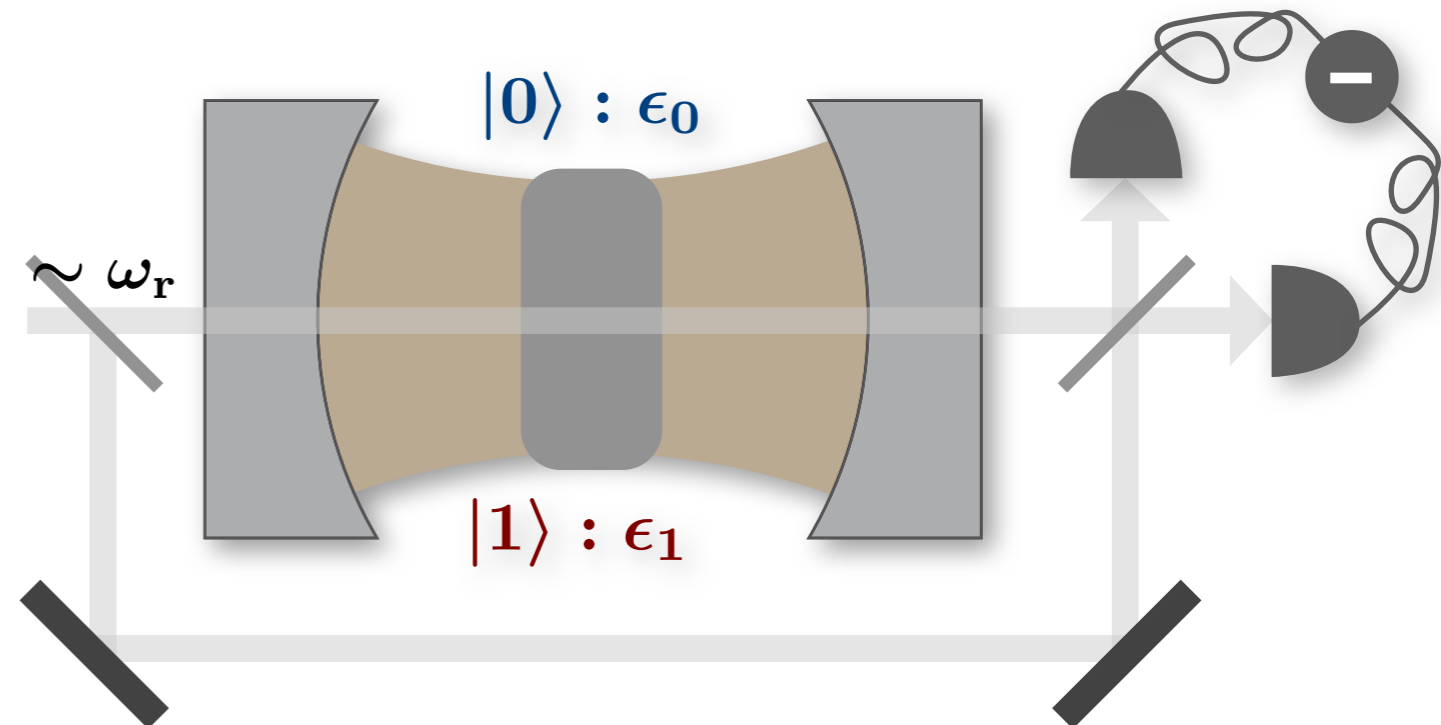
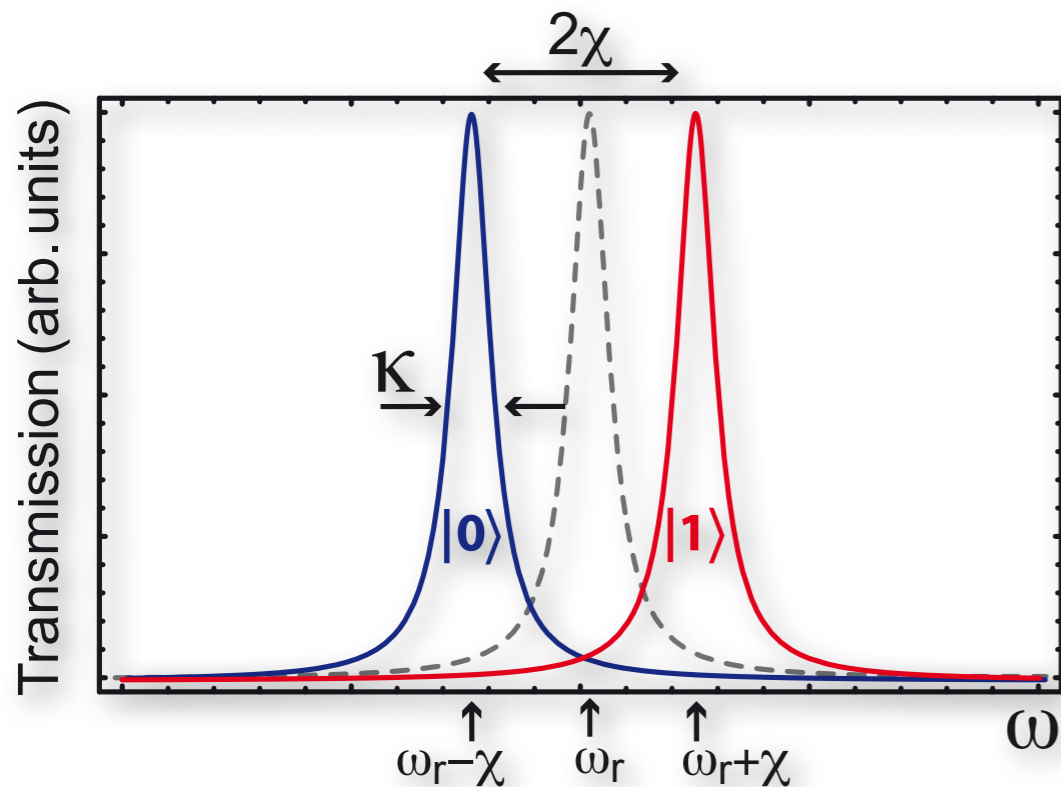
**Qubit readout in cQED**



# Dispersive regime: Qubit readout $|\Delta| = |\omega_{01} - \omega_r| \gg g$

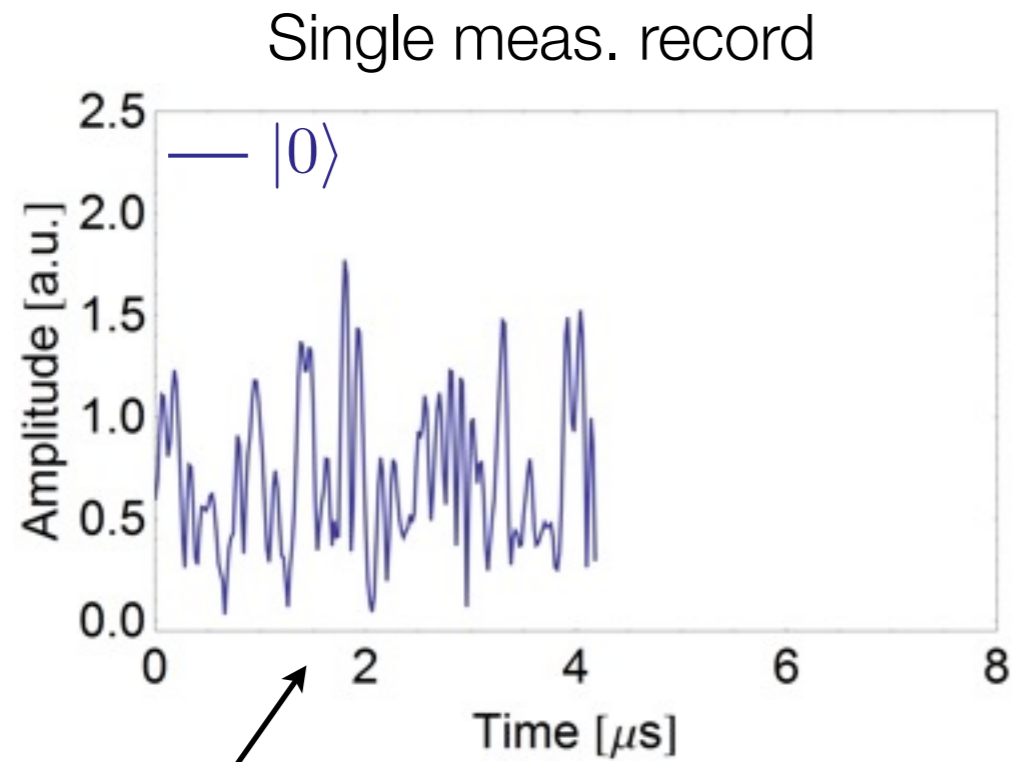
Dispersive interaction:  $H \approx (\omega_r + \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + \frac{\tilde{\omega}_{01}}{2} \hat{\sigma}_z$

$\swarrow$  Resonator frequency       $\nwarrow$  Dispersive interaction strength  $\chi = g^2 / \Delta$        $\swarrow$  Qubit transition frequency



# Signal-to-noise ratio

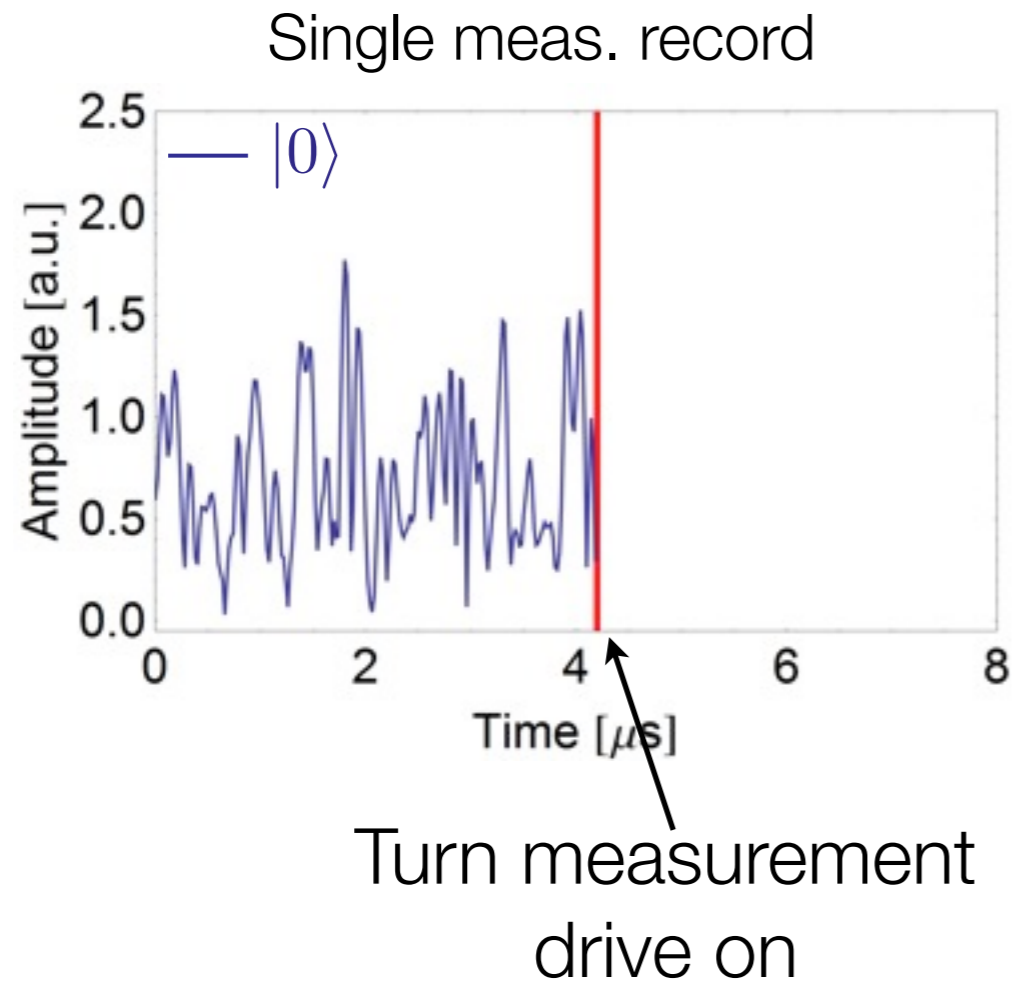
---



Measurement drive off:  
noise

# Signal-to-noise ratio

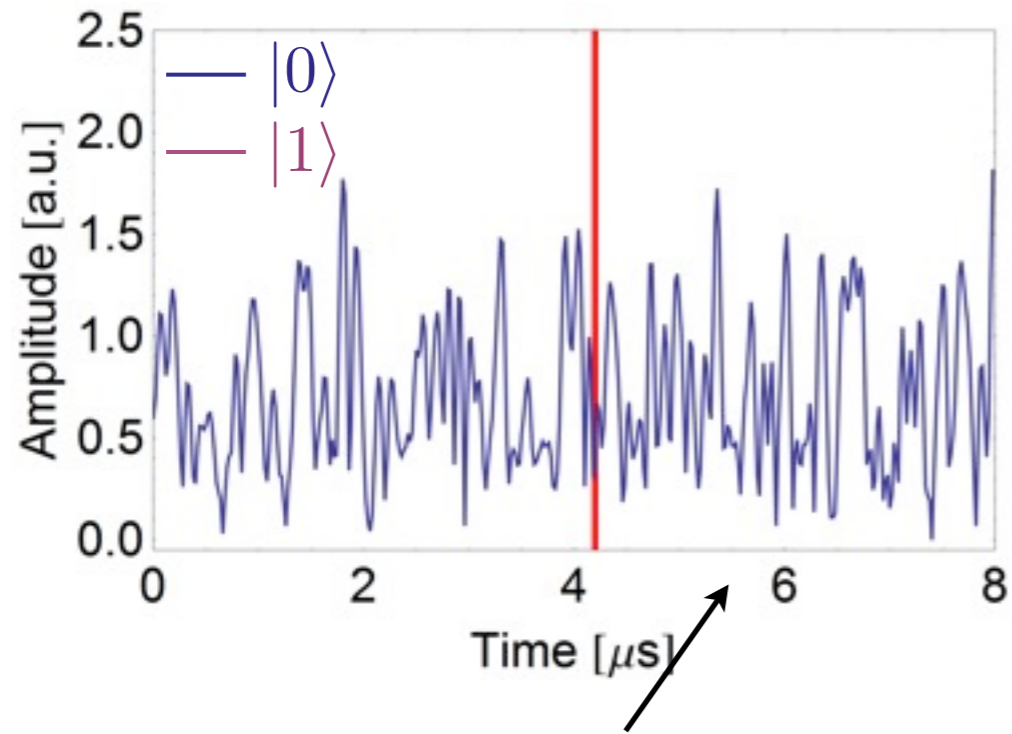
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# Signal-to-noise ratio

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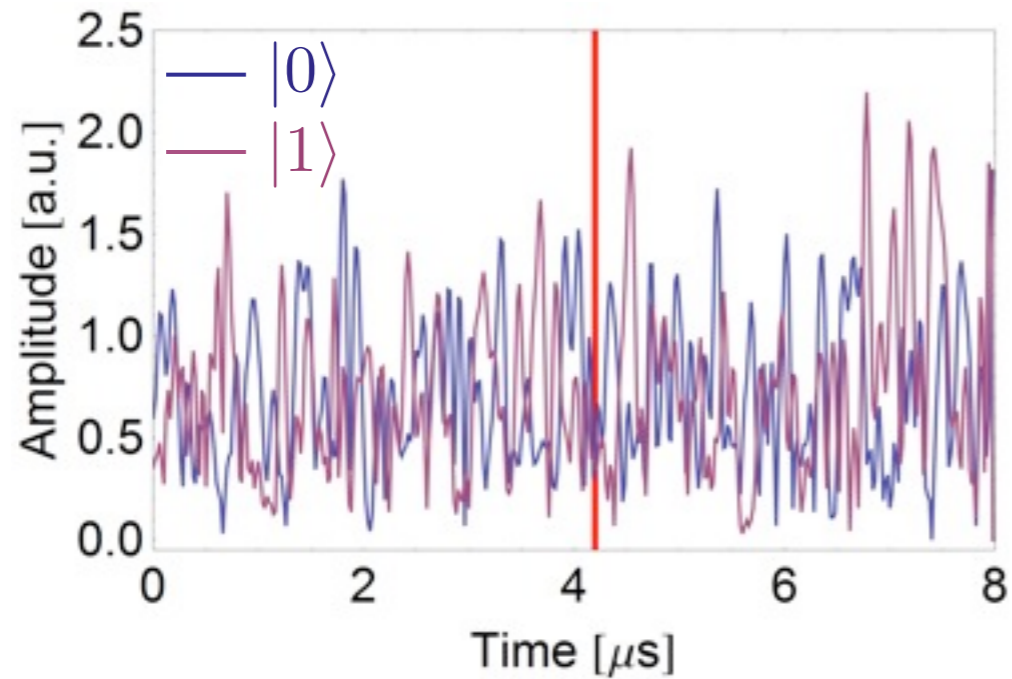
Single meas. record



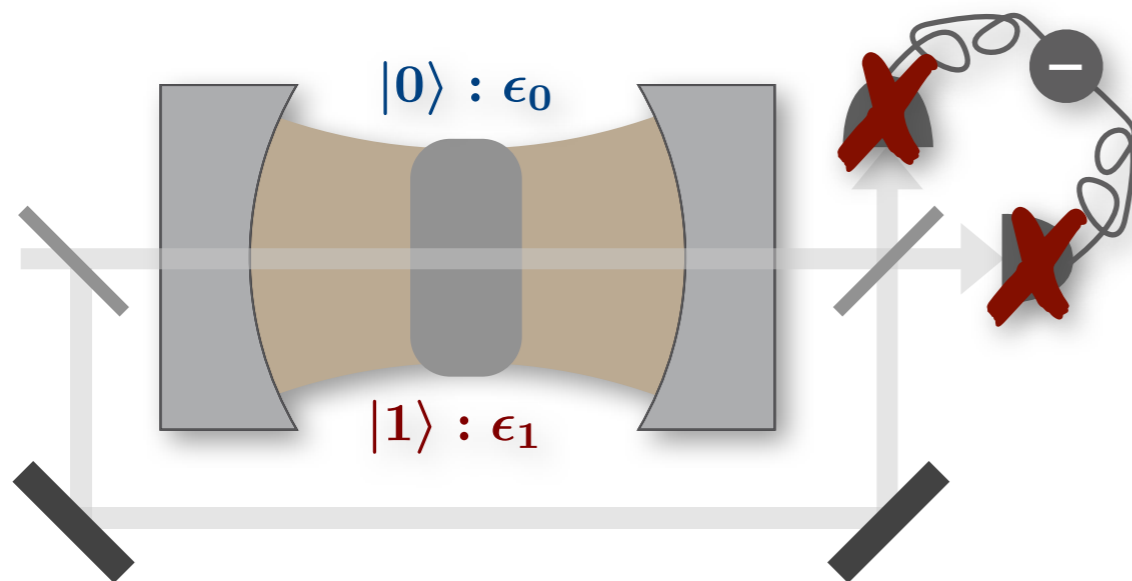
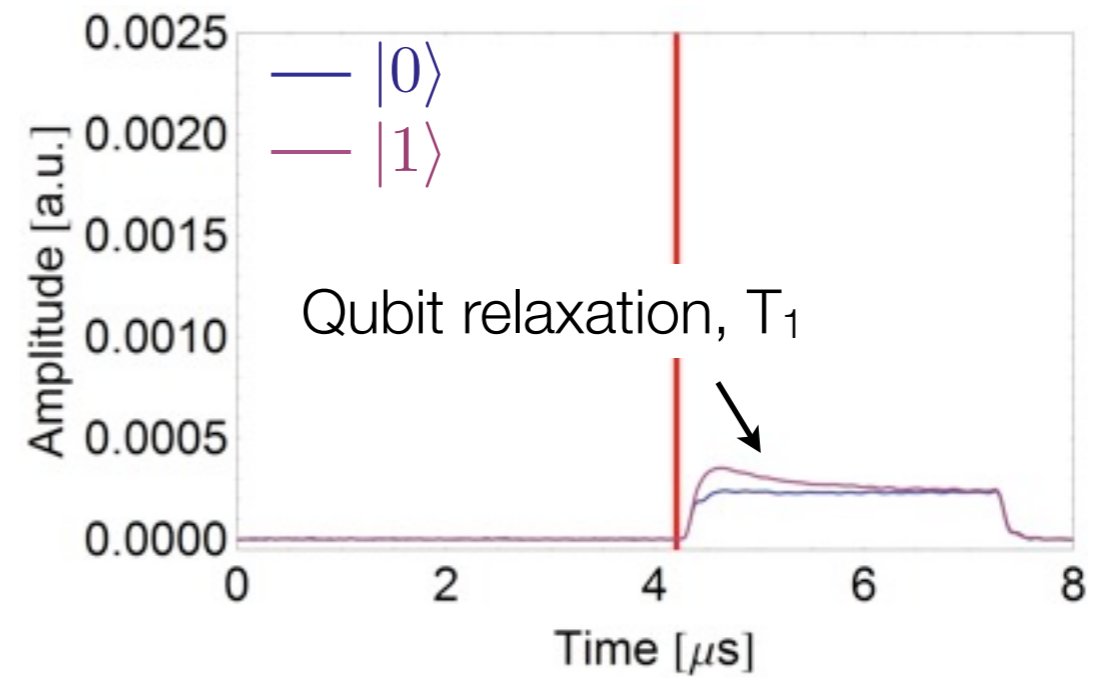
Measurement drive on:  
noise + signal (!?)

# Signal-to-noise ratio

Single meas. record



Ave. over 80K meas. records

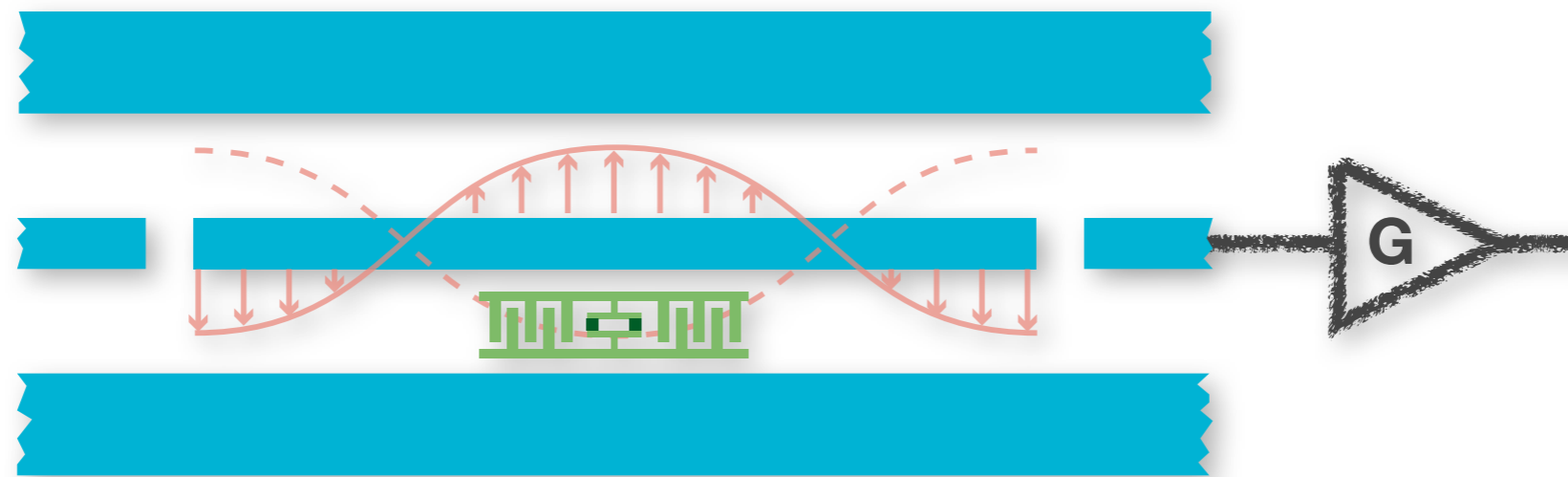


$\mu\text{w}$ -photons have too little energy...

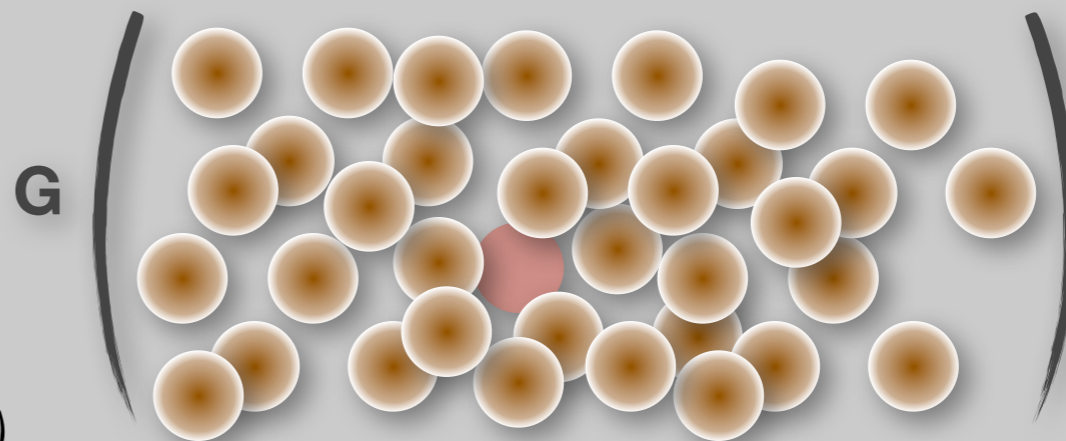
$$\frac{\nu_{\text{opt.}}}{\nu_{\mu\text{w}}} \sim \frac{500 \text{ THz}}{5 \text{ GHz}} \sim 10^5$$

# Linear amplifiers

---



*cryogenic high electron  
mobility transistor (HEMT)*



**~ 40 noise photons...**

# Phase-space representation

---

Coherent states:  $a|\alpha\rangle = \alpha|\alpha\rangle$ ,  $|\alpha\rangle = e^{i|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$  with  $a^\dagger a |n\rangle = n |n\rangle$

Both basis coincide for vacuum state:  $|\alpha = 0\rangle = |n = 0\rangle$

**Pictorial representation?**

State fidelity:  $F = |\langle\psi|\psi_0\rangle|^2$

(How close is  $\psi$  to  $\psi_0$ ?)

Q-function:  $Q_{\alpha_0}(\alpha) = \frac{1}{\pi} |\langle\alpha|\alpha_0\rangle|^2$

(How close is  $\alpha$  to  $\alpha_0$ ?)

# Phase-space representation

---

Q-function:  $Q$

$$\hat{Q} = i\sqrt{\frac{\hbar}{2Z_0}}(\hat{a}^\dagger - \hat{a}) \quad \hat{\Phi} = \sqrt{\frac{\hbar Z_0}{2}}(\hat{a}^\dagger + \hat{a})$$

$$\hat{a} = \sqrt{\frac{2}{\hbar Z_0}}(\hat{\Phi} + iZ_0\hat{Q})$$

$$\equiv \frac{1}{\sqrt{2}}(\hat{X} + i\hat{P})$$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \quad \rightarrow \quad \langle\alpha|\hat{a}|\alpha\rangle = \alpha$$

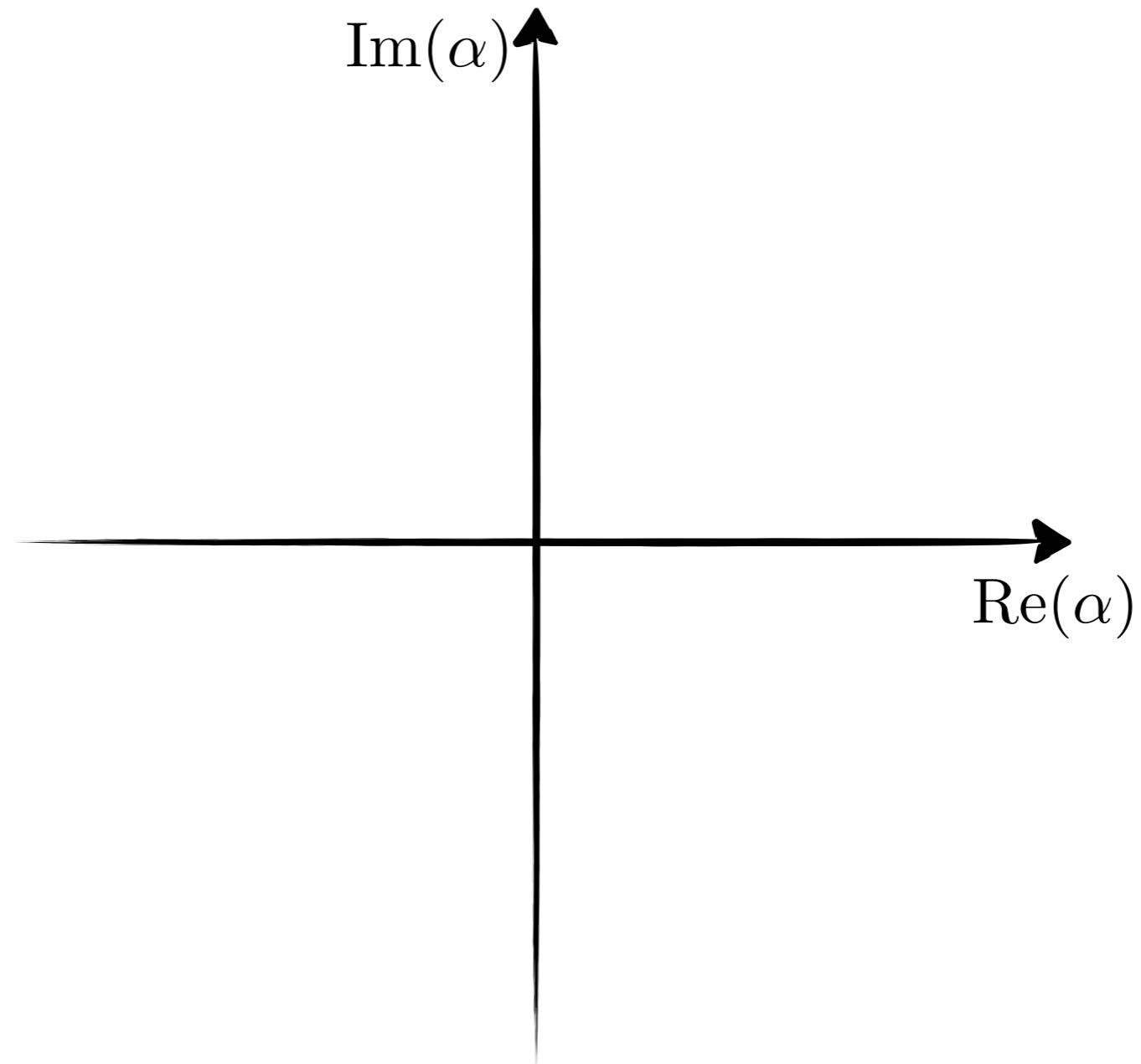
$$\Rightarrow \begin{cases} \text{Re}(\alpha) = \langle\hat{X}\rangle/\sqrt{2} \\ \text{Im}(\alpha) = \langle\hat{P}\rangle/\sqrt{2} \end{cases}$$



# Phase-space representation

---

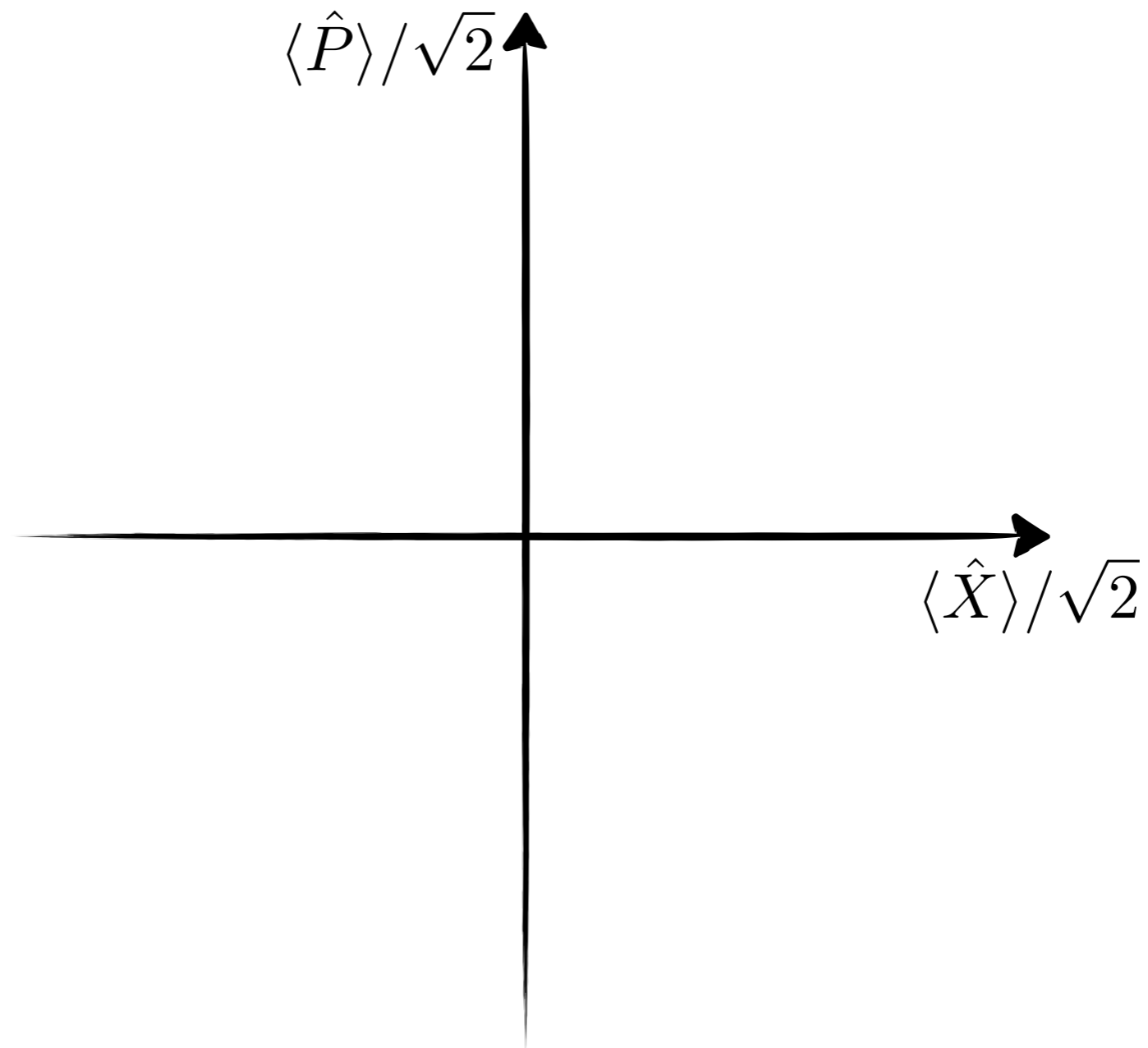
Q-function:  $Q_{\alpha_0}(\alpha) = \frac{1}{\pi} |\langle \alpha | \alpha_0 \rangle|^2$



# Phase-space representation

---

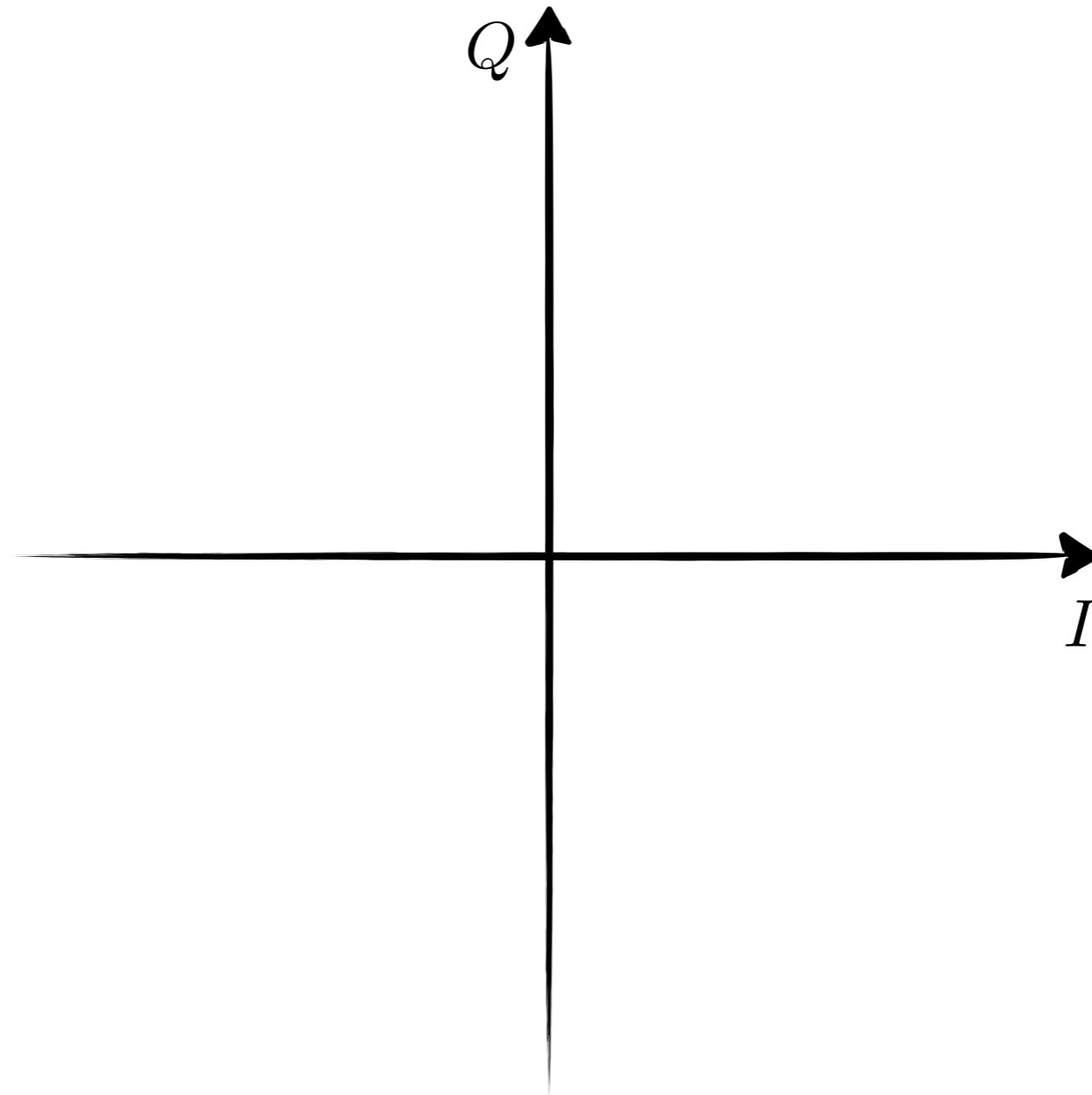
Q-function:  $Q_{\alpha_0}(\alpha) = \frac{1}{\pi} |\langle \alpha | \alpha_0 \rangle|^2$



# Phase-space representation

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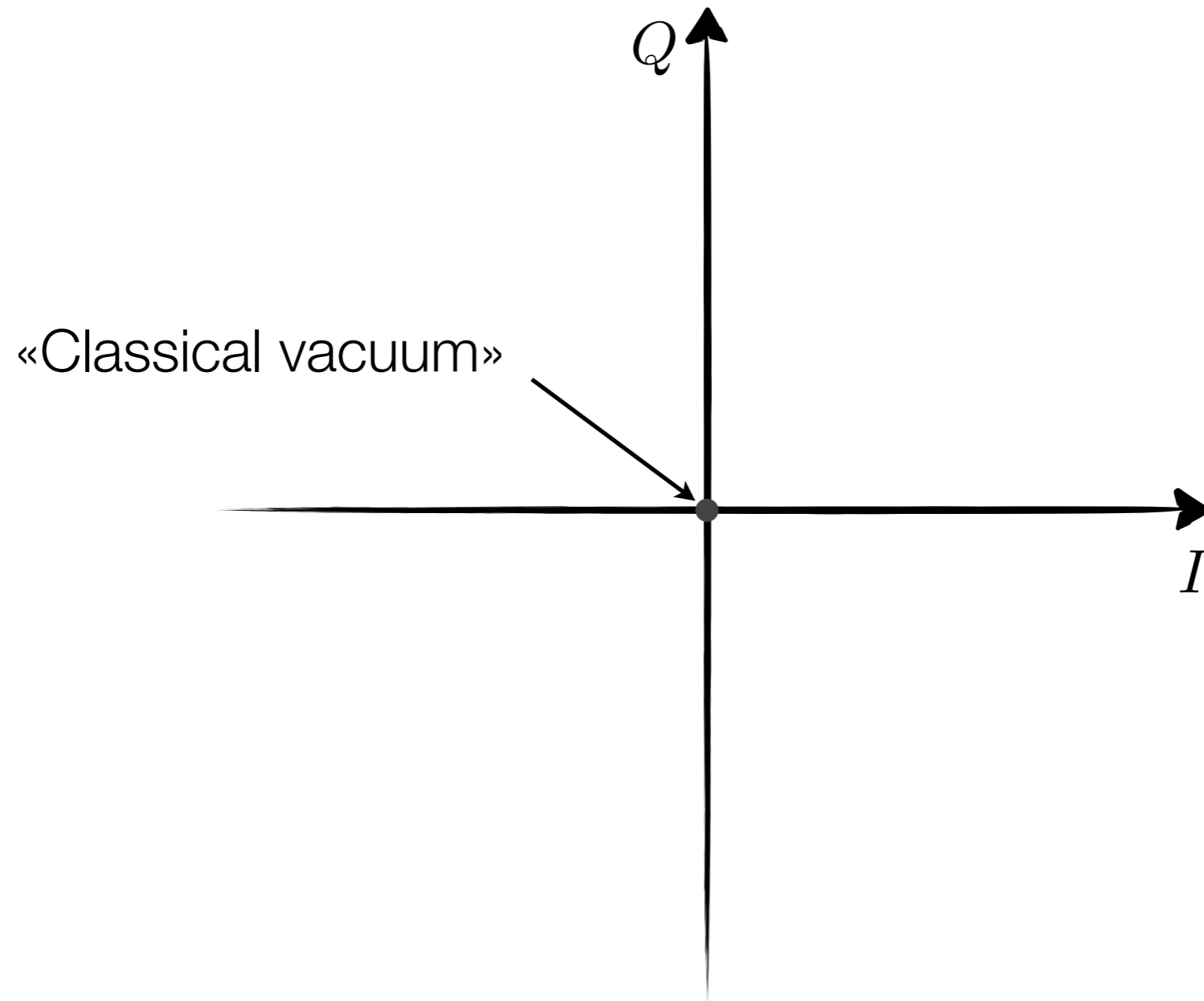
Q-function:  $Q_{\alpha_0}(\alpha) = \frac{1}{\pi} |\langle \alpha | \alpha_0 \rangle|^2$



# Phase-space representation

---

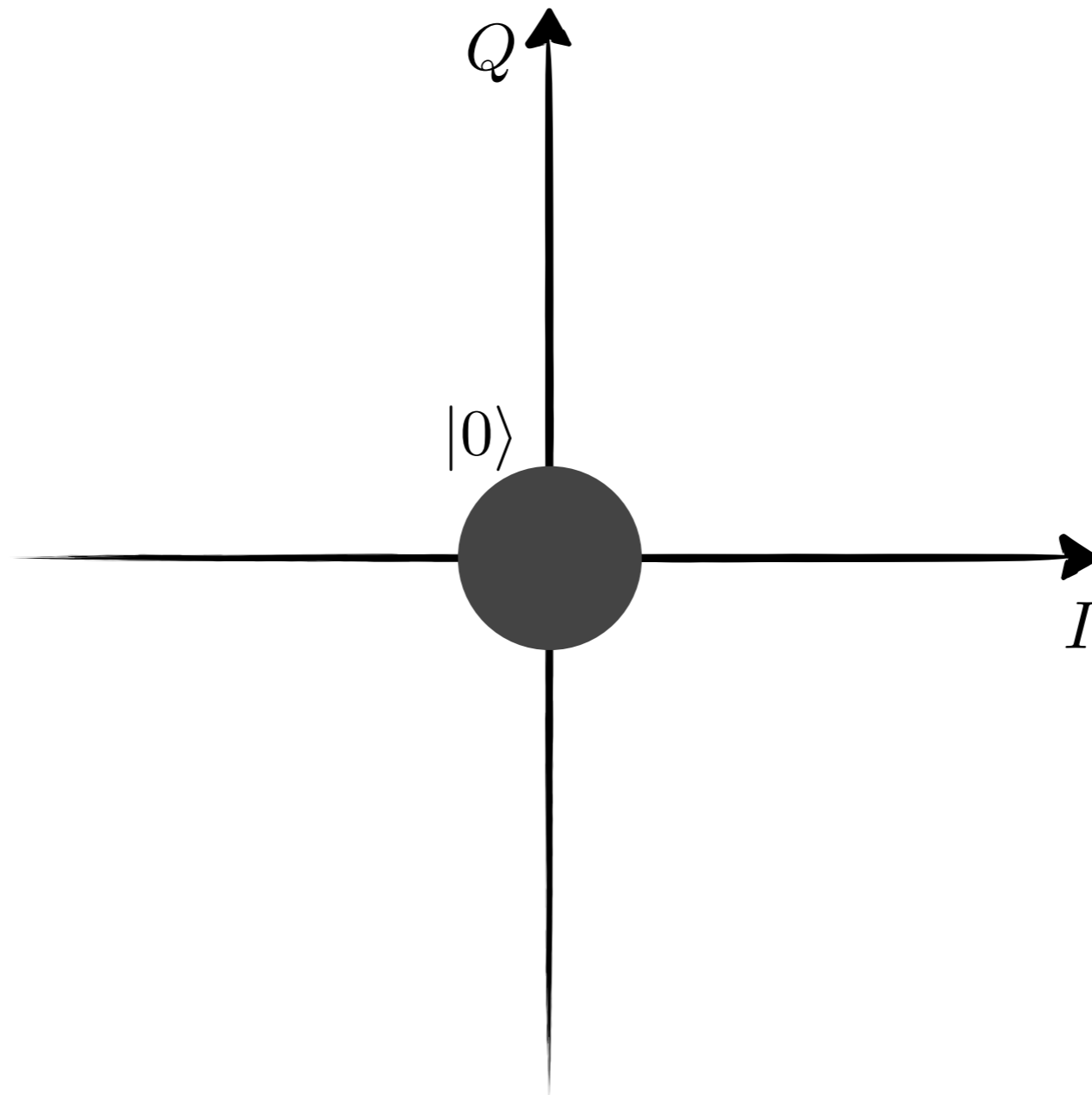
Q-function:  $Q_{\alpha_0}(\alpha) = \frac{1}{\pi} |\langle \alpha | \alpha_0 \rangle|^2$



# Phase-space representation

---

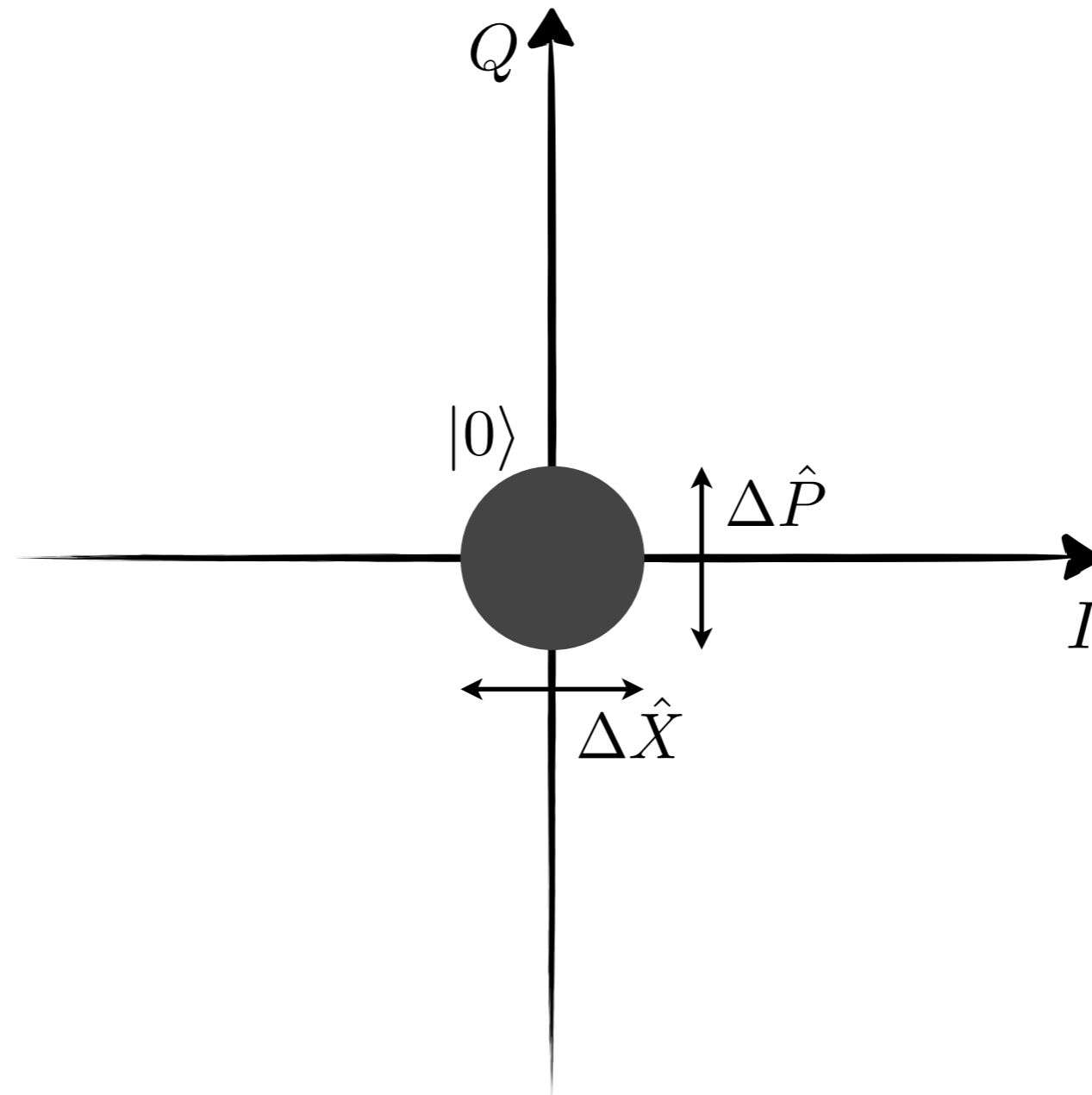
Q-function:  $Q_{\alpha_0}(\alpha) = \frac{1}{\pi} |\langle \alpha | \alpha_0 \rangle|^2$



# Phase-space representation

---

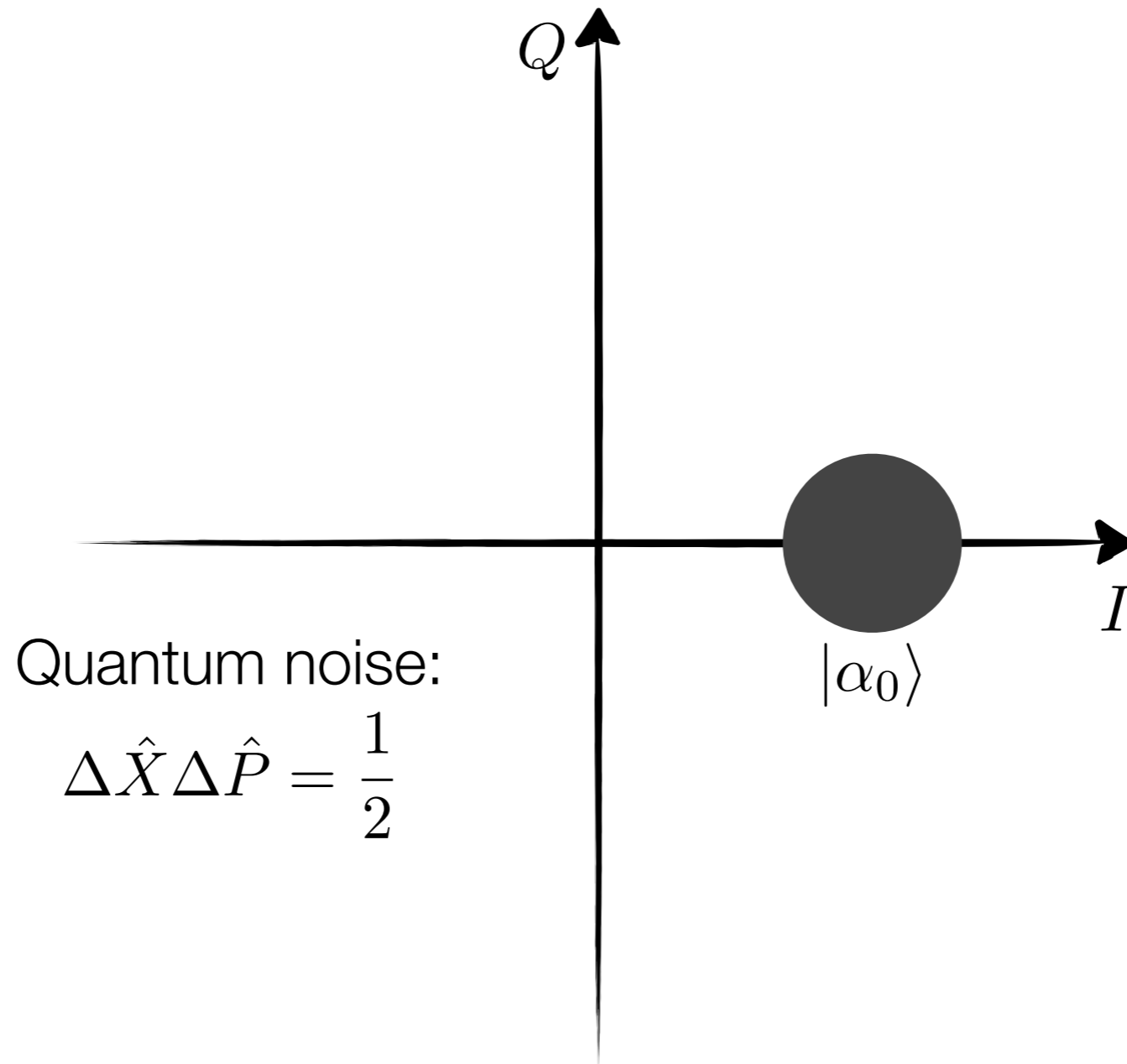
Q-function:  $Q_{\alpha_0}(\alpha) = \frac{1}{\pi} |\langle \alpha | \alpha_0 \rangle|^2$



# Phase-space representation

---

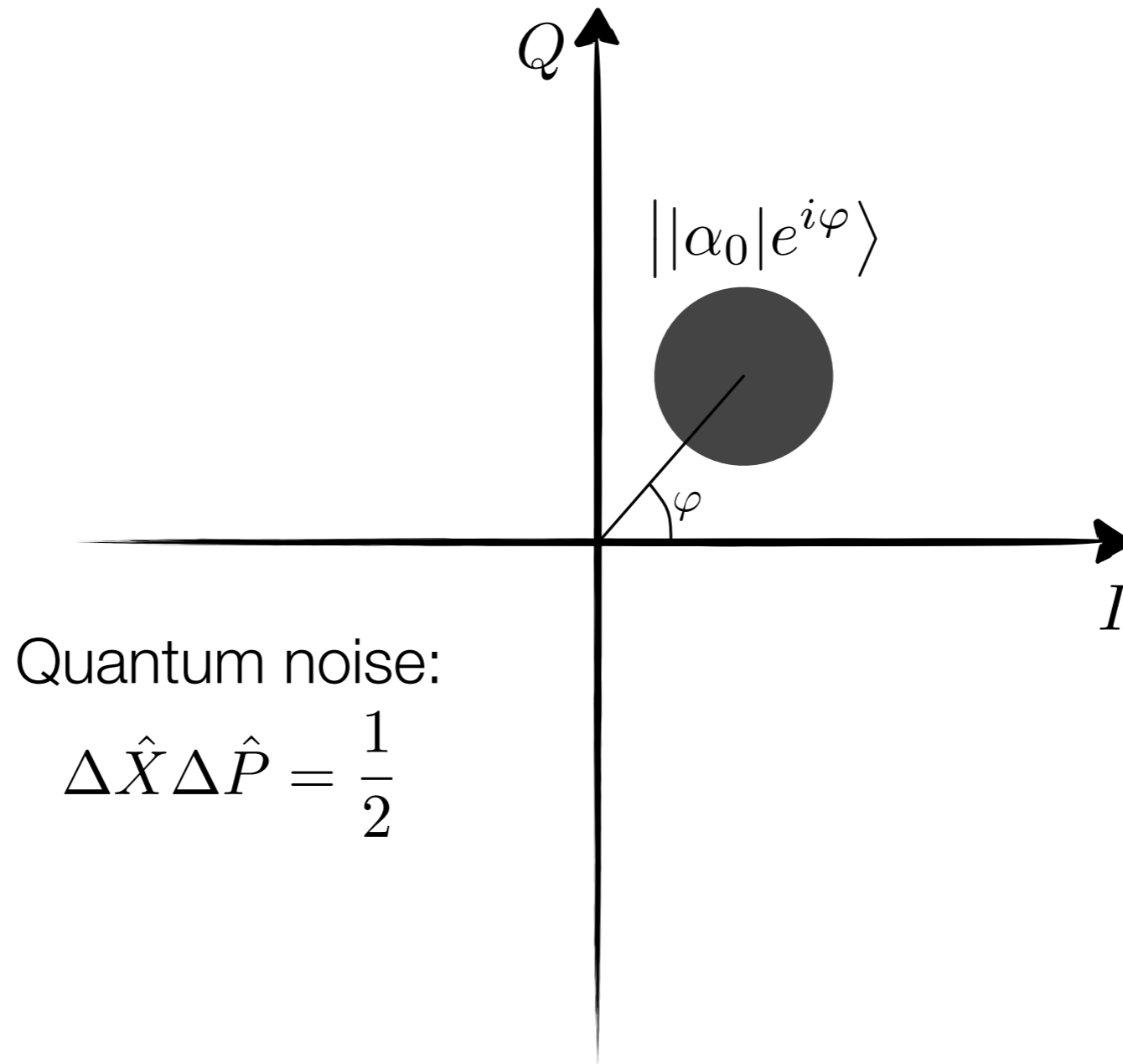
Q-function:  $Q_{\alpha_0}(\alpha) = \frac{1}{\pi} |\langle \alpha | \alpha_0 \rangle|^2$



# Phase-space representation

---

Q-function:  $Q_{\alpha_0}(\alpha) = \frac{1}{\pi} |\langle \alpha | \alpha_0 \rangle|^2$

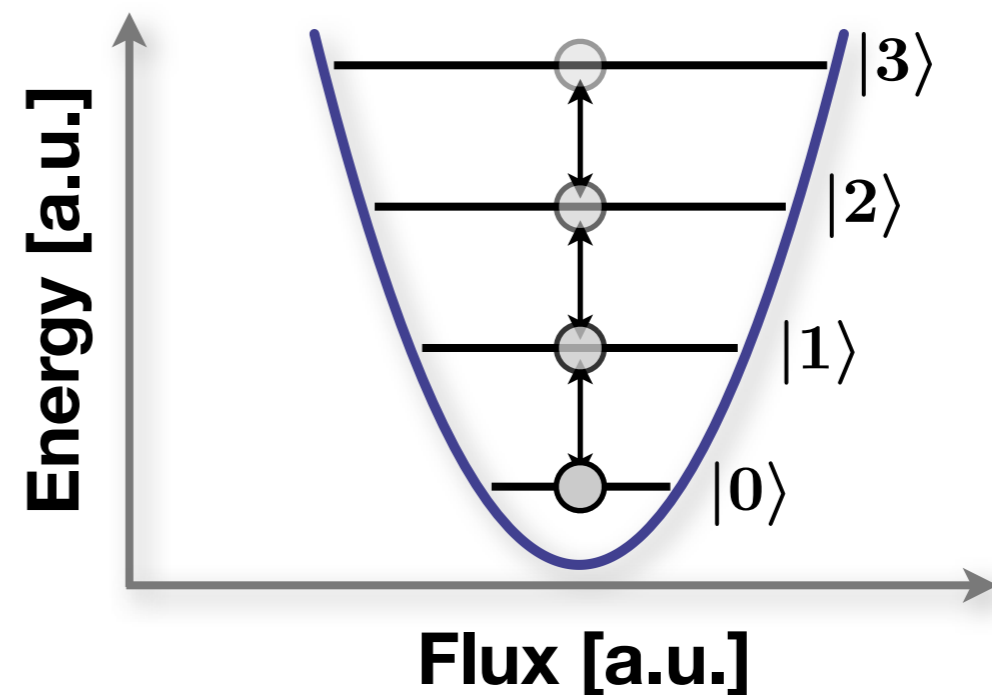
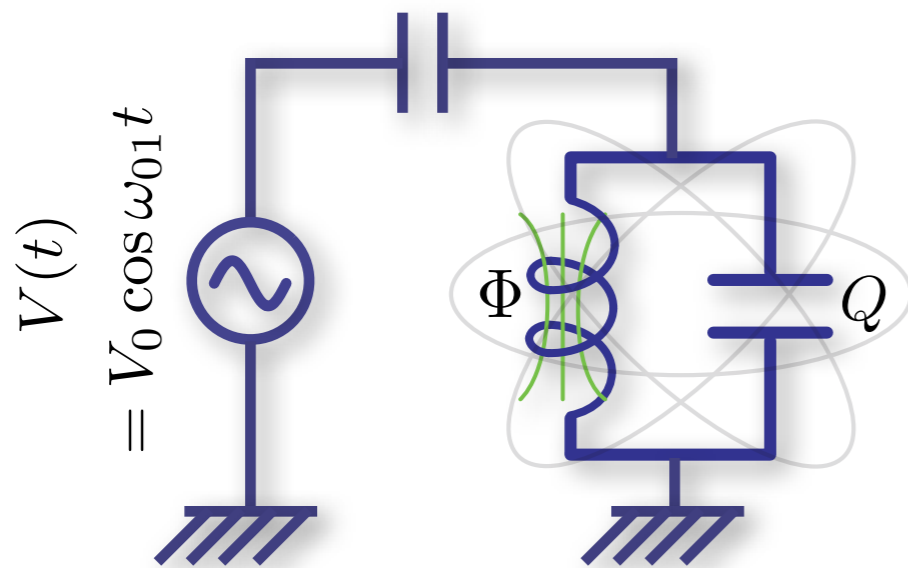


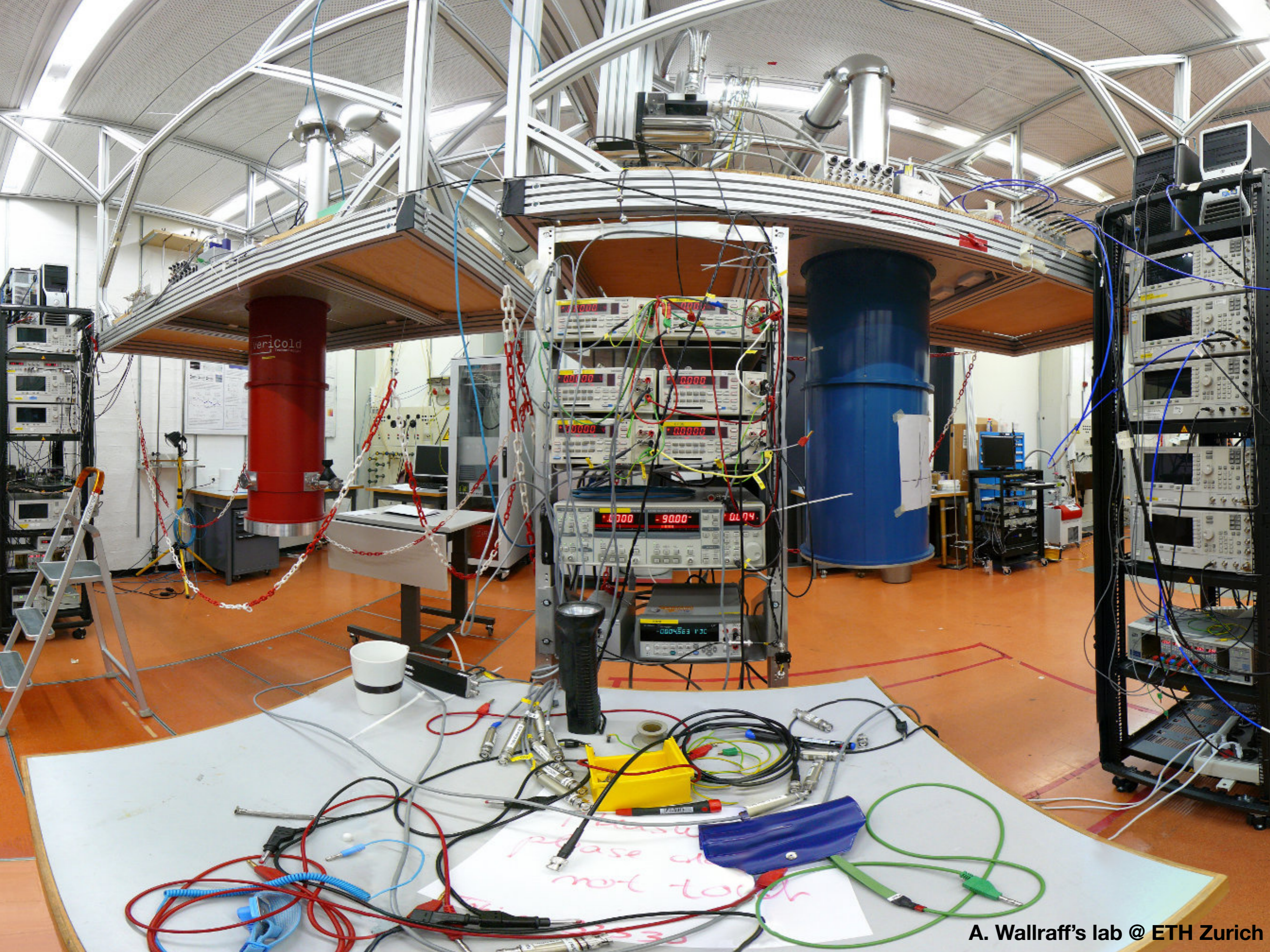


# How to prepare a coherent state of light?



Coherent state: Steady-state of driven damped harmonic oscillator





VeriCold

Please do not touch

# Dispersive regime: Qubit readout

---

Dispersive interaction:  $H \approx (\omega_r + \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + \frac{\tilde{\omega}_{01}}{2} \hat{\sigma}_z \rightarrow \hat{H}_{\text{int}} = \chi \hat{a}^\dagger \hat{a} \hat{\sigma}_z$

$$(c_0|0\rangle + c_1|1\rangle) \otimes |\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} (c_0|0\rangle|n\rangle + c_1|1\rangle|n\rangle)$$

$$\rightarrow e^{-i\chi t \hat{a}^\dagger \hat{a} \hat{\sigma}_z} e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} (c_0|0\rangle|n\rangle + c_1|1\rangle|n\rangle)$$

$$= e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} (c_0 e^{+i\chi t n} |0\rangle|n\rangle + c_1 e^{-i\chi t n} |1\rangle|n\rangle)$$

$$= c_0|0\rangle e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{(\alpha e^{+i\chi t})^n}{\sqrt{n!}} |n\rangle + c_1|1\rangle e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{(\alpha e^{-i\chi t})^n}{\sqrt{n!}} |n\rangle$$

$$= c_0|0\rangle|\alpha e^{+i\chi t}\rangle + c_1|1\rangle|\alpha e^{-i\chi t}\rangle$$

# Dispersive regime: Qubit readout

---

Dispersive interaction:  $H \approx (\omega_r + \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + \frac{\tilde{\omega}_{01}}{2} \hat{\sigma}_z \rightarrow \hat{H}_{\text{int}} = \chi \hat{a}^\dagger \hat{a} \hat{\sigma}_z$

$$(c_0|0\rangle + c_1|1\rangle) \otimes |\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} (c_0|0\rangle|n\rangle + c_1|1\rangle|n\rangle)$$

$$\rightarrow e^{-i\chi t \hat{a}^\dagger \hat{a} \hat{\sigma}_z} e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} (c_0|0\rangle|n\rangle + c_1|1\rangle|n\rangle)$$

**Qubit-state dependent coherent states!**

$$= e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} (c_0 e^{+i\chi t n} |0\rangle|n\rangle + c_1 e^{-i\chi t n} |1\rangle|n\rangle)$$

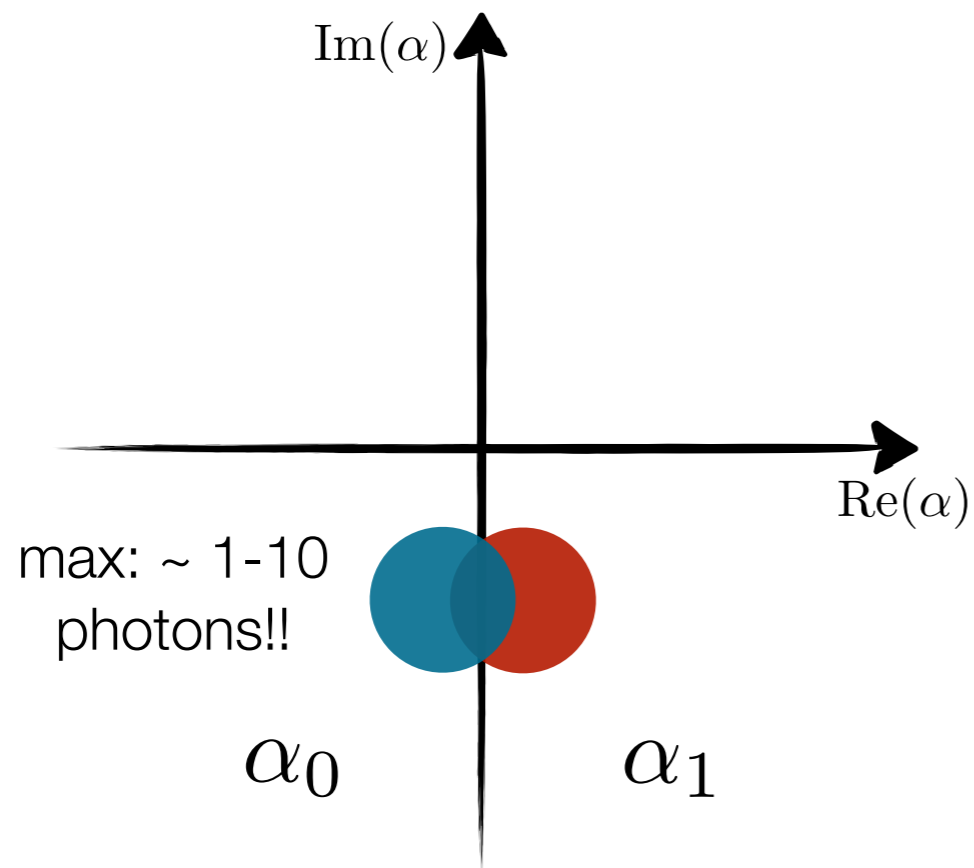
$$= c_0|0\rangle e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{(\alpha e^{+i\chi t})^n}{\sqrt{n!}} |n\rangle + c_1|1\rangle e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{(\alpha e^{-i\chi t})^n}{\sqrt{n!}} |n\rangle$$

$$= c_0|0\rangle |\alpha e^{+i\chi t}\rangle + c_1|1\rangle |\alpha e^{-i\chi t}\rangle$$

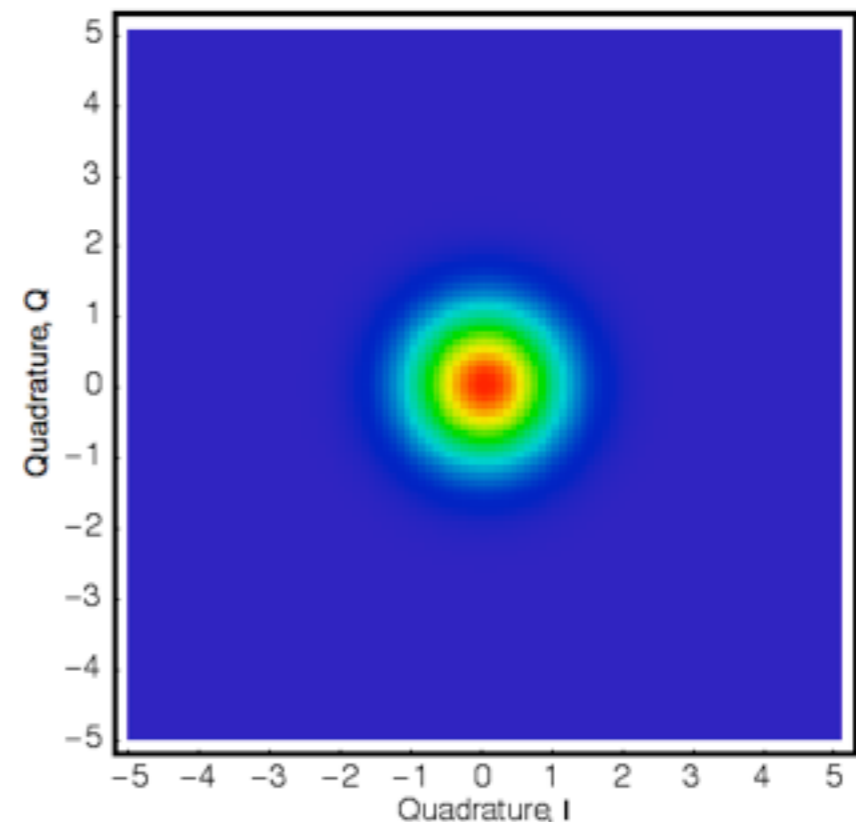
# Dispersive regime: Qubit readout

Dispersive interaction:  $H \approx (\omega_r + \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + \frac{\tilde{\omega}_{01}}{2} \hat{\sigma}_z \rightarrow \hat{H}_{\text{int}} = \chi \hat{a}^\dagger \hat{a} \hat{\sigma}_z$

$$(c_0|0\rangle + c_1|1\rangle) \otimes |\alpha\rangle \rightarrow c_0|0\rangle|\alpha e^{+i\chi t}\rangle + c_1|1\rangle|\alpha e^{-i\chi t}\rangle$$



Numerical simulation:

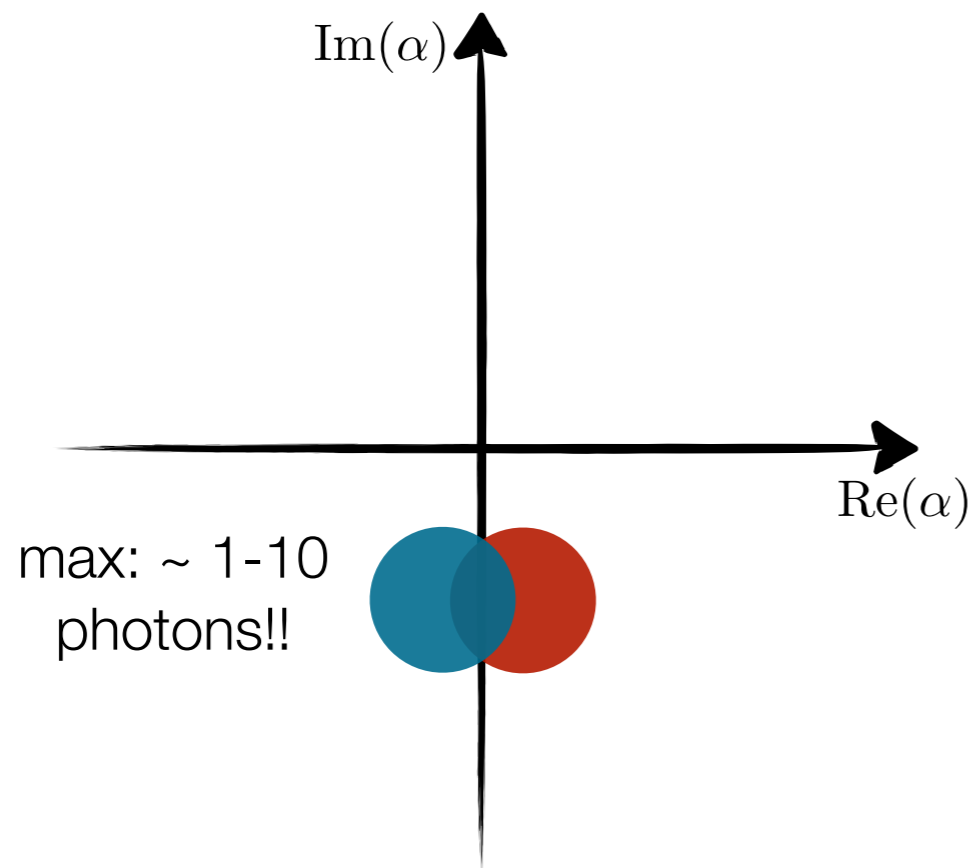


# Dispersive regime: Qubit readout

---

Dispersive interaction:  $H \approx (\omega_r + \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + \frac{\tilde{\omega}_{01}}{2} \hat{\sigma}_z \rightarrow \hat{H}_{\text{int}} = \chi \hat{a}^\dagger \hat{a} \hat{\sigma}_z$

$$(c_0|0\rangle + c_1|1\rangle) \otimes |\alpha\rangle \rightarrow c_0|0\rangle|\alpha e^{+i\chi t}\rangle + c_1|1\rangle|\alpha e^{-i\chi t}\rangle$$



Need to amplify:

- No «click» detectors
- Need to further separate the pointer states

# Quantum limit to amplification

---



Linear amplification:  $\langle \hat{a}_{\text{out}} \rangle = \sqrt{G} \langle \hat{a}_{\text{in}} \rangle$

$$\Rightarrow \hat{a}_{\text{out}} \stackrel{?}{=} \sqrt{G} \hat{a}_{\text{in}} + \hat{h} \quad [\hat{a}_{\text{out}}, \hat{a}_{\text{out}}^\dagger] = 1 \quad \langle \hat{h} \rangle = 0 \quad (\text{it's noise})$$

$$[\hat{h}, \hat{h}^\dagger] = 1 - G$$

Noise at the amplifier output:

$$(\Delta a_{\text{out}})^2 = G(\Delta a_{\text{in}})^2 + \frac{1}{2} \left\langle \left\{ \hat{h}, \hat{h}^\dagger \right\} \right\rangle \geq G(\Delta a_{\text{in}})^2 + \frac{1}{2} |G - 1|$$

Noise at the output as equivalent noise at the input; large gain limit :

$$(\Delta a_{\text{out}})^2 / G \geq (\Delta a_{\text{in}})^2 + \frac{1}{2}$$

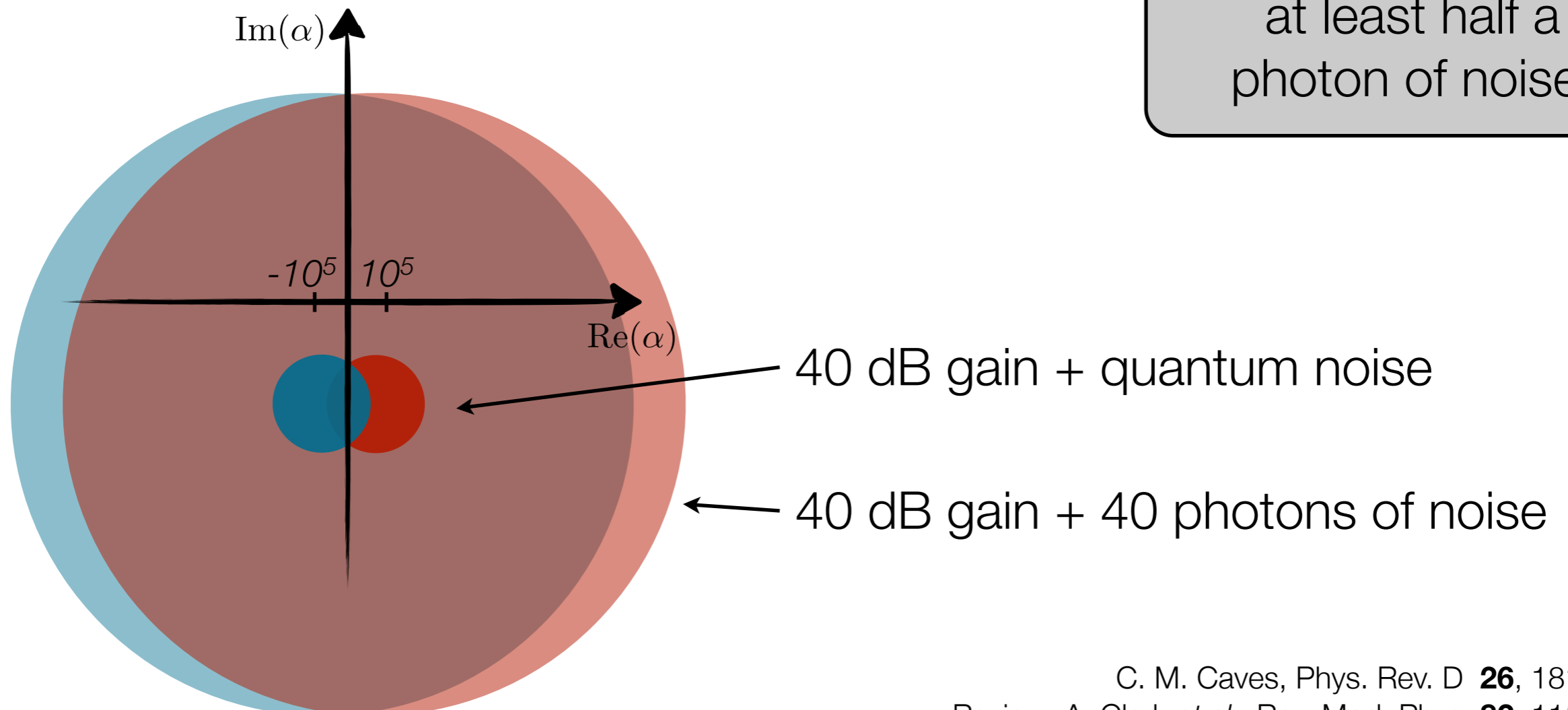
# Quantum limit to amplification

$$\hat{a}_{\text{out}} = \sqrt{G}\hat{a}_{\text{in}} + \hat{h}$$

Noise at the output as equivalent noise at the input; large gain limit :

$$(\Delta a_{\text{out}})^2 / G \geq (\Delta a_{\text{in}})^2 + \frac{1}{2}$$

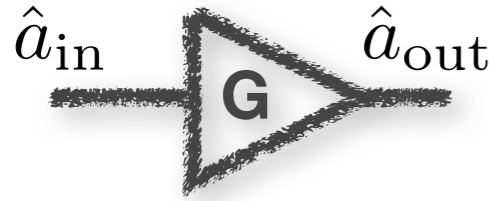
Linear amplifier with large gain must add at least half a photon of noise.





# Quantum limit to amplification

---



Linear amplification:  $\langle \hat{a}_{out} \rangle = \sqrt{G} \langle \hat{a}_{in} \rangle$

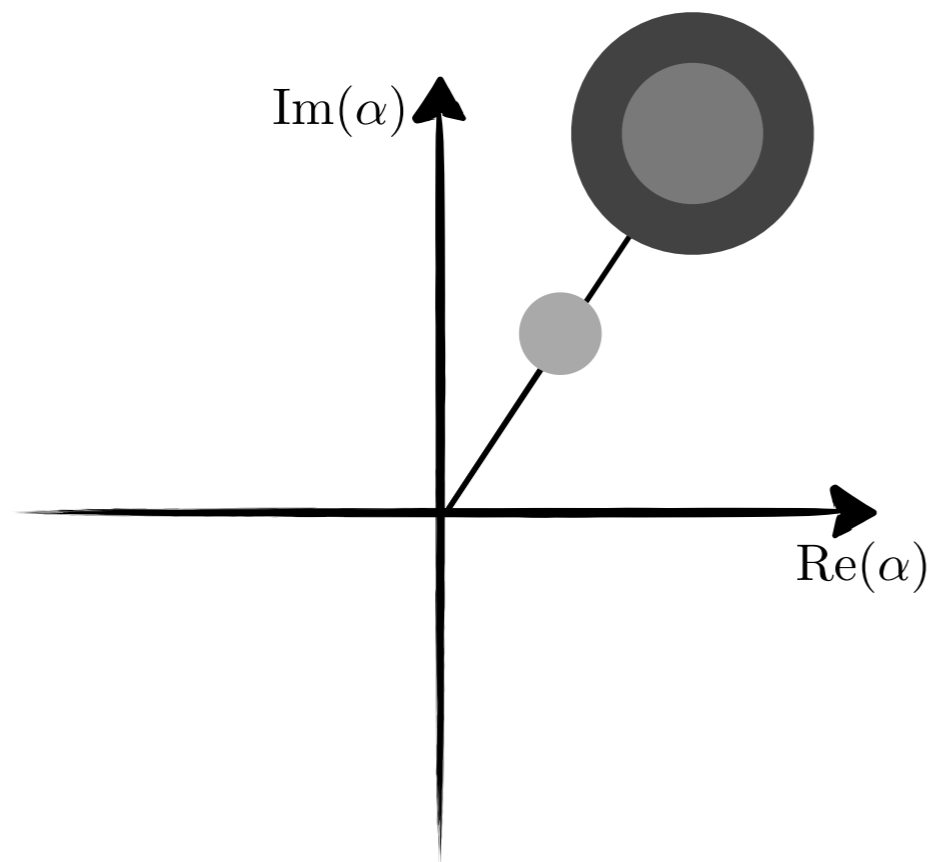
$$\Rightarrow \hat{a}_{out} = \sqrt{G} \hat{a}_{in} + \hat{h} \quad [\hat{a}_{out}, \hat{a}_{out}^\dagger] = 1 \quad \langle \hat{h} \rangle = 0 \quad (\text{it's noise})$$

---

$$\hat{a}_{out} = \sqrt{G} \hat{a}_{in} \quad \hat{a}_{out}^\dagger = \hat{a}_{in}^\dagger / \sqrt{G} \quad [\hat{a}_{out}, \hat{a}_{out}^\dagger] = 1$$

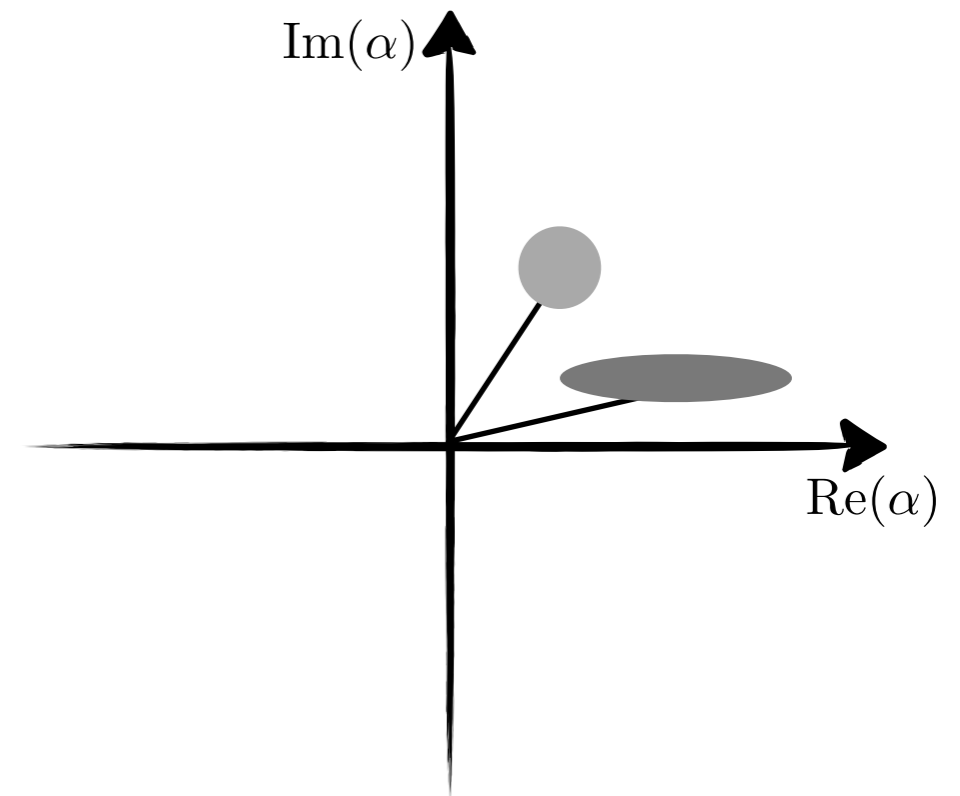
# Quantum limit to amplification

$$\hat{a}_{\text{out}} = \sqrt{G}\hat{a}_{\text{in}} + \hat{h}$$



Phase insensitive  
(phase preserving)

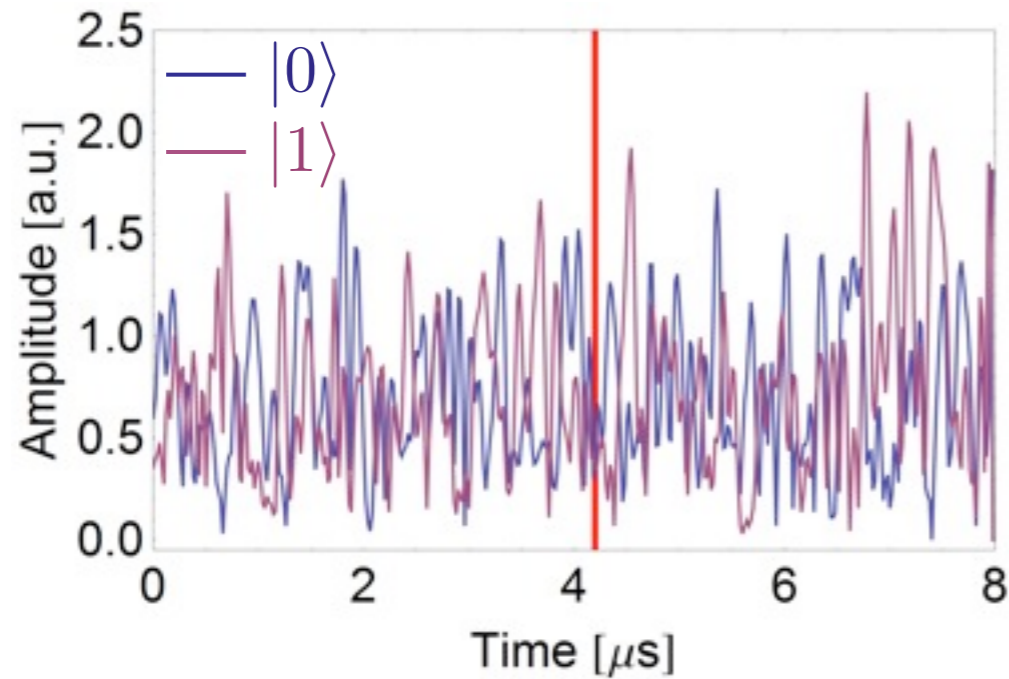
$$\hat{a}_{\text{out}} = \sqrt{G}\hat{a}_{\text{in}} \quad \hat{a}_{\text{out}}^\dagger = \hat{a}_{\text{in}}^\dagger / \sqrt{G}$$



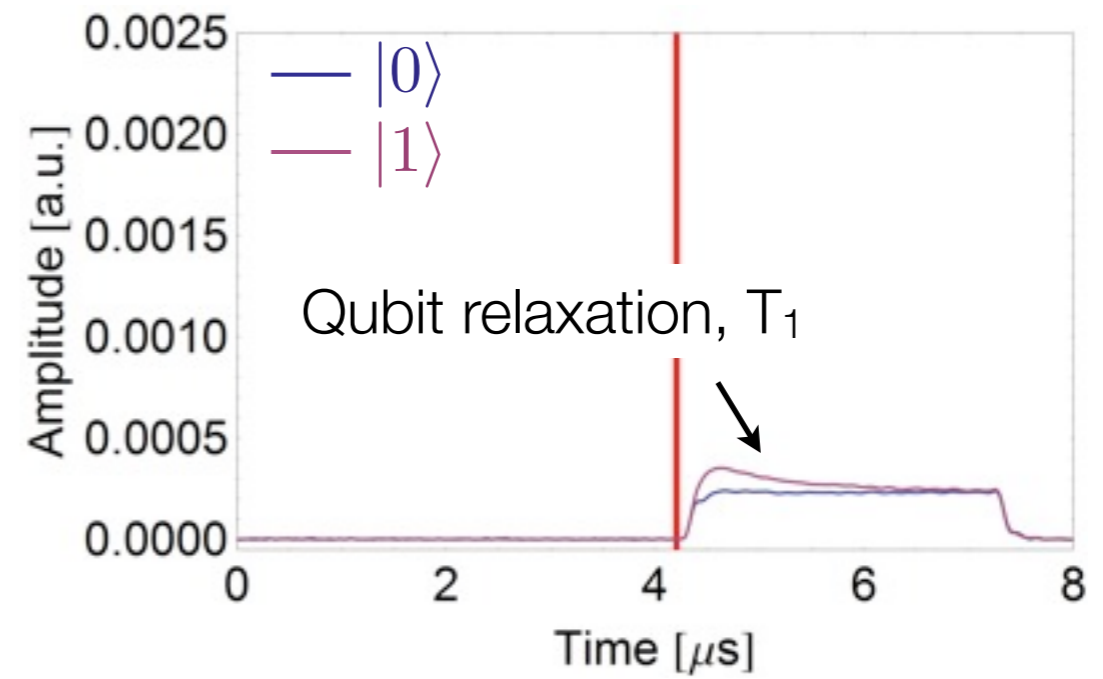
Phase sensitive  
(phase non-preserving)

# Signal-to-noise ratio

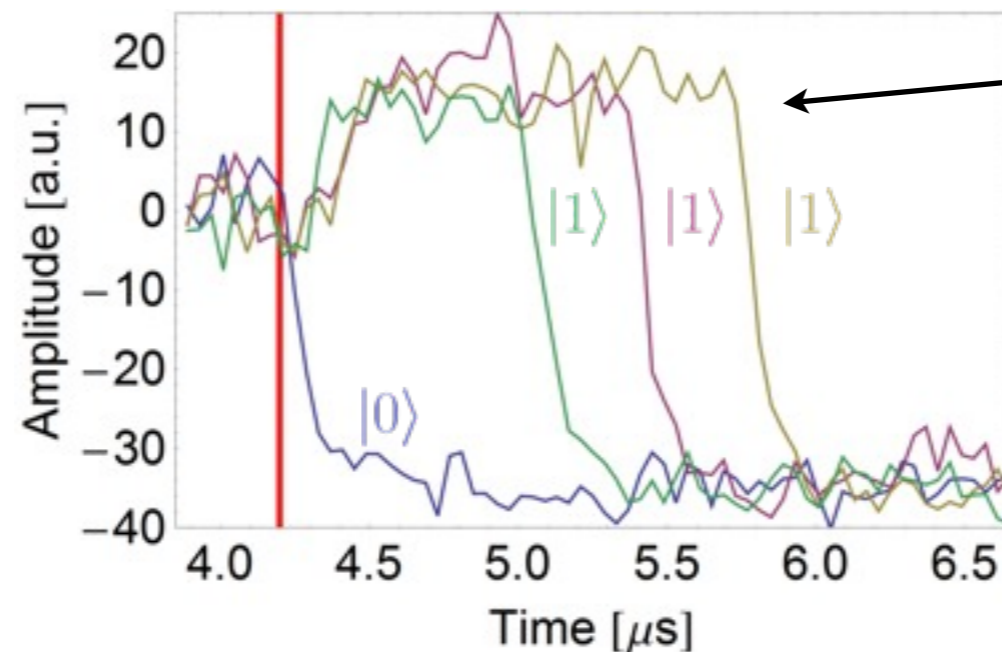
Single meas. record



Ave. over 80K meas. records



Single meas.  
with JJ amp



Quantum jumps!

First observation:  
R. Vijay et al. PRL  
**106**, 110502 (2011)



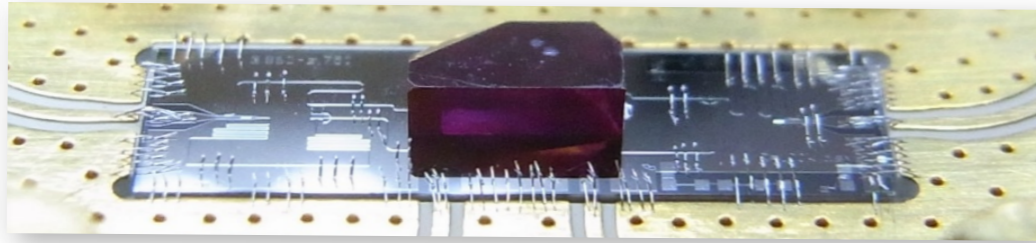
# **Challenges and future directions**

# Future directions and challenges

## Hybrid systems

- Coupling to NV centers, rare earths, quantum dots...
- Quantum memory + fast gates

Credit:  
Quantronics group



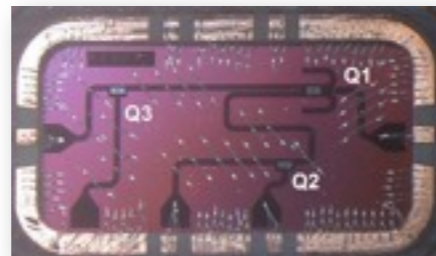
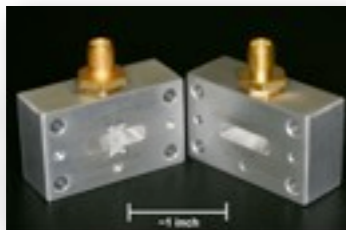
## Strong nonlinearities

- **New** regimes of quantum optics
- ▶ Single photon Kerr nonlinearities several orders of magnitude larger than in optics
- Improving qubit readout

## Quantum information processing

- Keep improving  $T_1$
- Understand and improve qubit readout
- Scalable architecture: multi-resonators

Credit: IBM

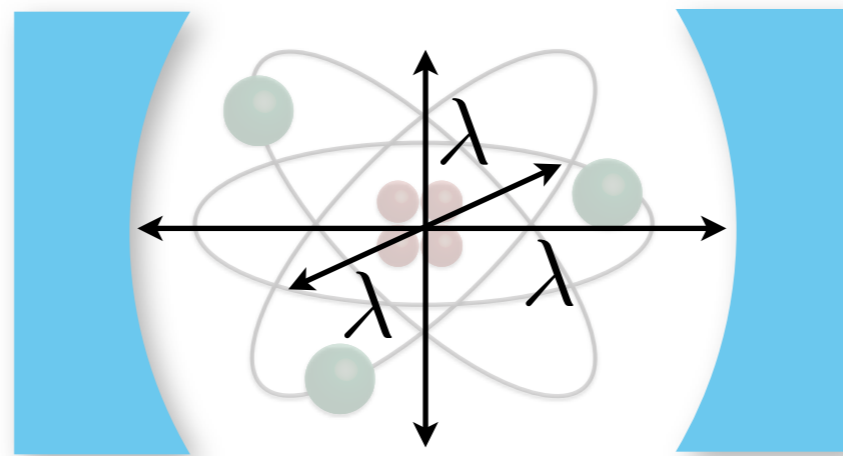


## New tools for discoveries

- Probes of noise at high frequencies
- Quantum sensing: sensitive magnetometers, ...
- What else?!

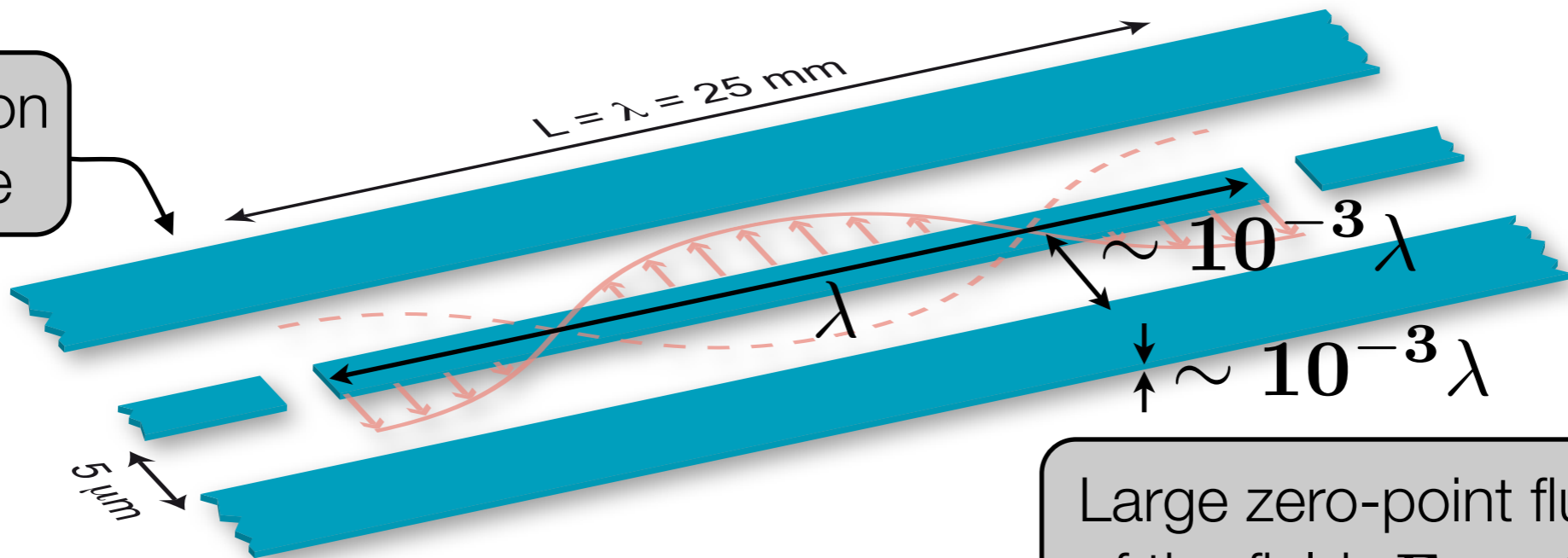
# From cavity to *circuit* QED

- E-field can be tightly confined



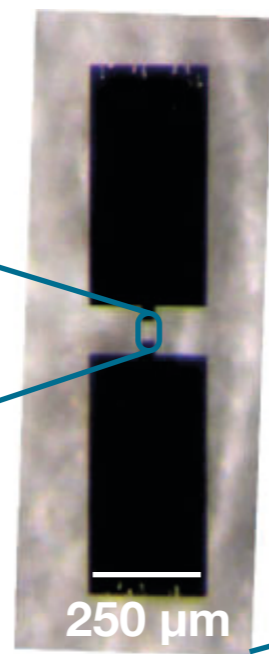
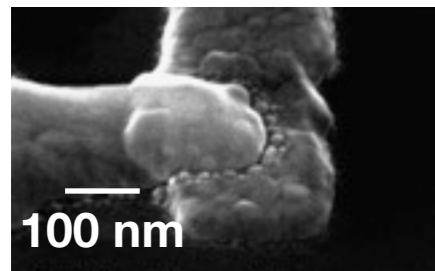
$$\mathbf{E} \propto \sqrt{1/\lambda^3}$$

Al or Nb on sapphire

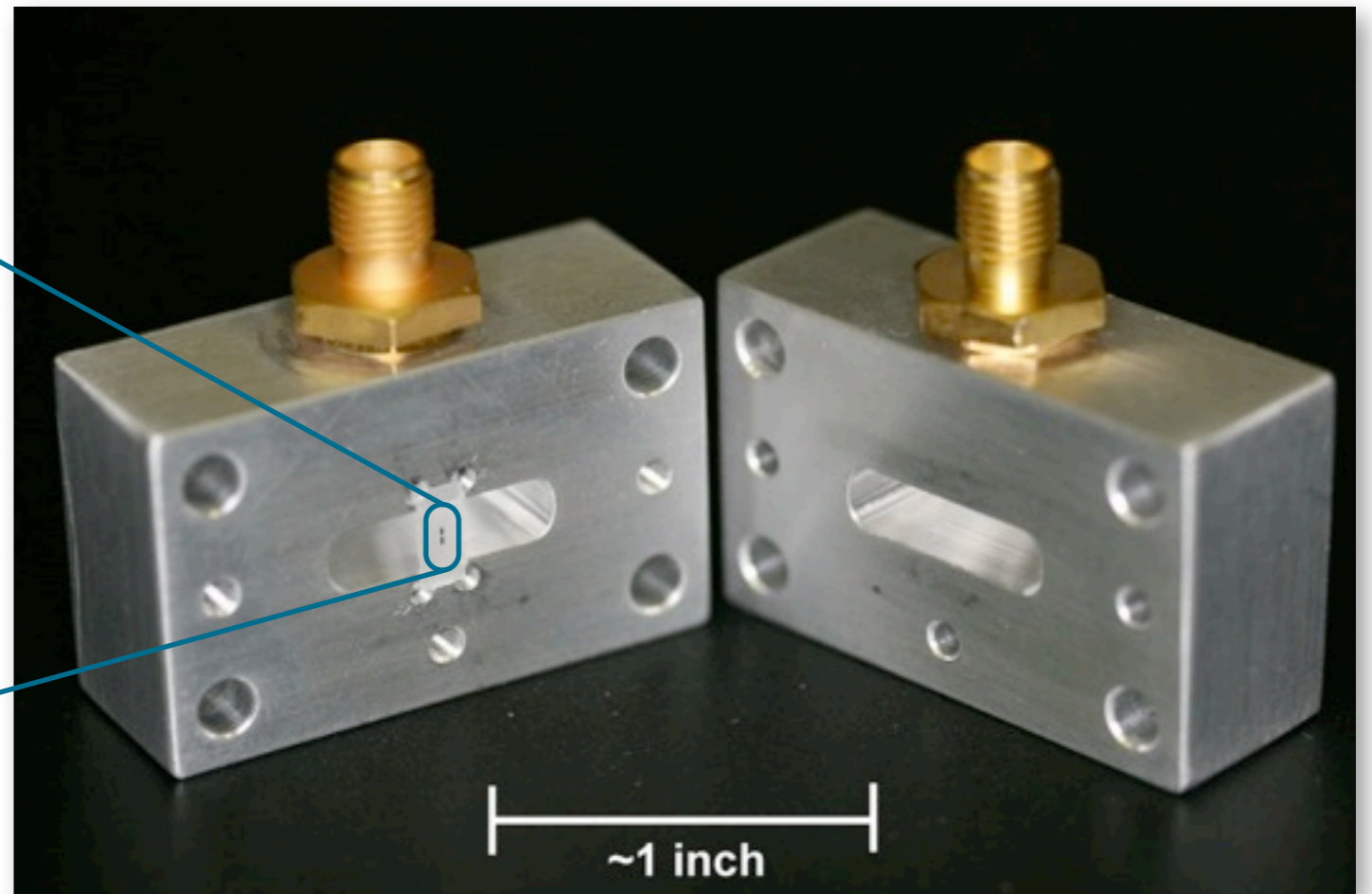


Large zero-point fluctuations of the field:  $\mathbf{E}_0 \sim 0.2 \text{ V/m}$

# Circuit QED goes **3D**



**Large** dipole moment



**Weak** electric fields

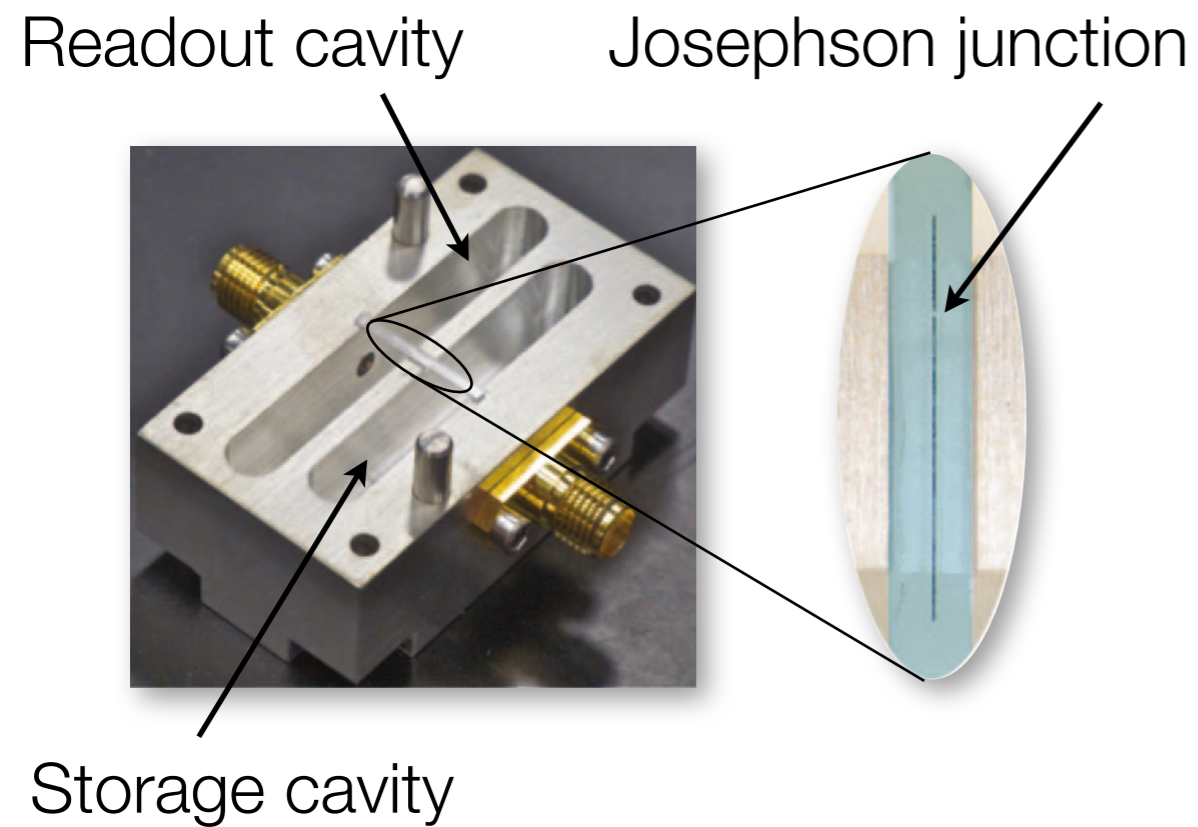
*Light-matter interaction:  $g = d \cdot E$  unchanged!*

*Current best values:  $T_1 = 70 \mu\text{s}$  and  $T_2^* = 95 \mu\text{s}$*

# Circuit QED goes **3D**

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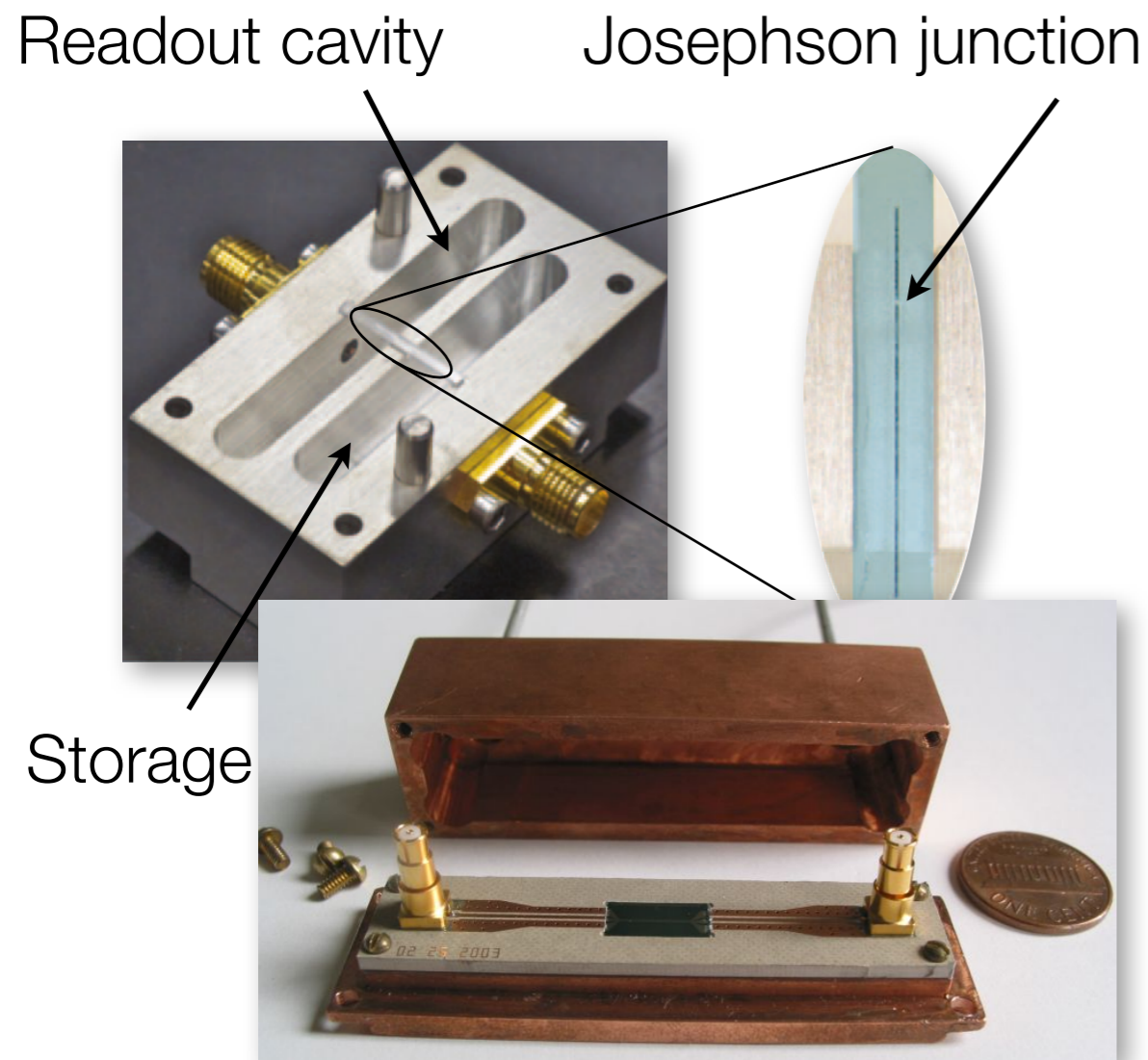
## First steps towards scaling-up



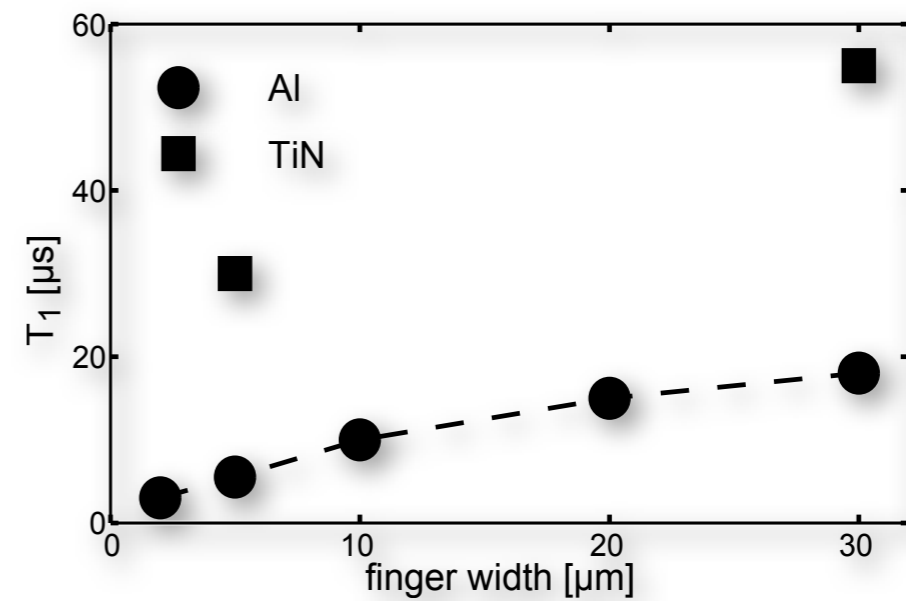
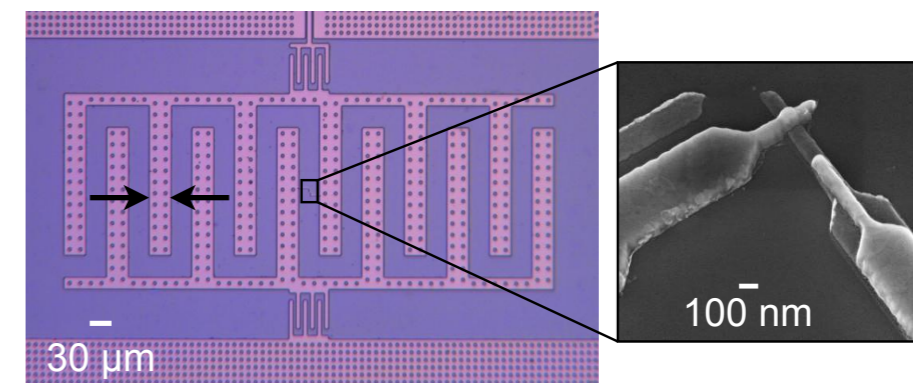


# Circuit QED goes **3D**

## First steps towards scaling-up

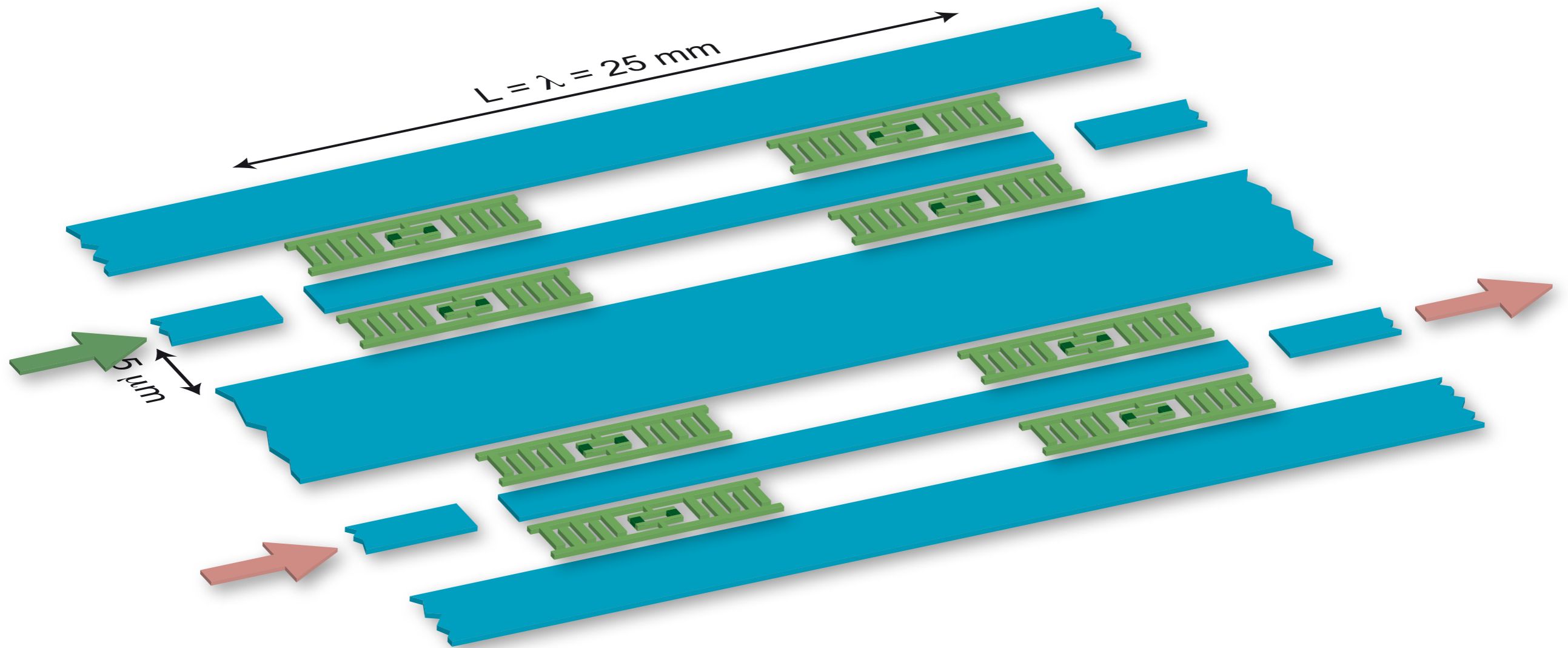


## ... and back to 2D



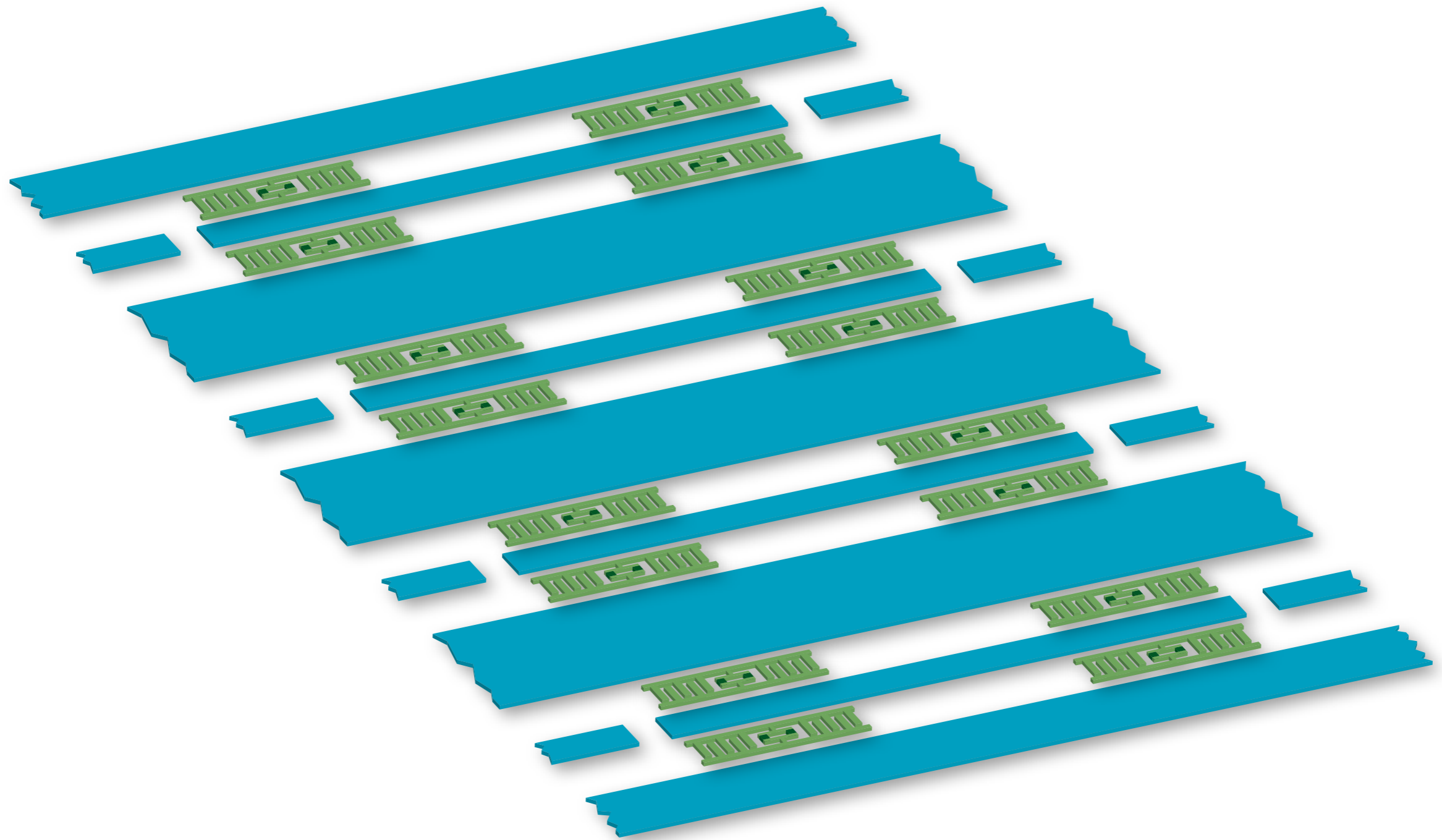
# Towards a scalable architecture

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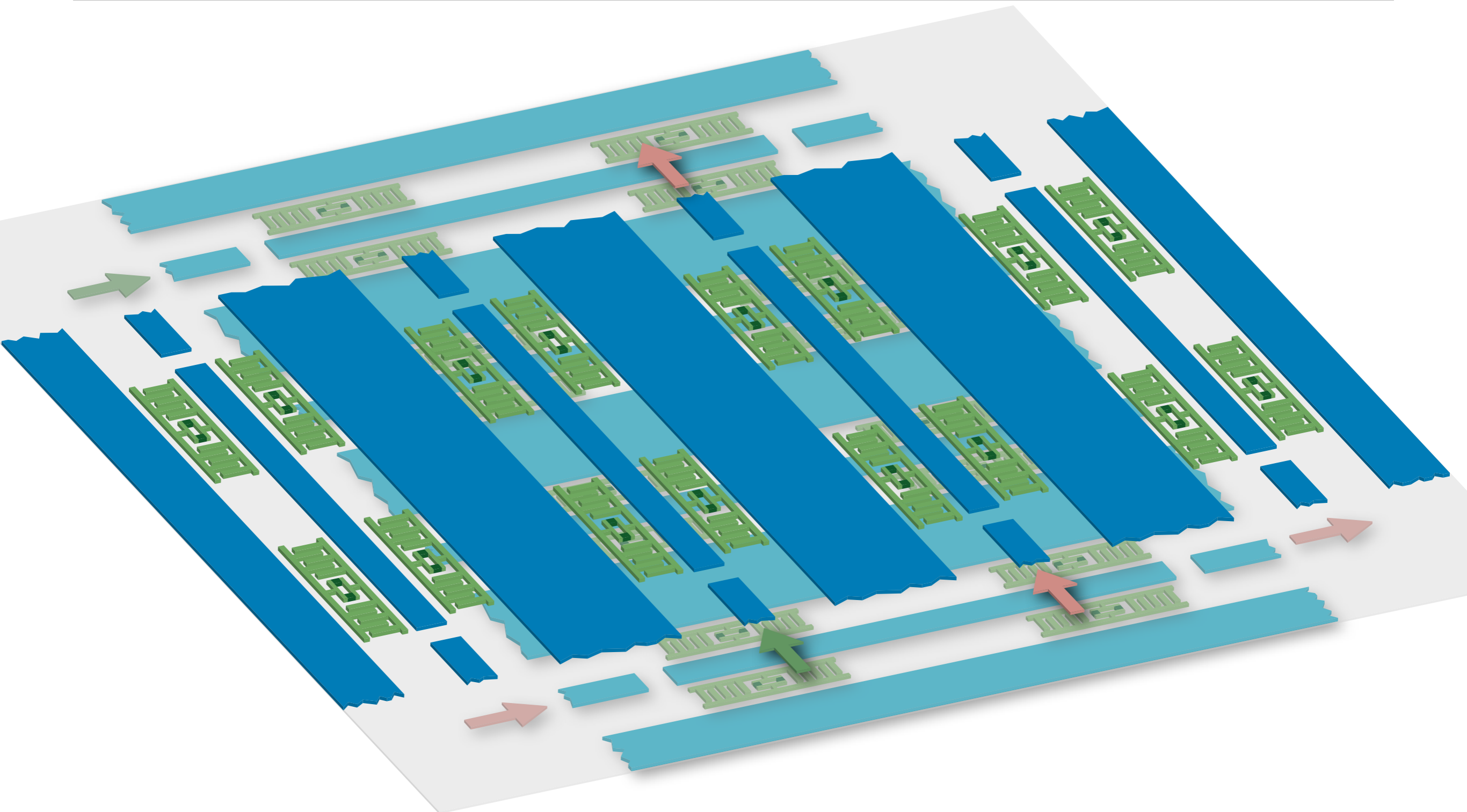
# Towards a scalable architecture

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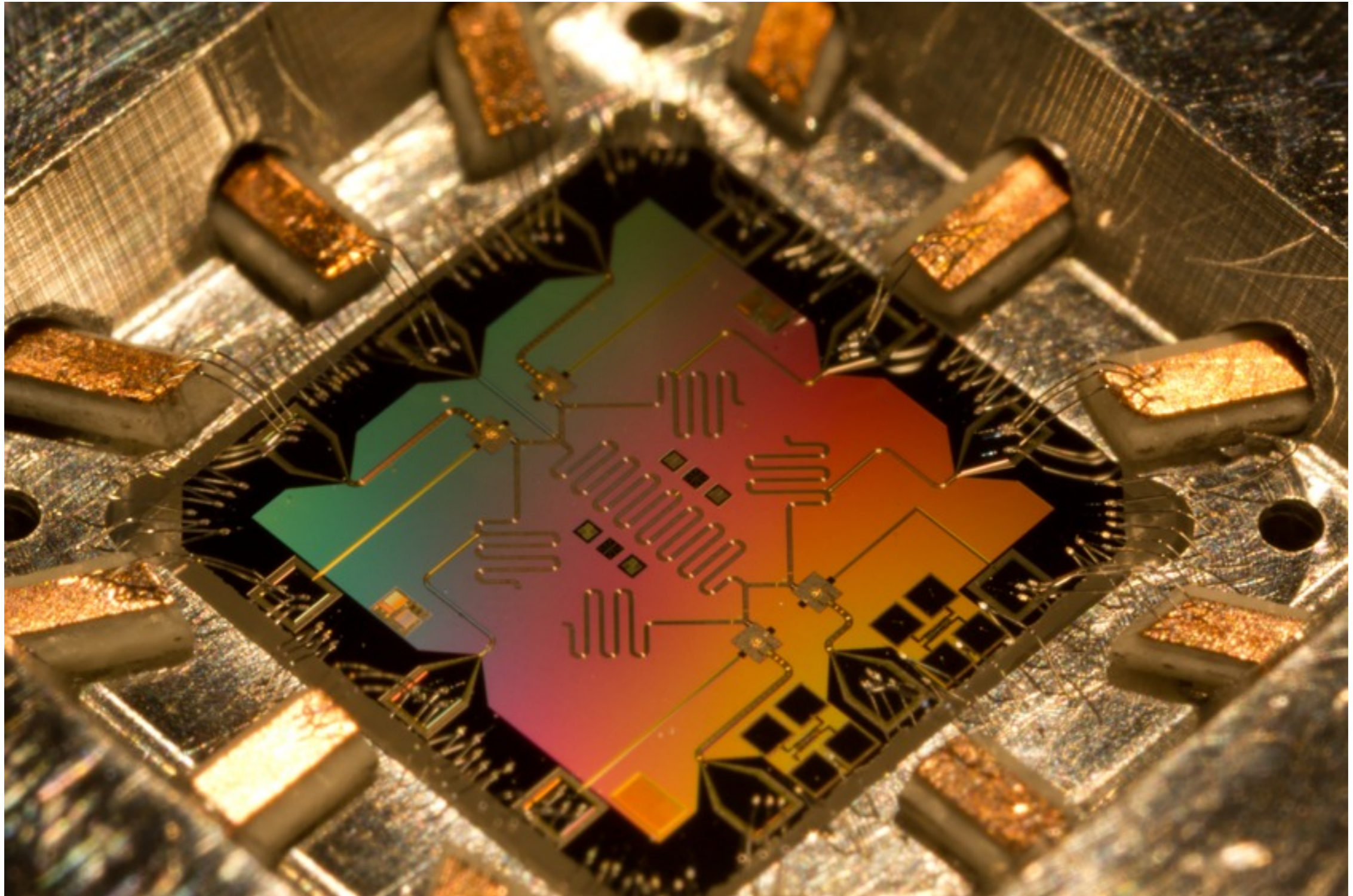
# Towards a scalable architecture

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# Towards a scalable architecture

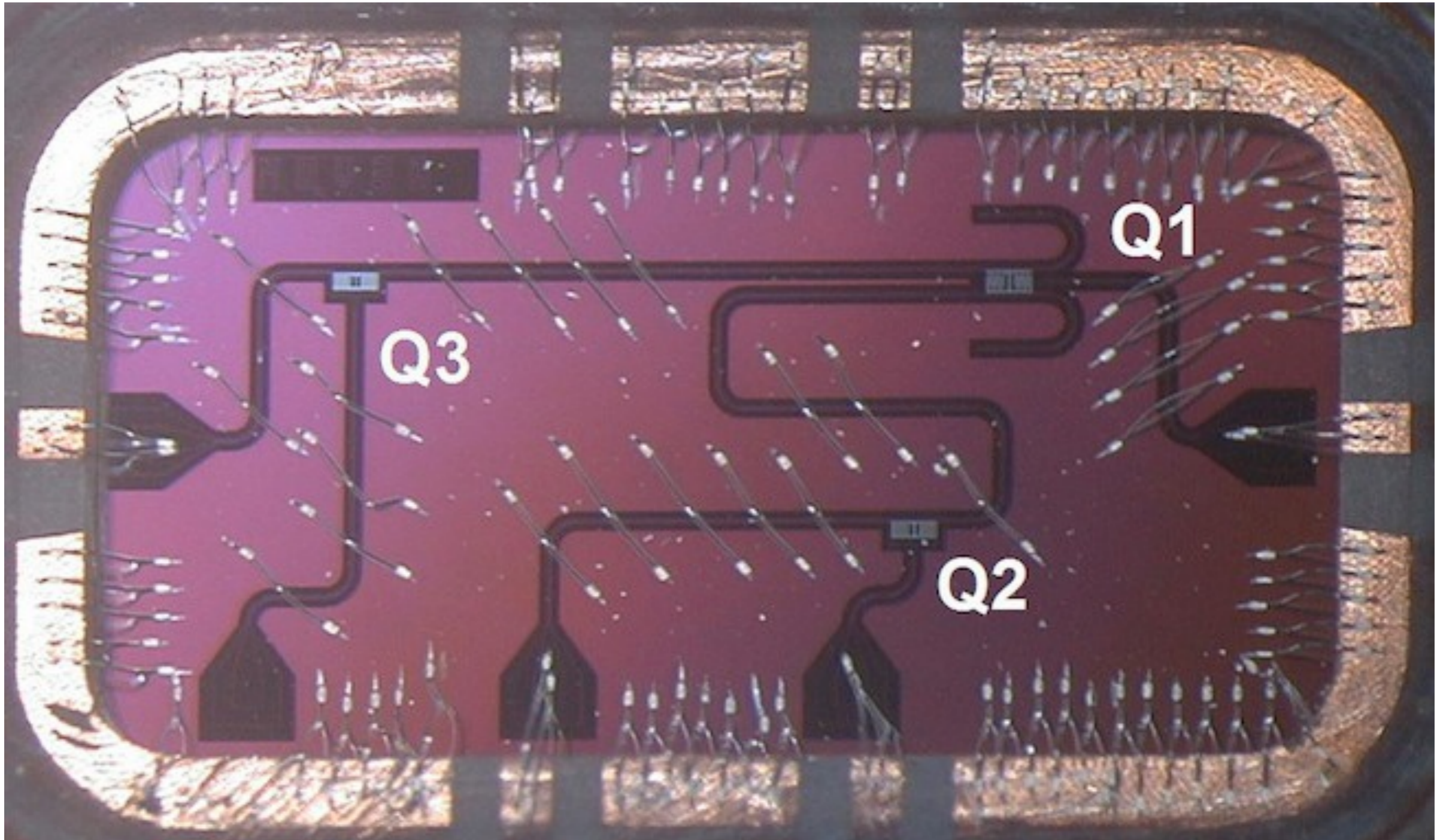
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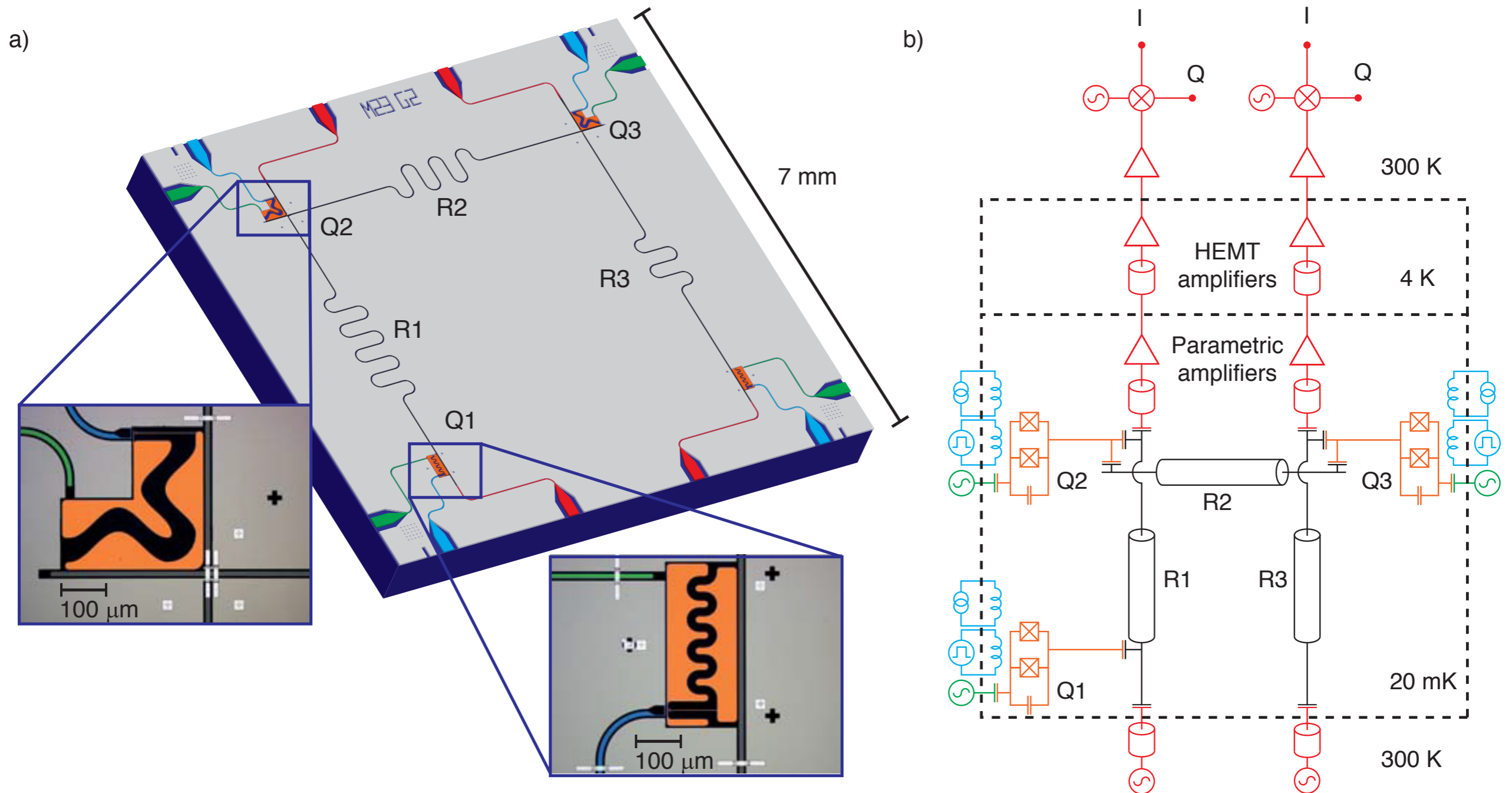
Credit: UCSB

# Towards a scalable architecture

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# Towards a scalable architecture



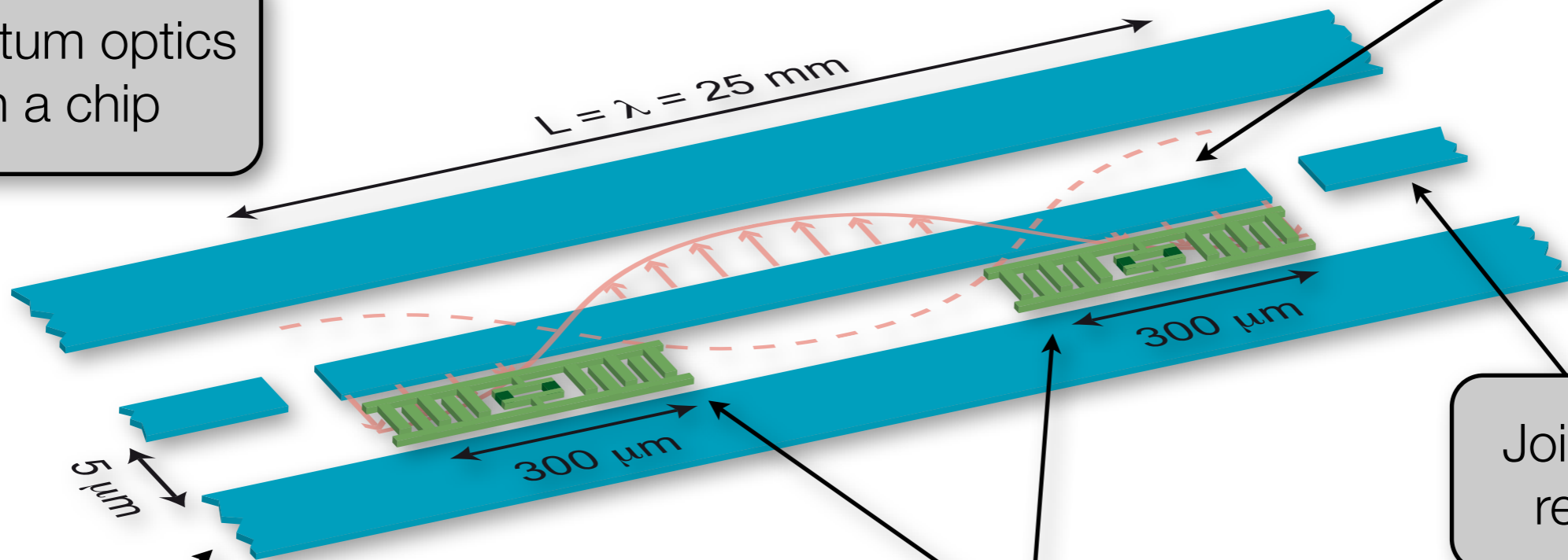
# Summary

Exploring quantum mechanics on large length scales

Quantum protocols implemented in solid-state quantum processor

Quantum bus mediating qubit-qubit interaction

Quantum optics on a chip



Superconducting high-Q resonator

Superconducting qubits with long coherence times

Joint qubit readout