

# Signatures expérimentales des interactions électroniques dans le transport mésoscopique

**Introduction** : ordres de grandeur, rappels transport cohérent

**Interactions et cohérence de phase**

Désordre, dimensionnalité, environnement électromagnétique, géométrie  
Comment déterminer le temps de cohérence de phase dans un système fini

**Interactions et propriétés thermodynamiques**

Correction à la densité d'états d'un système diffusif ; rôle de la dimensionalité  
Système localisé : gap de Coulomb  
Magnétisme orbital

**Transport hors d'équilibre**

Rectification mésoscopique et interactions

## **Que se passe t'il à la limite 1D ?**

Instabilité du liquide de Fermi,

Exemple fils quantiques et nanotubes de carbone

Peut on parler de comportement liquide de Luttinger dans les conducteurs 1D

Blocage de Coulomb dynamique et environnement électromagnétique

## **Interactions dans les boîtes quantiques**

Du blocage de Coulomb à l'effet Kondo

Contacts supraconducteurs : compétition entre effet Kondo et effet Josephson

# Interactions et densité d'états

$$G_{\epsilon}^{R,A} = (\epsilon - H \pm i0)^{-1} \quad \delta(\epsilon - H) = \text{Im}(G_{\epsilon}^R)/2\pi = (G_{\epsilon}^R - G_{\epsilon}^A)/2\pi$$

Régime diffusif sans interactions:  $\nu = \text{Tr}(\delta(\epsilon - H)) \sim N/E_F$   
Pas de correction quantique à la DOS moyenne

En présence d'interactions:  
Perturbation des niveaux électroniques à l'ordre 1:

$$\delta\epsilon_i^H = \sum_i f(\epsilon_j) \int U(r - r') \phi_j^*(r') \phi_j(r') dr' \phi_i^*(r) \phi_i(r) dr$$

Terme de Hartree

$$\delta\epsilon_i^F = \sum_j f(\epsilon_j) \int U(r - r') \phi_j^*(r') \phi_j(r) \phi_i^*(r) \phi_i(r') dr dr'$$

Terme de Fock (échange)

- Interactions et densité d'états

$$\Delta_\epsilon = \frac{1}{\nu_0} \sum_i \delta(\epsilon - \epsilon_i) \delta\epsilon_i \quad \frac{\delta\nu}{\nu_0} = -\partial\Delta_\epsilon/\partial\epsilon$$

Terme de Fock

$$\delta\epsilon_i^F = \sum_j f(\epsilon_j) \int U(r - r') \phi_j^*(r') \phi_j(r) \phi_i^*(r) \phi_i(r') dr dr'$$

$$\Delta_\epsilon^F = -\frac{1}{\nu_0} \int_{-\infty}^{+\infty} f(\epsilon - \omega) d\omega \sum_q U_q \operatorname{Re} \frac{1}{i\omega + Dq^2}$$

Diffuson: contribution dominante

$$\delta\nu^F = -U \left[ \frac{\epsilon}{D} \right]^{d/2} \Omega/\epsilon$$

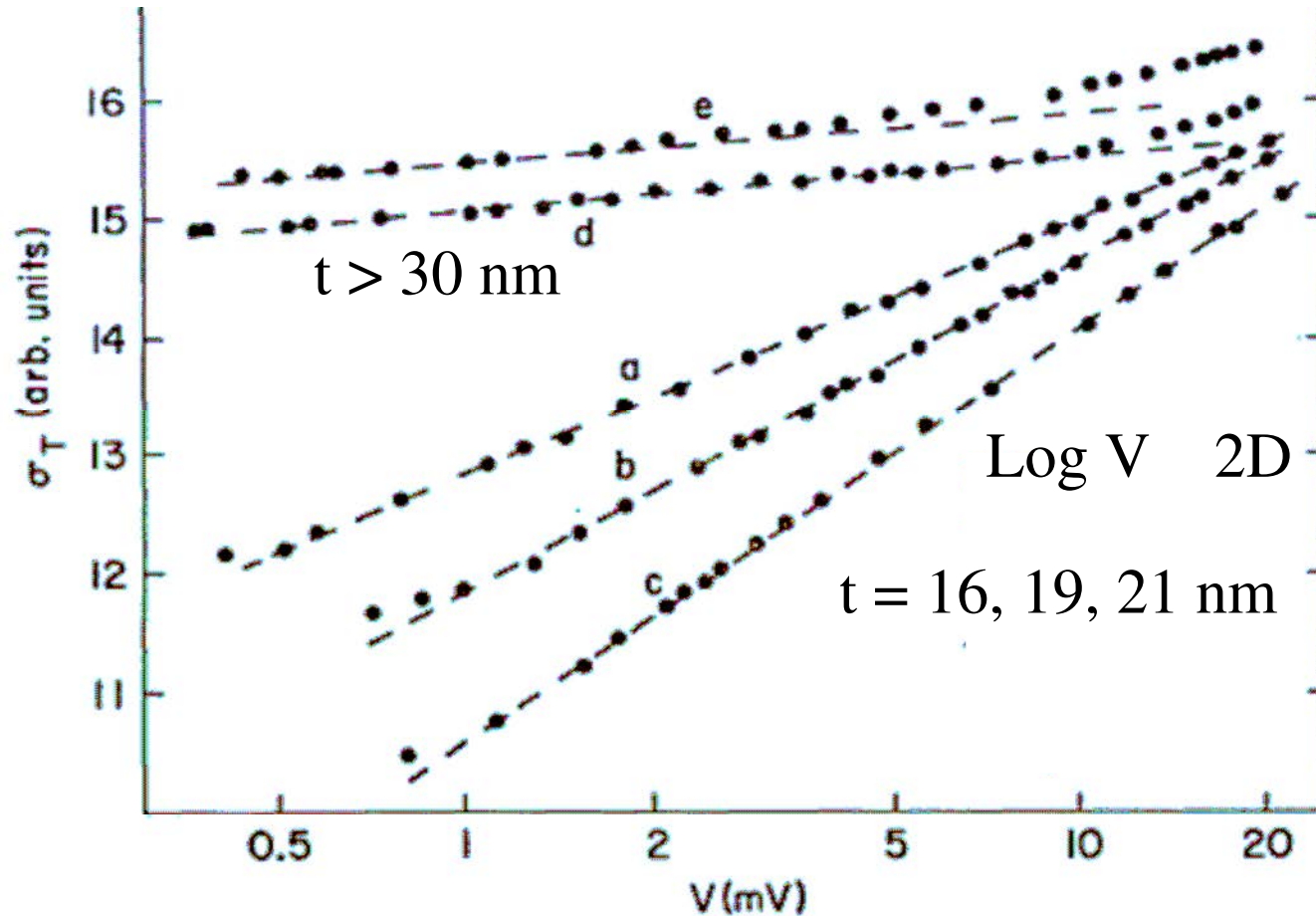
Anomalie en:  $\left\{ \begin{array}{ll} -\epsilon^{1/2} & \text{à 3D} \\ -\operatorname{Log}(\epsilon) & \text{à 2D} \\ -\epsilon^{-1/2} & \text{à 1D} \end{array} \right.$

Longueur caractéristique

$$L_c = (\hbar D / \epsilon)^{1/2}$$

$\delta\nu^H < \delta\nu^F$  si écrantage modéré

## Observation expérimentale: Conductance tunnel

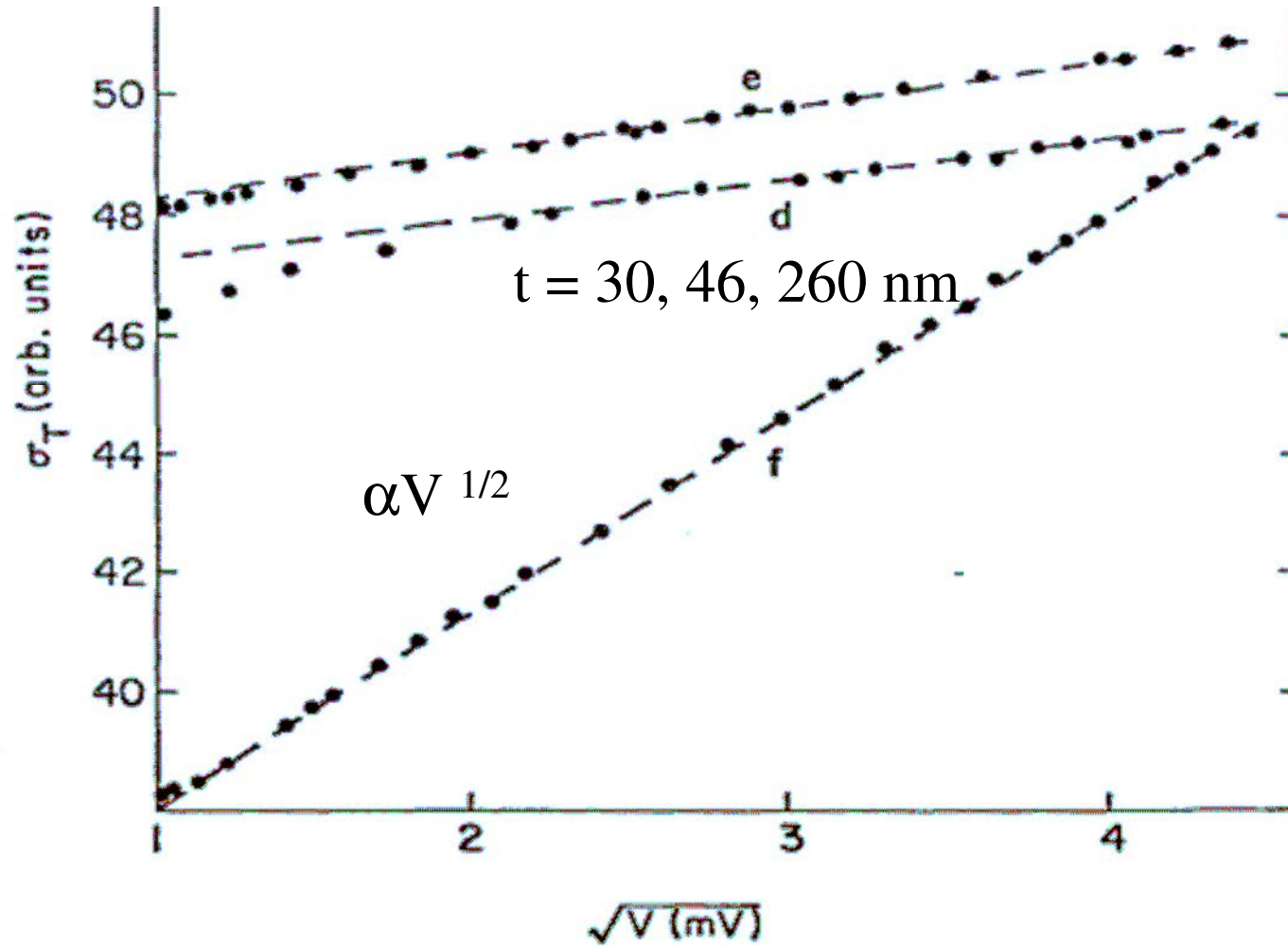


Films In ox épaisseur  $t$ : Imry, Ovadyau PRL 1982

$eV > k_B T$

Crossover 2D 3D  $t > L_V = (\hbar D / eV)^{1/2}$

## Observation expérimentale: Conductance tunnel

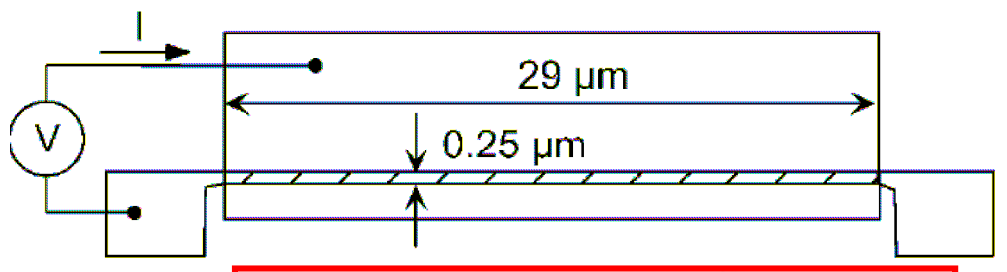
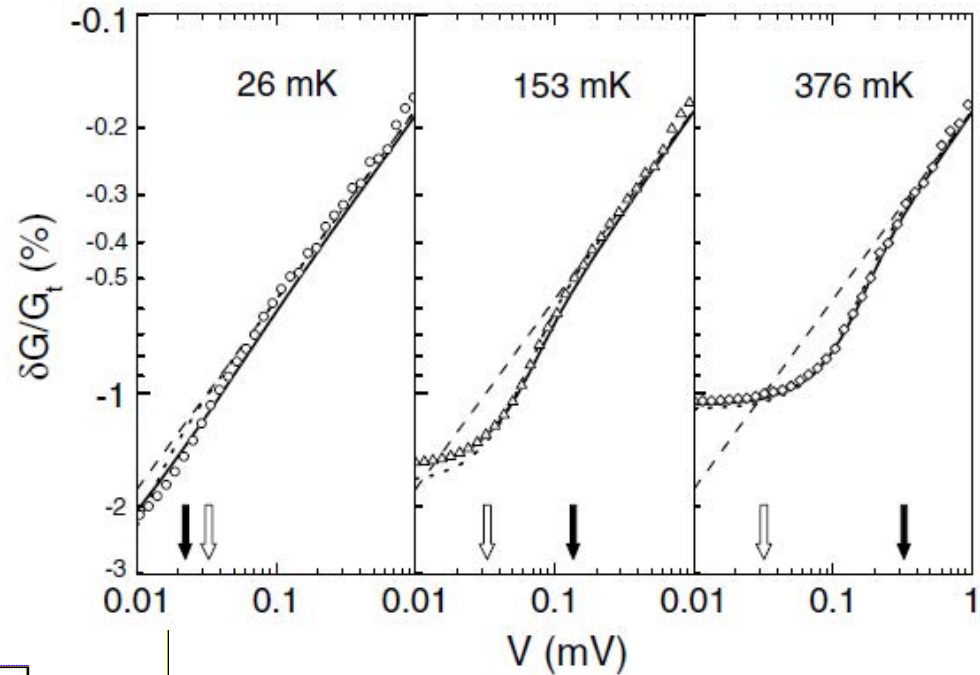


Films In ox épaisseur  $t$ : Imry, Ovadyau PRL 1982

# Tunneling into a 1D wire

*Pierre et al. PRL 2001*

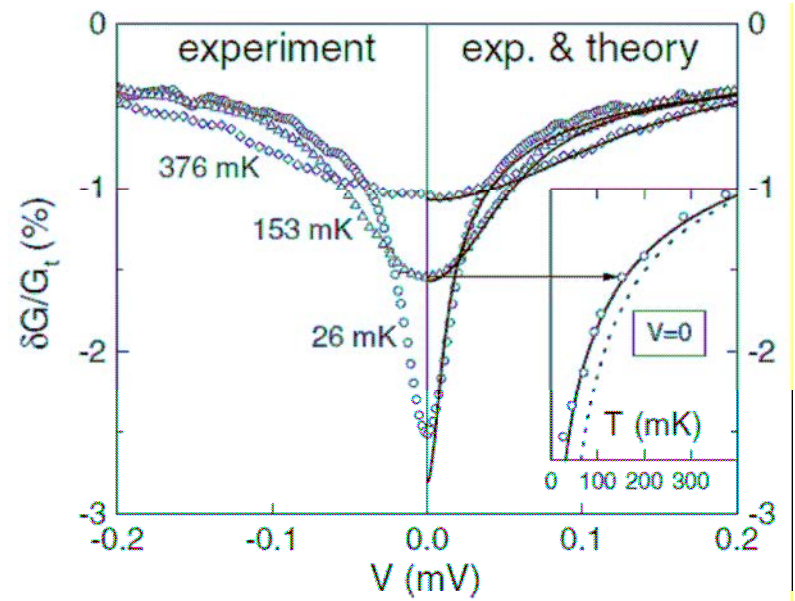
Influence de l'environnement électromagnétique



$$D^* = (R_{\square} C_s)^{-1} \gg D$$

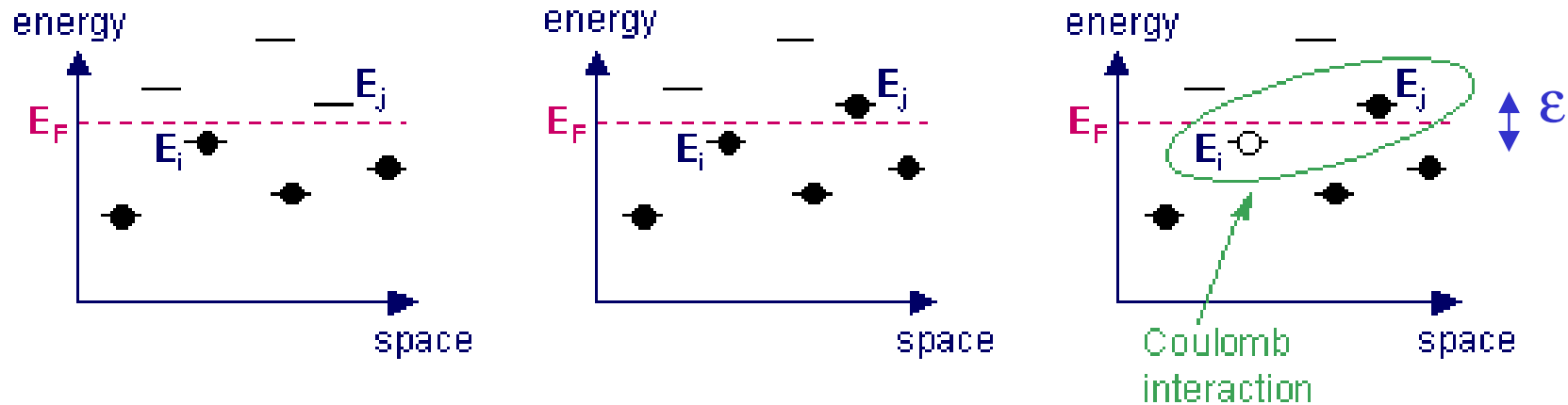
$$\delta G/G_t \approx -1.56 \frac{R_l}{R_K} \sqrt{\frac{\hbar D^*}{k_B T}} \quad \text{for } \frac{eV}{k_B T} \rightarrow 0$$

$$\delta G/G_t = -\sqrt{2} \frac{R_l}{R_k} \sqrt{\frac{\hbar D^*}{eV}} \quad \text{for } \frac{eV}{k_B T} \rightarrow \infty$$



# Localisation et interactions électroniques

*Efros Shklovskii J phys C 75*



Limite  $k_F l_e \sim 1$

La conductance s'effectue par sauts entre états localisés.

variation d'énergie correspondant à un saut de  $i$  vers  $j$  au voisinage de  $E_F$

$$\Delta E = E_j - E_i - e^2/R_{ij} \quad R_{ij} \sim (\epsilon v(\epsilon_F)/V)^{1/d}$$

La condition  $\Delta E > 0$  implique  $e^2/R_{ij} < \epsilon$   $v(\epsilon_F) = 0$  et  $v(\epsilon) \sim |\epsilon - \epsilon_F|^{d-1}$

Forte perturbation de la densité d'états par les interactions

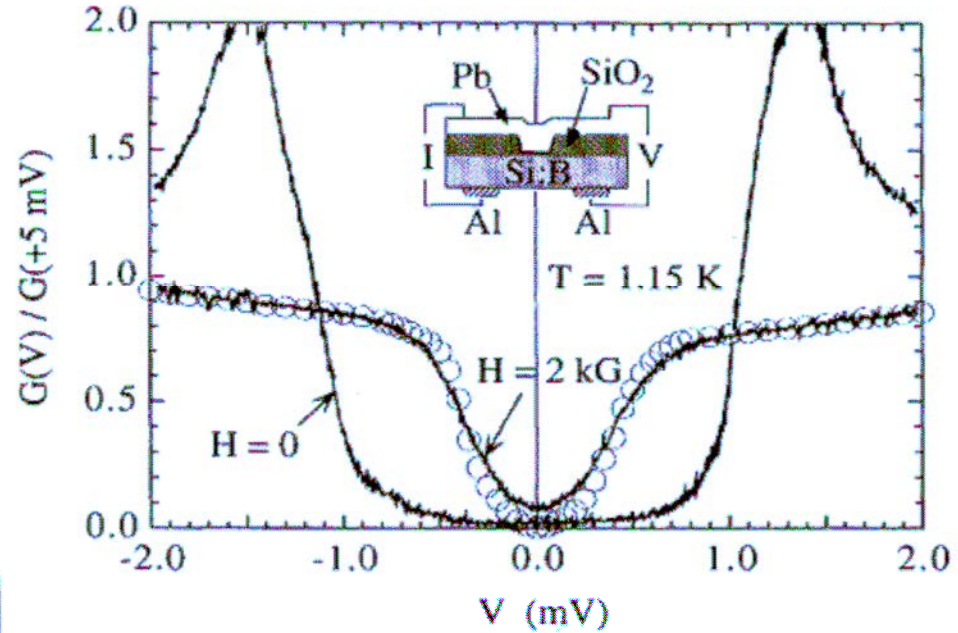


# Mise en évidence du gap d'Efros et Shklovskii

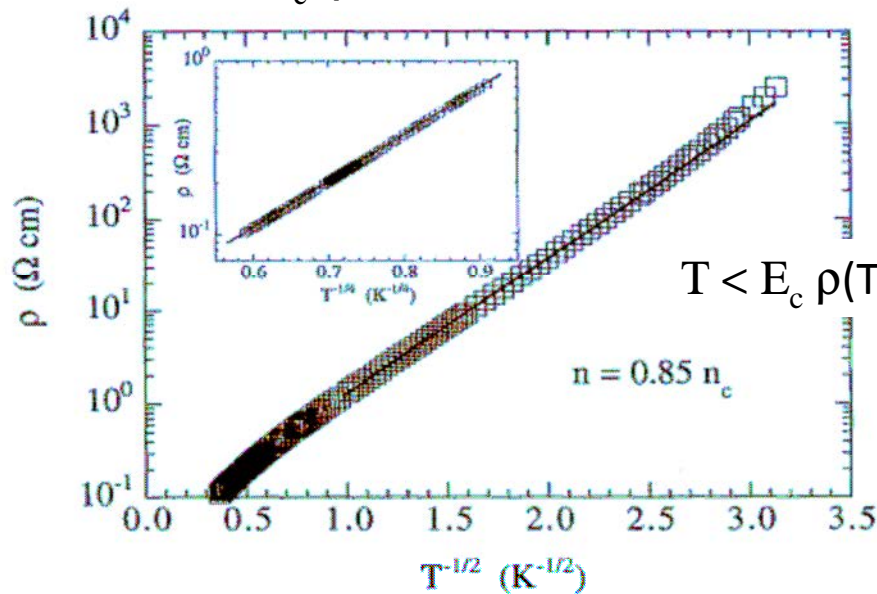
## 3D

$$\frac{G(V)}{G_0} = \int_{-0}^{+\infty} \frac{N(\epsilon)}{N_0} \left[ -\frac{\partial f(\epsilon - eV)}{\partial(eV)} \right] d\epsilon$$

Mesures de conductance tunnel sur Si:B  
Electrode Pb



$T > E_c$   $\rho(T) \sim \exp -b/T^{1/4}$



$T < E_c$   $\rho(T) \sim \exp -a/T^{1/2}$

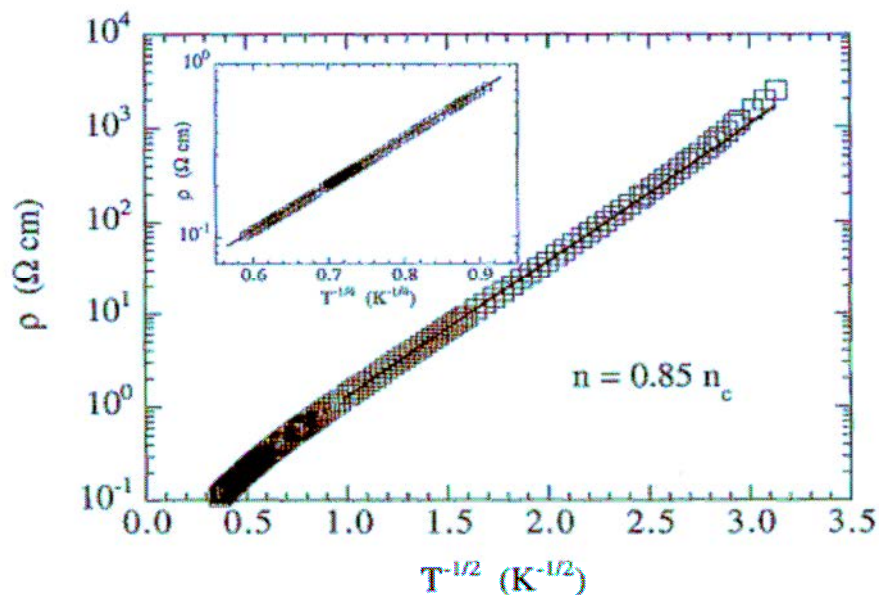
$$N(\epsilon) = \frac{3}{\pi} \left( \frac{\kappa}{e^2} \right)^3 (\epsilon - \epsilon_F)^2$$

# Mise en évidence du gap d'Efros et Shklowskii

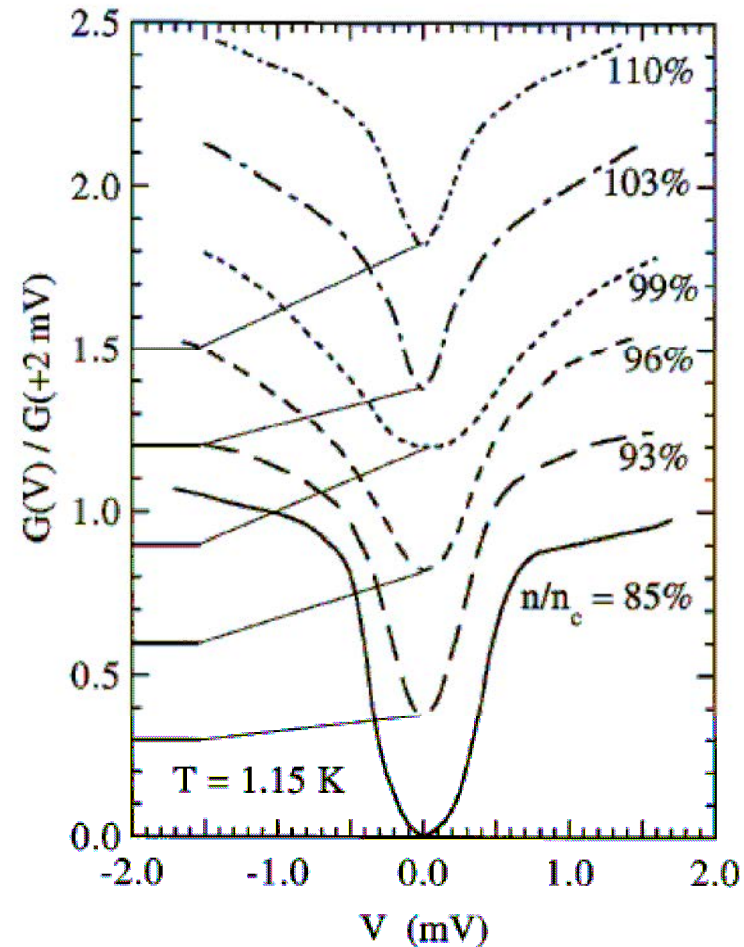
3D

$$\frac{G(V)}{G_0} = \int_{-0}^{+\infty} \frac{N(\varepsilon)}{N_0} \left[ -\frac{\partial f(\varepsilon - eV)}{\partial(eV)} \right] d\varepsilon,$$

Mesures de conductance tunnel sur Si:B Electrode Pb



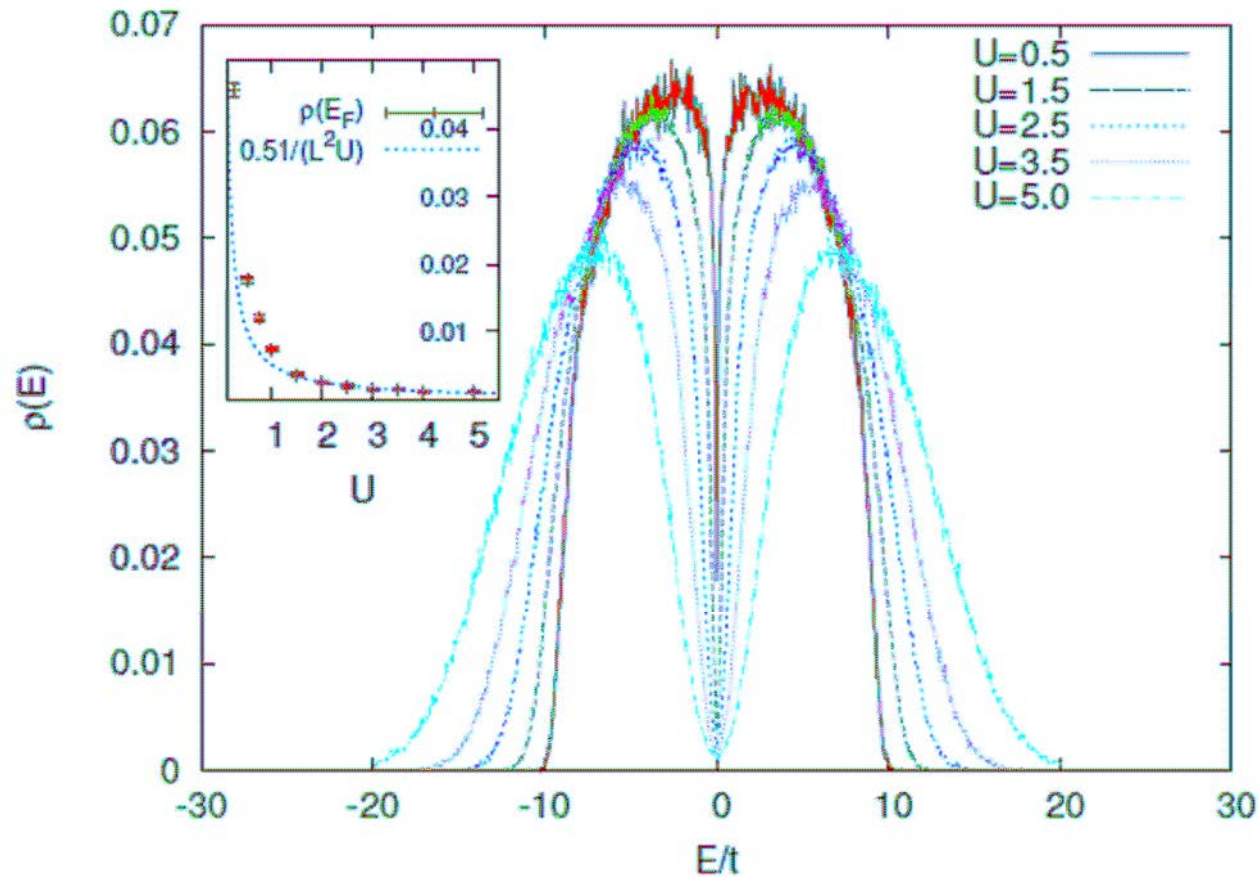
Fort dopage: Anomalie Altshuler Aronov LogV



Faible dopage: gap de Coulomb

Massey, Lee PRL 96

# Simulations numériques

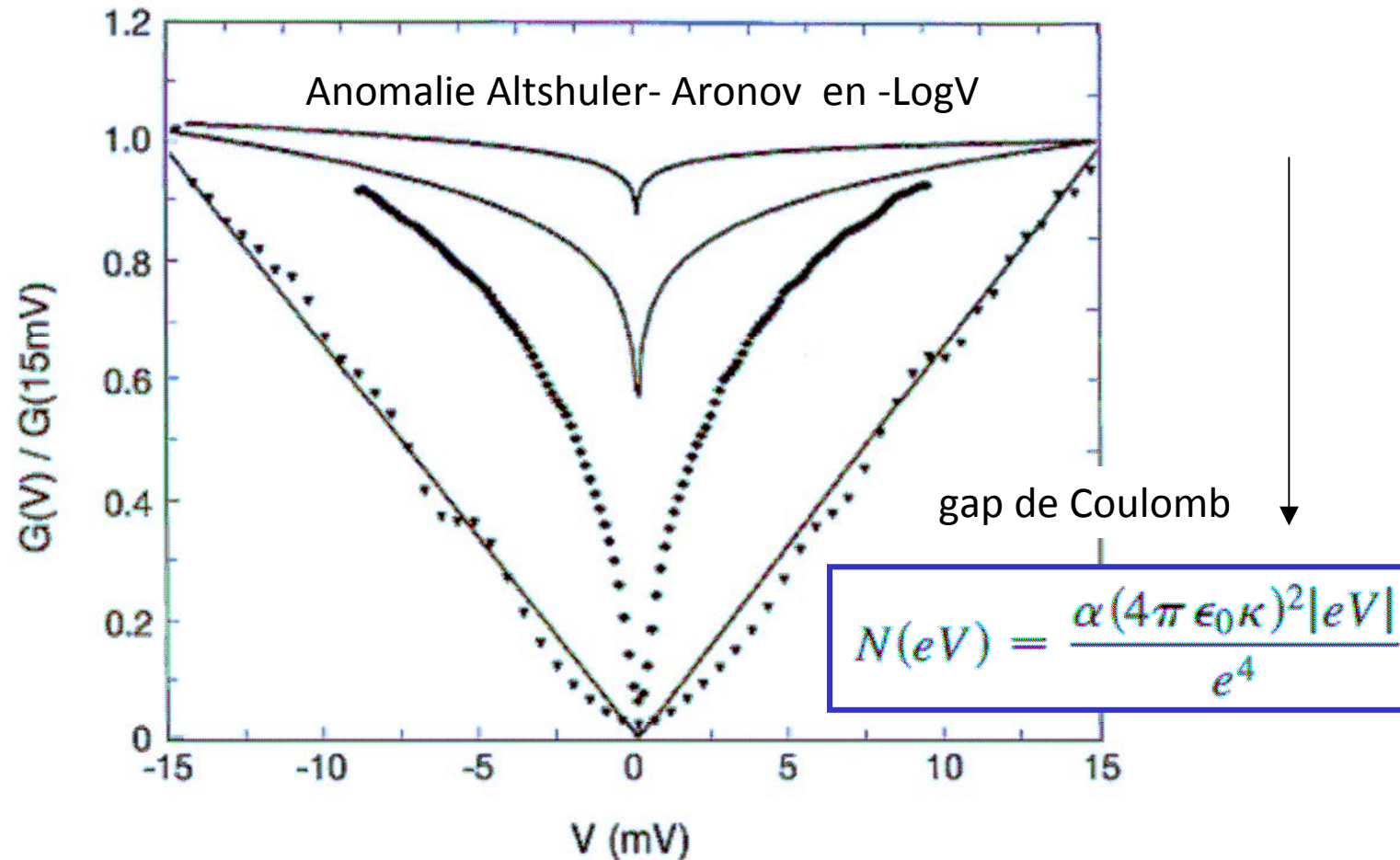


*Amini et al. 2013*

$$H_0 = \sum_i (\epsilon_i - \mu) c_i^\dagger c_i - t \sum_{\langle ij \rangle} c_i^\dagger c_j + \text{h.c.}, \quad H_1 = \frac{U}{2} \sum_{i,j} \frac{n_i n_j}{r_{ij}}.$$

# Mise en évidence du gap d'Efros et Shklowskii

2D



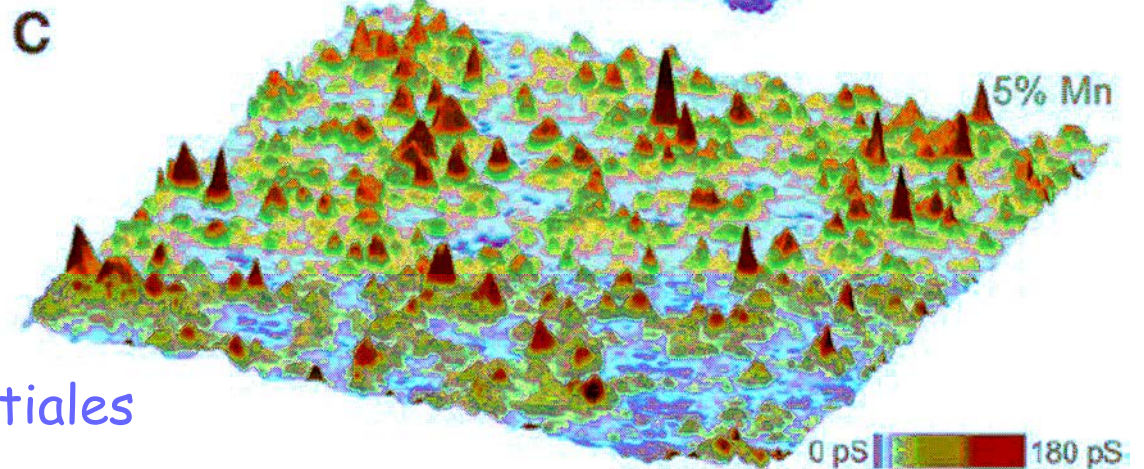
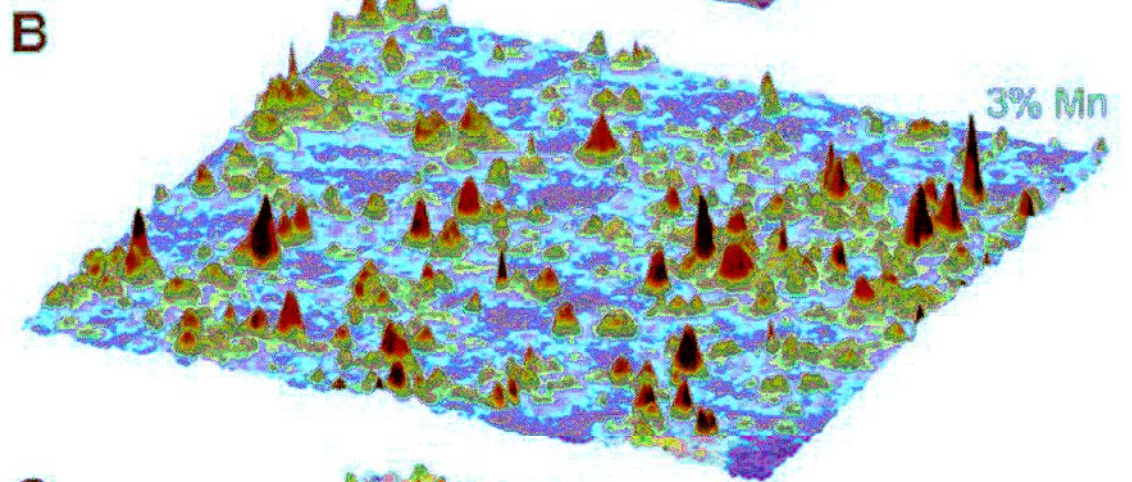
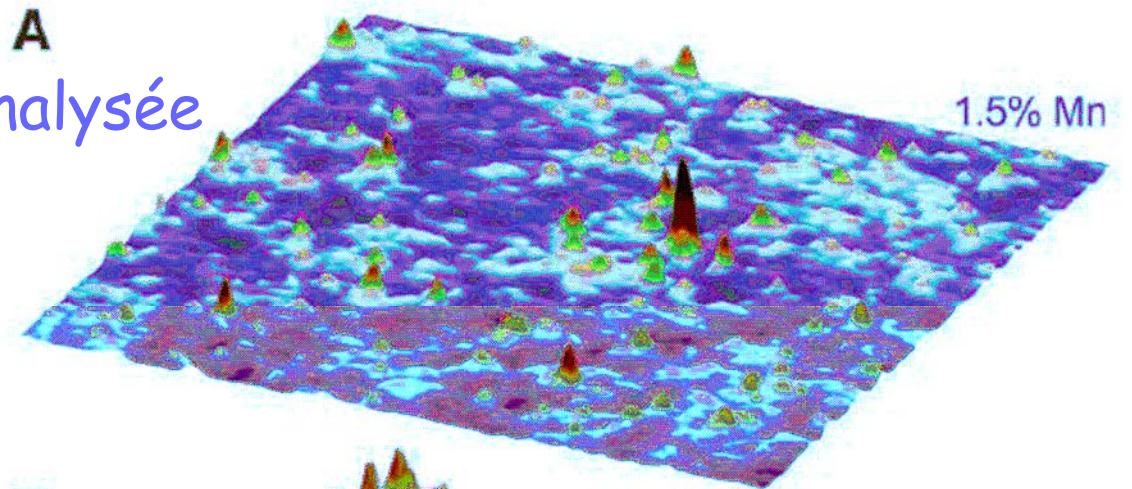
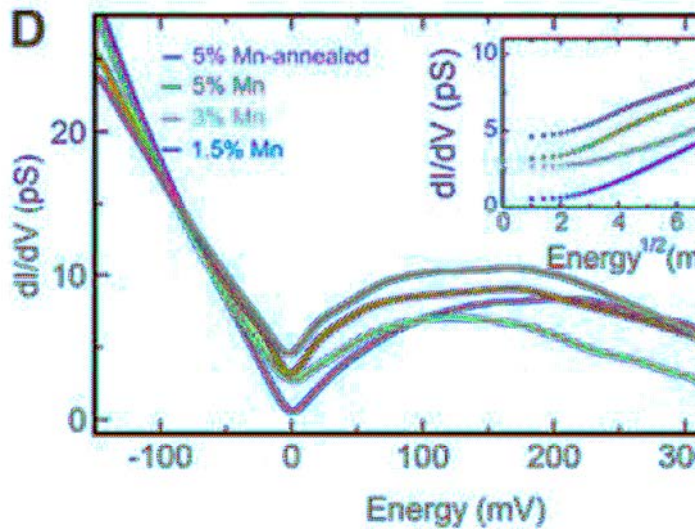
Films de Be différentes épaisseurs

Butko et al. 2000

# Transition métal isolant analysée par spectroscopie tunnel

GaAsMn

*Richardella et al. Science 2012*



Analyse des corrélations spatiales

# Interactions et magnétisme orbital

Courants permanents et effet Aharonov Bohm

Problème de la moyenne d'ensemble

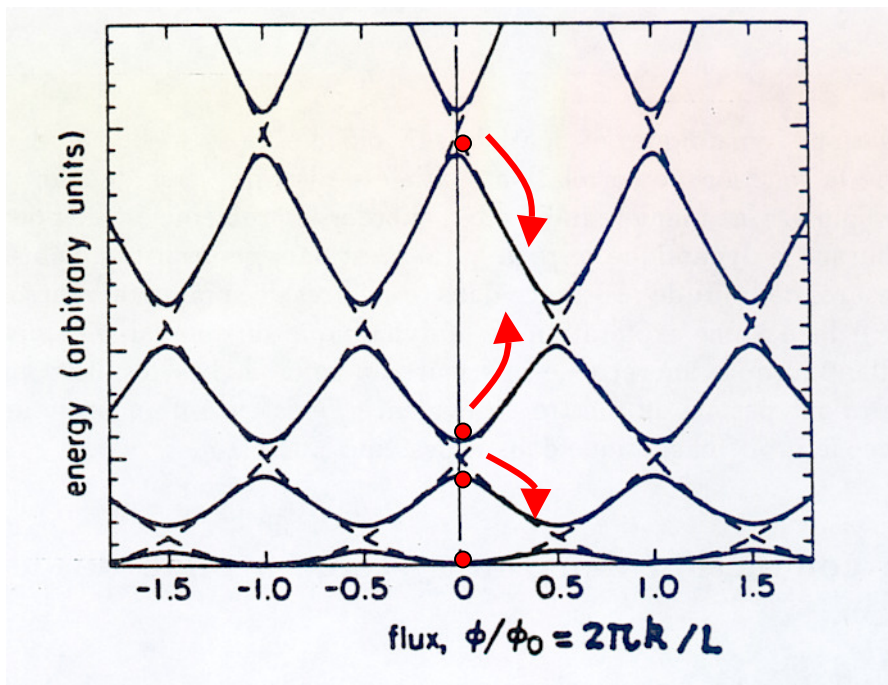
Rôle des interactions en relation avec les corrections quantiques à la DOS (Cooperon)

# Spectre d'un anneau 1D

$$k = (2\pi/L)(\Phi/\Phi_0)$$

Ondes de Bloch sur une maille du réseau réciproque  $2\pi/L$ ,  
niveaux d'énergie de l'anneau décrit les bandes

$$E_n[k = (2\pi/L)(\Phi/\Phi_0)]$$



Chacun des niveaux d'énergie de l'anneau porte  
un courant  $\mathbf{i}_n = e\mathbf{v}_n/L$

$$\begin{aligned} \text{où } v_n &= \langle n' | \mathbf{P} / m | n' \rangle = \langle n | (\mathbf{P} - e\mathbf{A}) / m | n \rangle \\ &= \partial E_n / \partial k \quad |k(\Phi) = -(2\pi/L)(\Phi/\Phi_0) \end{aligned}$$

$$\text{d'où } i_n(\Phi) = ev_n / L = -\partial E_n / \partial \phi$$

Ondes de Bloch

dans un potentiel de période  $L=2\pi R$

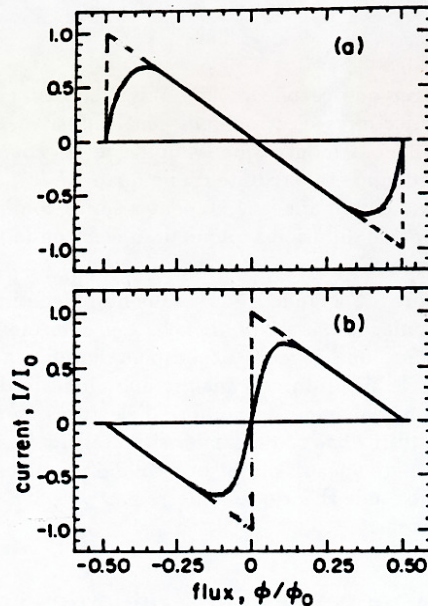
## Courant permanent

$I(N) = \sum_1^N s i_n(\Phi) = -\partial E(N)/\partial \phi$  où  $E(N)$  est l'énergie totale de l'anneau contenant  $N$  électrons,

$M(N) = I(N) \times S$  cette relation thermodynamique est équivalente à celle reliant l'aimantation au champ magnétique dans un système :

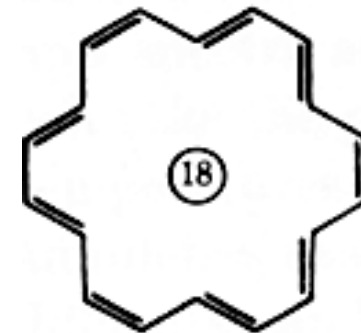
$$M(N) = -\partial E(N)/\partial B$$

Molécules Aromatiques!



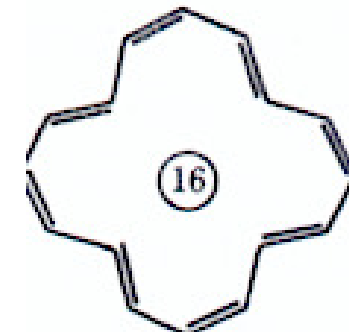
$$N=4n+2$$

Diamagnétisme



$$N=4n$$

Paramagnétisme



$$I = -\frac{\partial E}{\partial \Phi}$$

$$\sim ev_F/L$$



## Courant permanent

$I(N) = \sum_1^N s i_n(\Phi) = -\partial E(N)/\partial \phi$  où  $E(N)$  est l'énergie totale de l'anneau contenant  $N$  électrons,

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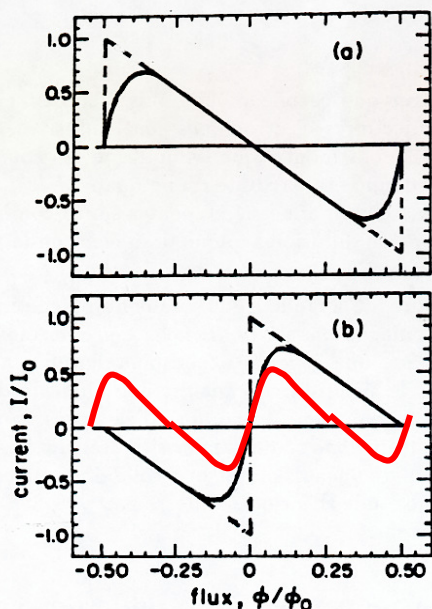
$$M(N) = -\partial E(N)/\partial B$$

Molécules Aromatiques!

$$I = -\frac{\partial E}{\partial \Phi}$$

$$\sim e v_F / L$$

$$I_{N+1}(\Phi) = I_N(\Phi + \Phi_0/2)$$



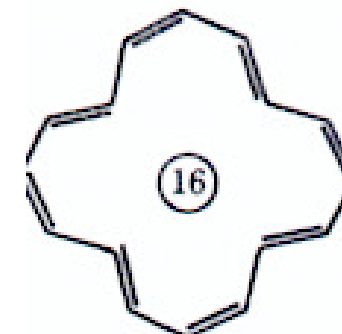
$$N=4n+2$$

Diamagnétisme



$$N=4n$$

Paramagnétisme

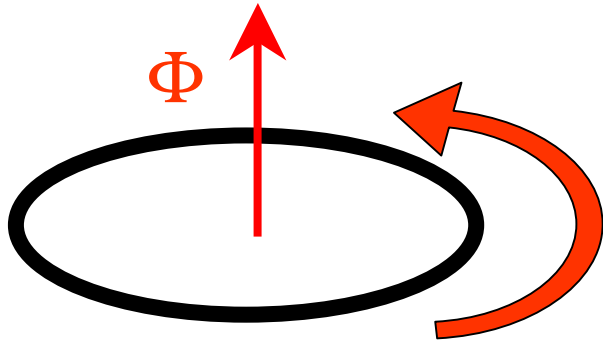


Moyenne non nulle  
périodique en  $\Phi_0/2$

# Courant Permanent d'un anneau mésoscopique

Energie totale :  $E(\Phi) = \sum_{n \text{ occupé}} E_n(\Phi)$  oscille avec le flux

Courant permanent  $I = \langle I \rangle = \langle -\partial H / \partial \phi \rangle$



$$I = -\frac{\partial E}{\partial \Phi} = I_0 \sin(2\pi\Phi/\Phi_0) + \text{harm}$$

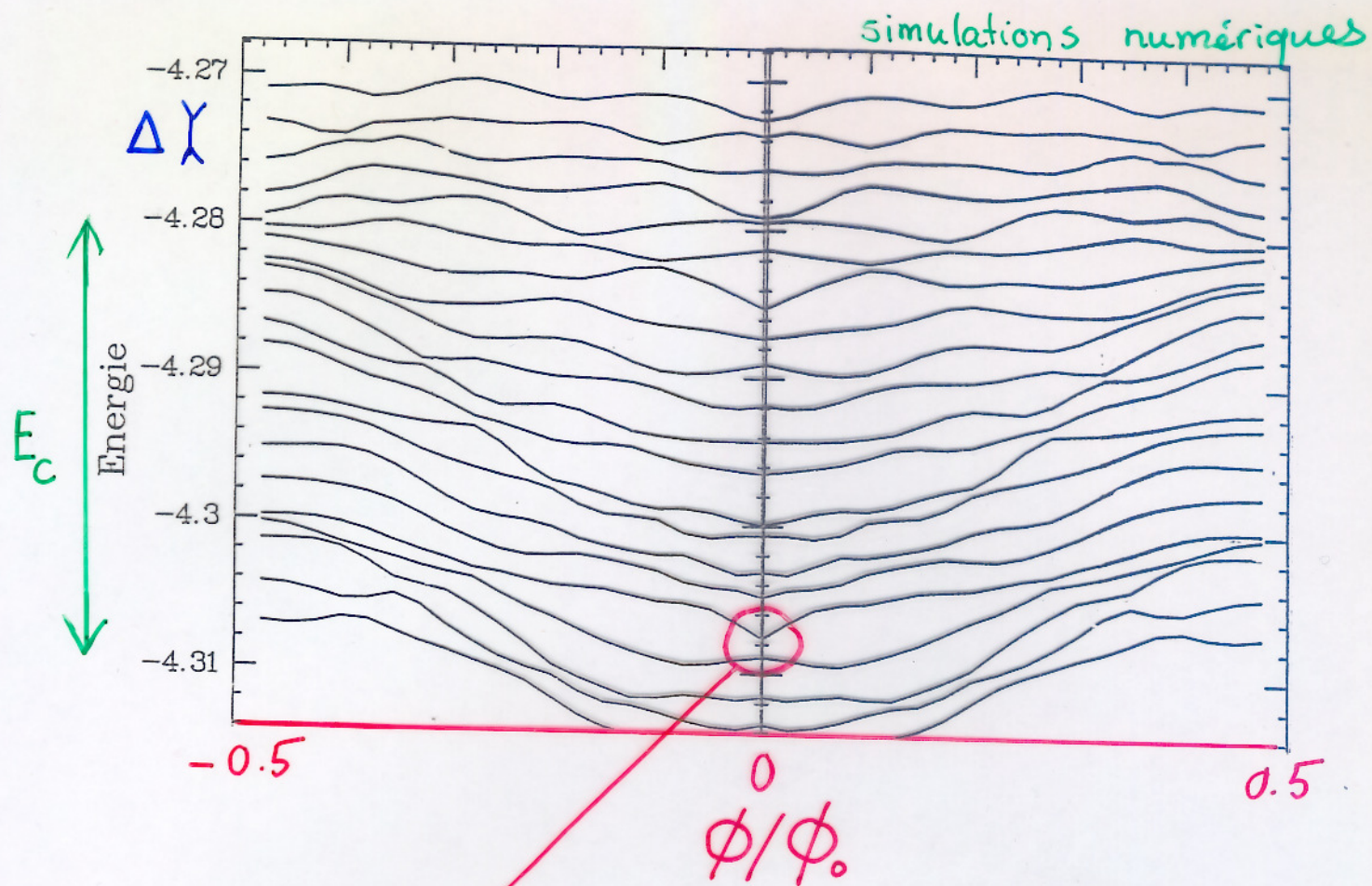
Résiste au désordre !

Existe dans un anneau de résistance finie!

Moment magnétique:

$$M = -dE/dB = I S \sim 100 \text{ spins mesurable!}$$

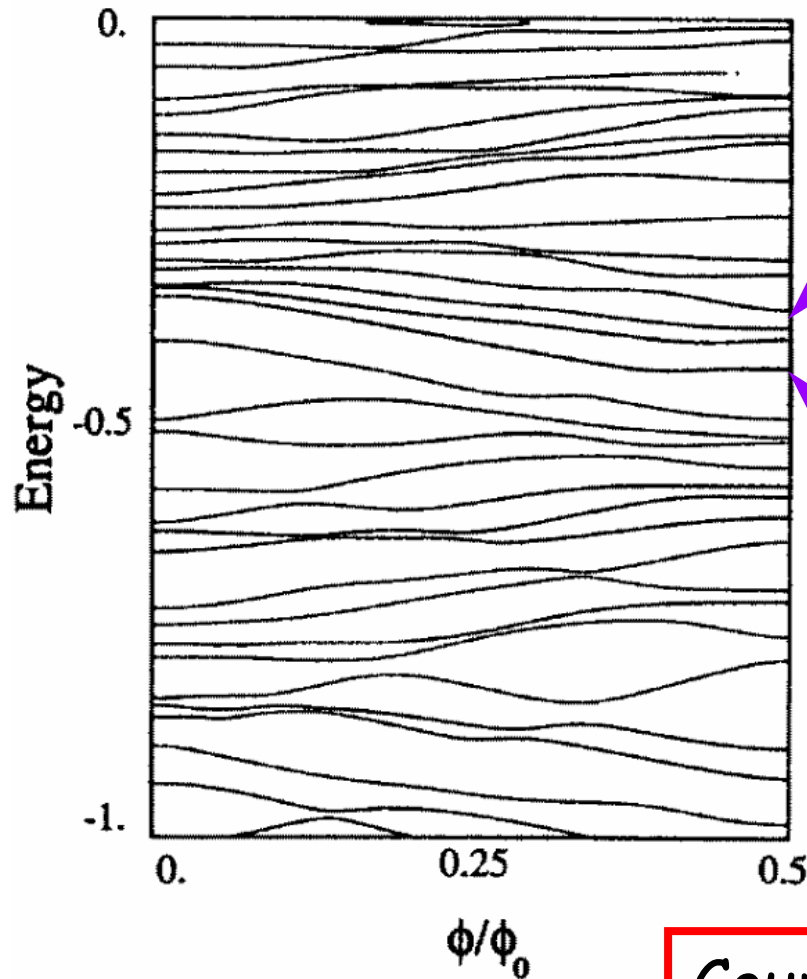
# SPECTRE D'UN ANNEAU DIFFUSIF 3D $l_e < L$



ENERGIE DE CORRELATION:

$$E_C = h/\tau_D = hD/L^2$$

# Spectre d'un anneau désordonné



Spectre corrélé

Energie de Thouless

$$E_c = h D / L^2 = h / \tau_D$$

$\tau_D = L^2 / D$  temps de diffusion autour de l'anneau

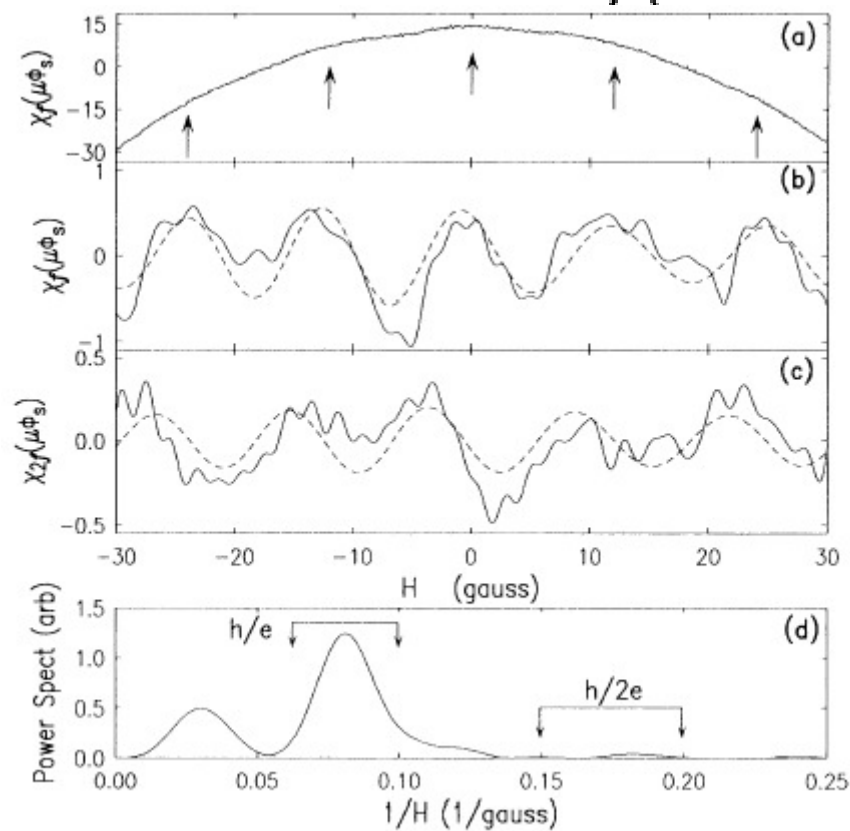
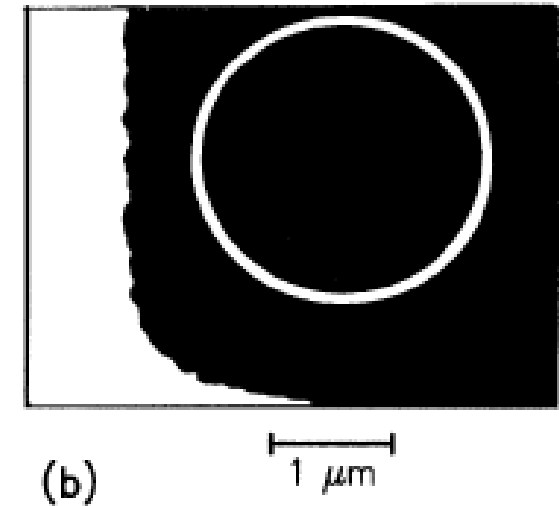
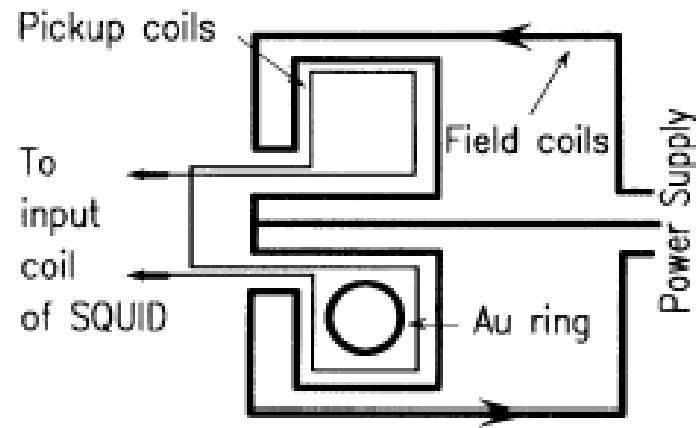
Décrit par la théorie des matrices aléatoires sur cette échelle d'énergie

Courant permanent: variable aléatoire

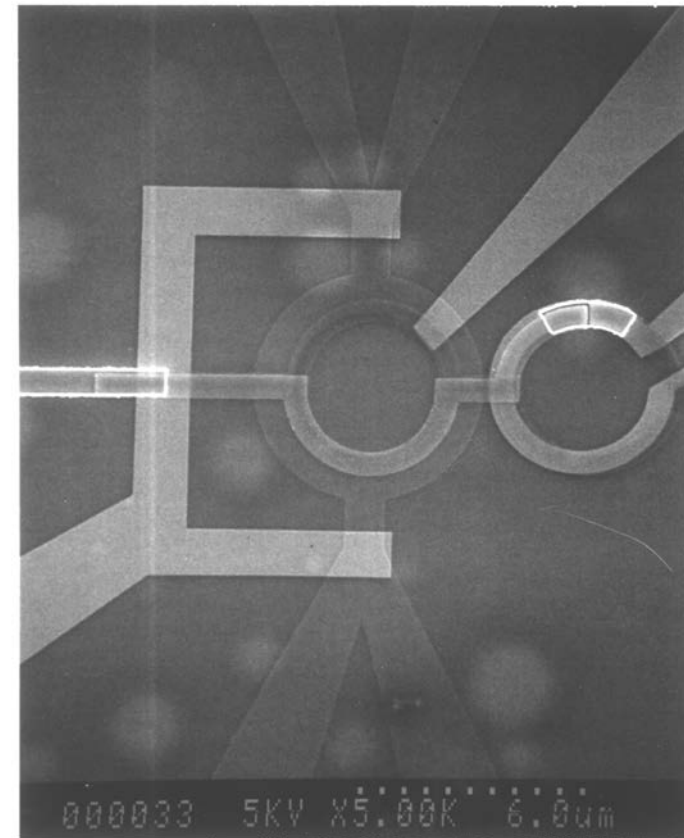
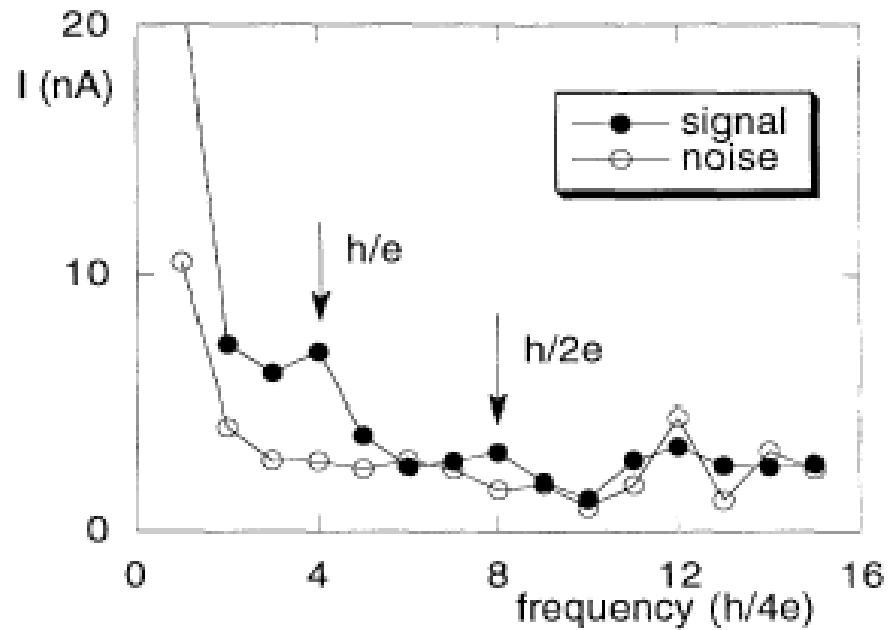
$$\langle I^2 \rangle^{1/2} = e / \tau_D = I_0 |e / L$$

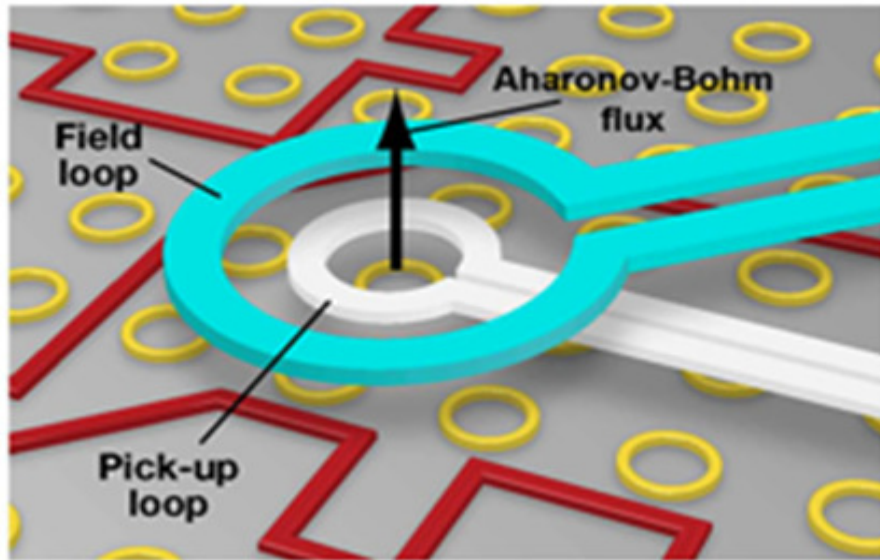
# Magnetic moment of an individual ring:

Chandrasekar, Webb (1991)

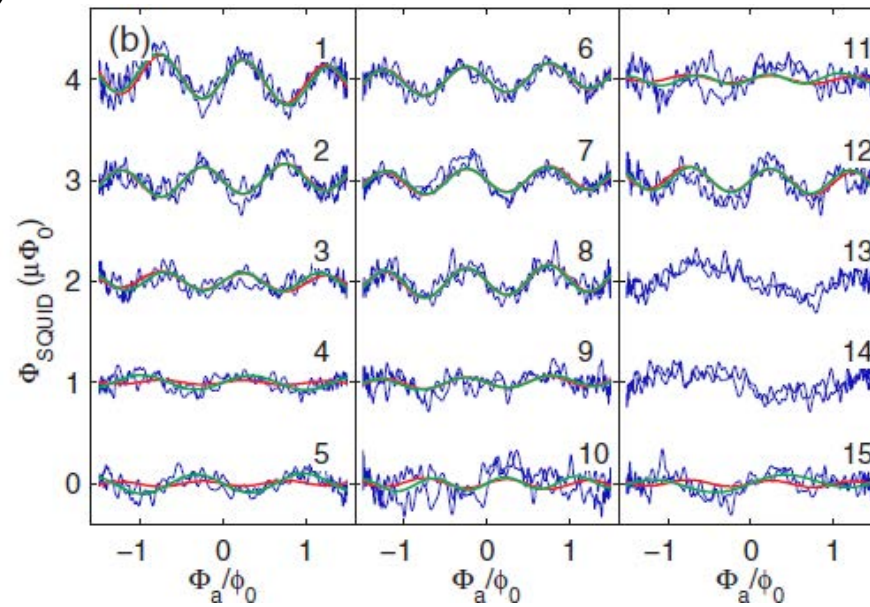


*Experimental set up for the detection  
of the magnetic moment of an individual  
mesoscopic ring (D.Mailly et al. 1993)*

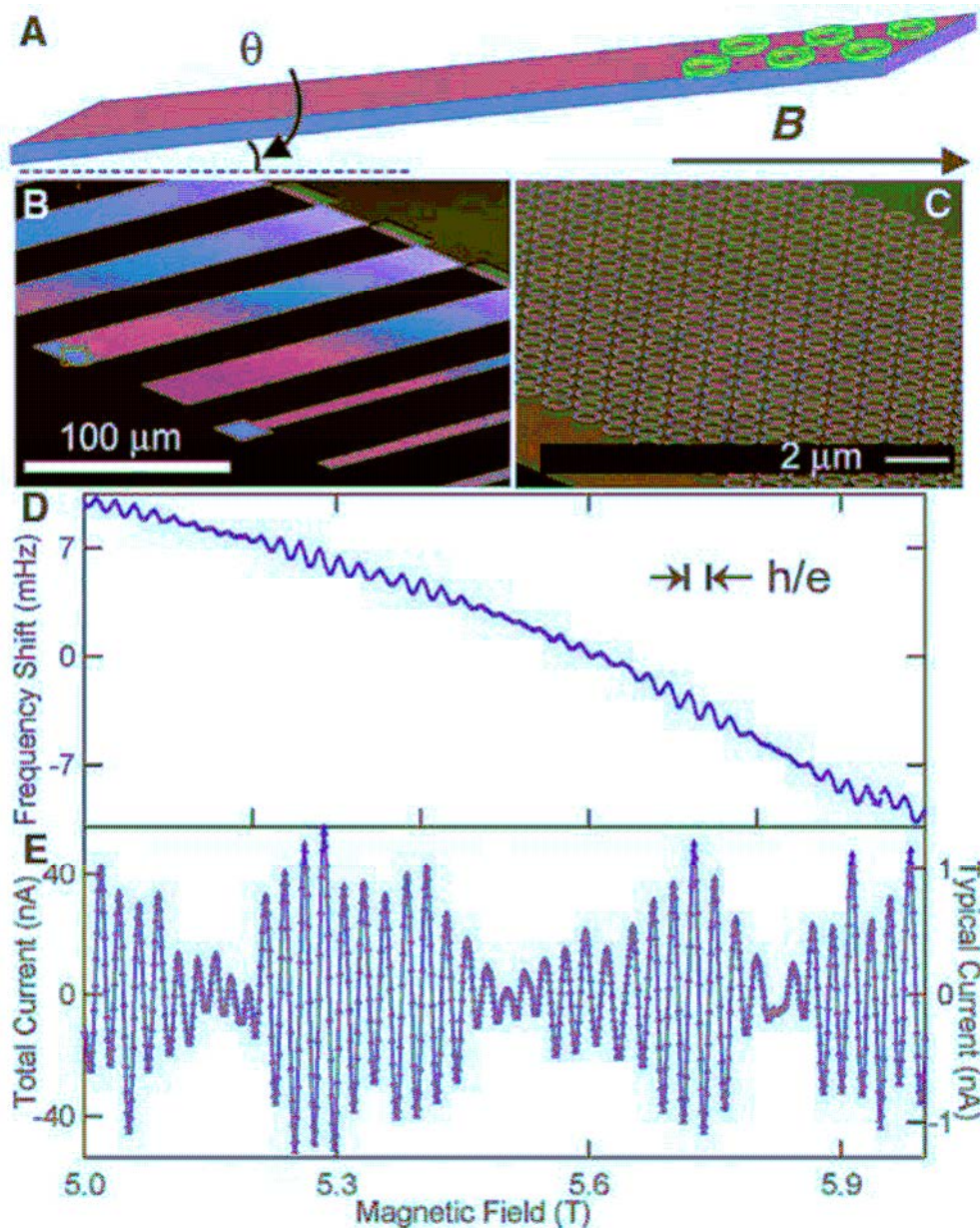




*Set up for magnetic detection of persistent currents  
(Bluhm et al. PRL 2008)*



# Mesures de couple en fort champ magnétique



Oscillateur mécanique  
fréquence de résonance

$$\propto \mathbf{M} \times \mathbf{B}$$

Signal mesuré  $\propto N^{1/2}$

Périodique en  $h/e$

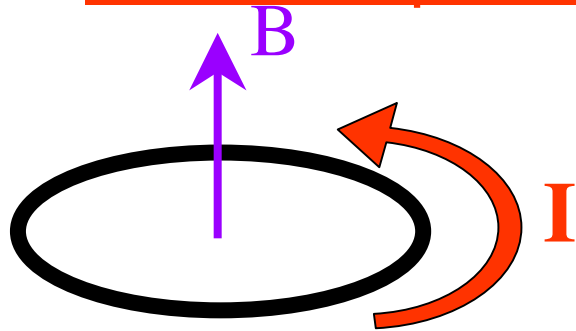
Correspond à

$\pm e/\tau_D$  par anneau

*A. Beya Bleszynski et al. 2009*



# Courants permanents et Moyenne d'ensemble



Echelle de courant :  $e/\tau_D$   
avec  $\tau_D = L^2/D$

Mesure sur un échantillon

$\neq$   
Moyenne

Courant typique :  $I_{typ} = \sqrt{\langle I^2 \rangle}$

- mesure sur **un anneau**
- périodique en  $\Phi_0$

Courant moyen :  $I_{moy} = \langle I \rangle$

- mesure sur **une assemblée d'anneaux**
- périodique en  $\Phi_0/2$

2 contributions :

1- Moyenne « canonique » N fixe ind de F =  $\langle I \rangle = \Delta/\Phi_0 \sin 4\pi\Phi/\Phi_0$

**Anneau métallique**  $\Delta \sim \text{qq}\mu\text{K}$

2-Interactions e-e

$$\delta E = e^2 \int V(\mathbf{r}-\mathbf{r}') n(\mathbf{r})n(\mathbf{r}') d\mathbf{r}d\mathbf{r}'$$

$$V(\mathbf{r}) = U \delta(\mathbf{r})$$

$$\langle I \rangle = - e^2 U \frac{d}{d\Phi} \left[ \int \langle n(\mathbf{r})^2 \rangle d\mathbf{r} \right]$$

# Magnétisme orbital et interactions électroniques

$$M_{ee} = -\frac{\nu U}{2\pi} \partial_B \int_0^{+\infty} \omega \underbrace{\text{Re} P_c(\omega, B)}_{\text{Cooperon}} d\omega = -\partial_B \delta E_{ee}(B)$$

Correction quantiques à l'énergie liée aux interactions  
**Cooperon**

Dans un anneau :  $\sum_n \text{Re} \frac{1}{i\omega + Dq_n^2 + 1/\tau_\phi}$

$$q_n = n2\pi/L + 4\pi\phi/\phi_0$$

Ressemble à la correction de localisation faible à la conductance moyenne:

$$\delta G_Q(\omega) = (4e^2/h) \frac{D}{L^2} \sum_n \text{Re} \frac{1}{i\omega + Dq_n^2 + 1/\tau_\phi} \propto \int_0^{+\infty} \exp(-t/\tau_\phi) P_c(r, r, B, t) dt$$

$$I_{ee} = M_{ee}/S = \nu U E_{Th} / \phi_0 \sin(4\pi\phi/\phi_0) + \text{harm...}$$

$$E_c = \hbar/\tau_D = \hbar D/L^2$$

Susceptibilité magnétique  $\chi_{ee} = dI_{ee}/dB |_{B=0} = \nu U (e^2/4\pi^2 m) k_{F1e} \sim \chi_L k_{F1e}$

$$\chi_L = -e^2/12\pi m = -2/3 \mu_B^2 \rho_0$$

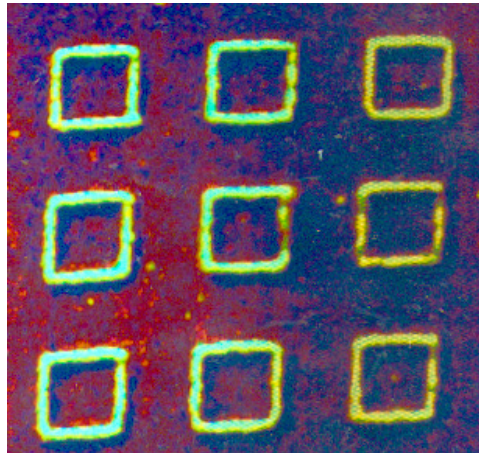
Signe donné par celui des interactions

*Ackermans, Montambaux*

Expérimentalement signe diamagnétique!

Rôle des fluctuations supraconductrices?

# Mesure du courant moyen



0.5  $\mu\text{m}$

Magnétométrie à SQUID

Lévy et al., PRL (1990)

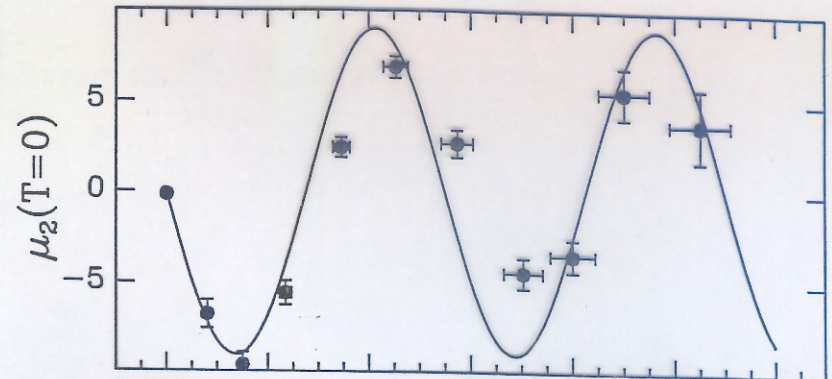
$10^7$  anneaux de cuivre :

- courant moyen ( $\Phi_0/2$ )
- courant de l'ordre de  $e/\tau_D$
- signe diamagnétique

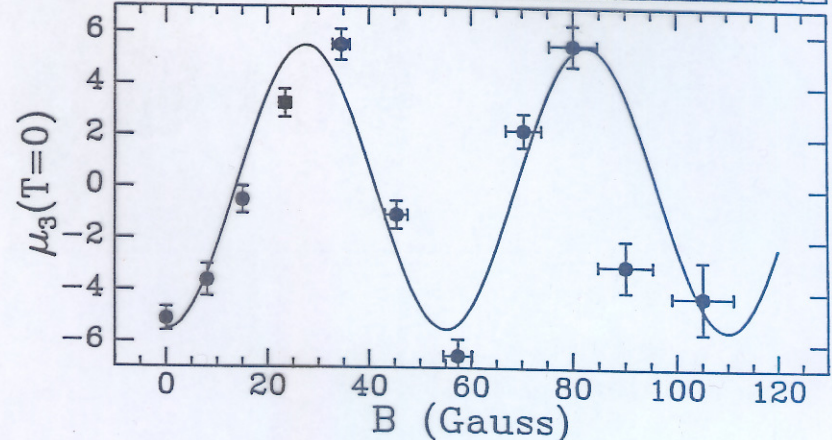
MOMENT MAGNETIQUE DE  $10^7$  ANNEAUX DE CUIVRE

$T = 10\text{mK}$

$\frac{\partial^2 M}{\partial H^2}$



$\frac{\partial^3 M}{\partial H^3}$

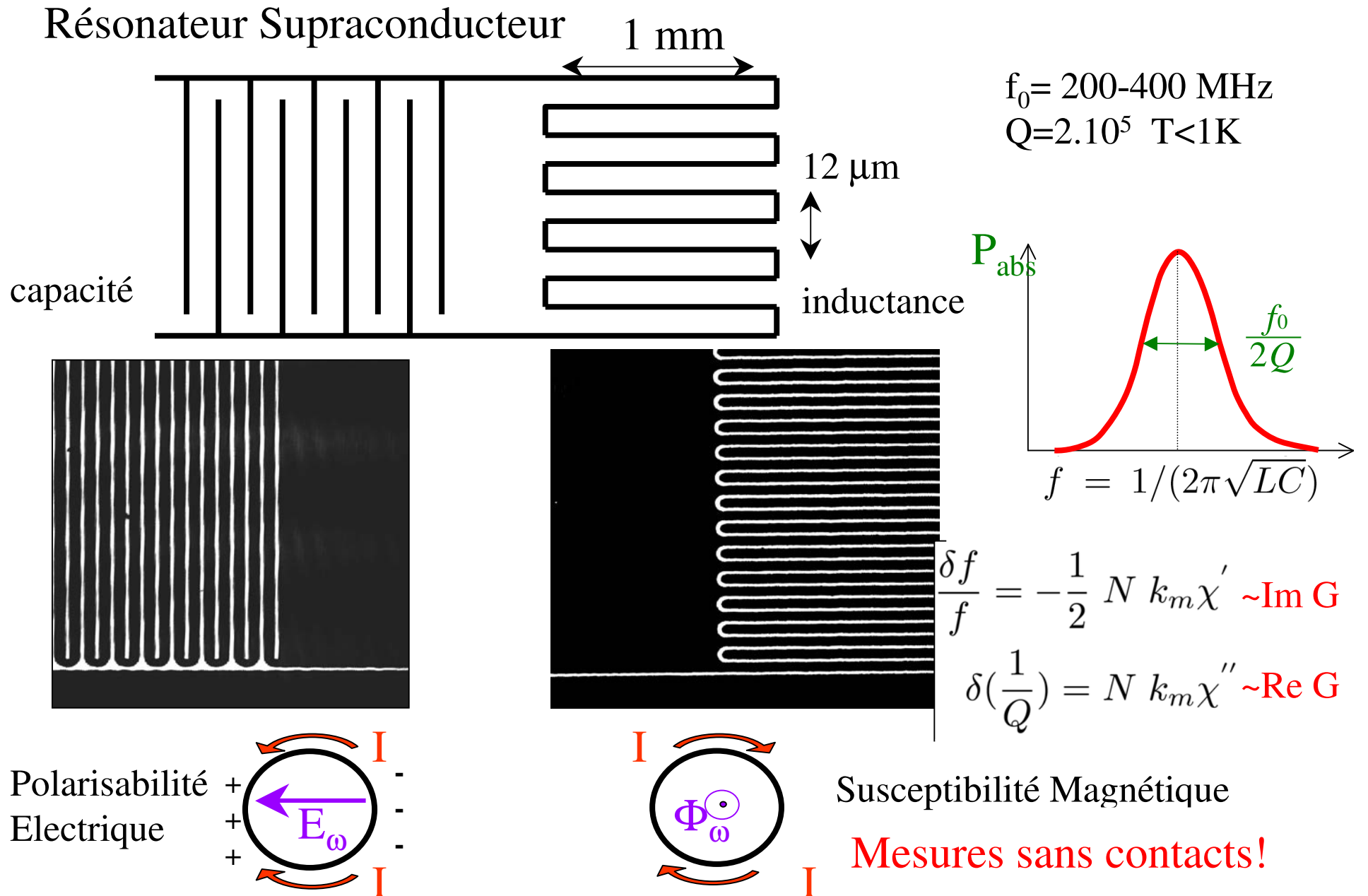


PERIODICITE :  $\phi_0/2$

AMPLITUDE  $\sim 0.3 e/\tau_D$  PAR ANNEAU

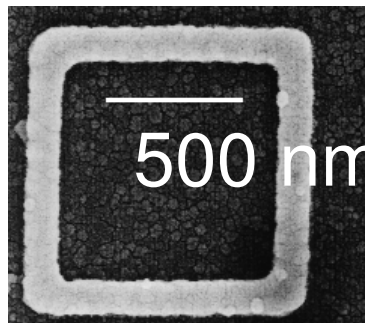
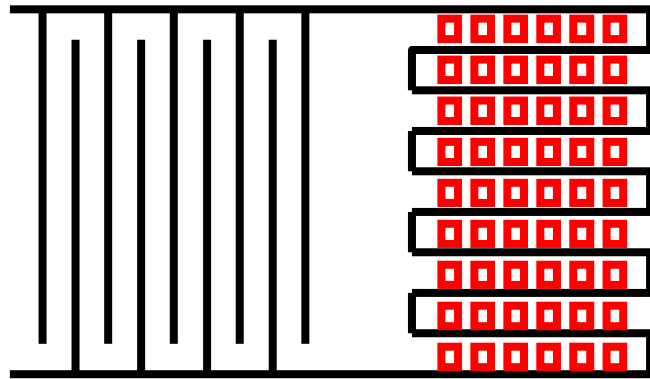
> prédictions théoriques

# Mesures ac sur des anneaux isolés



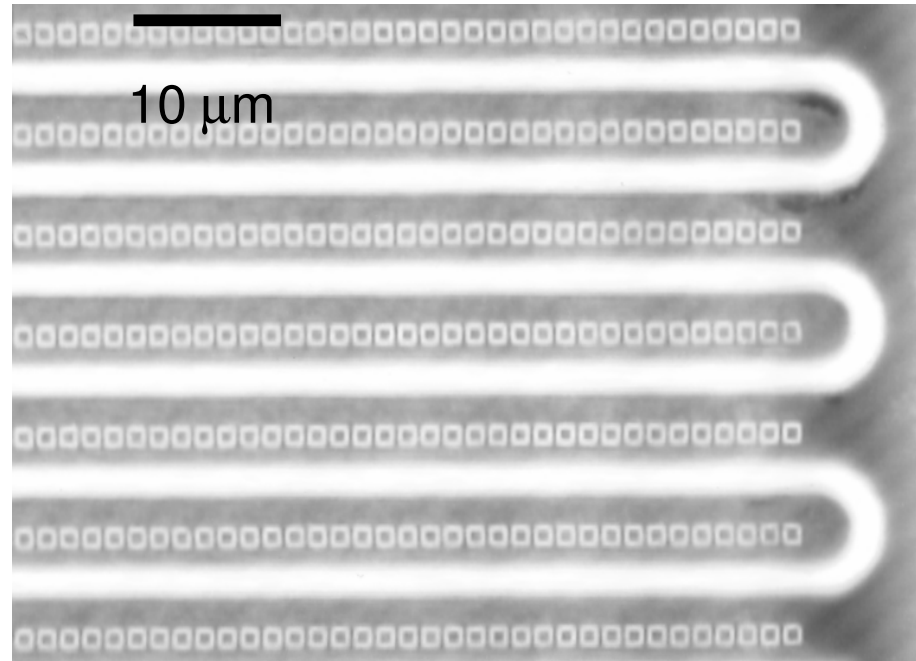
# Réponse magnétique d'Anneaux d'Argent

résonateur



Deblock et al. 2002

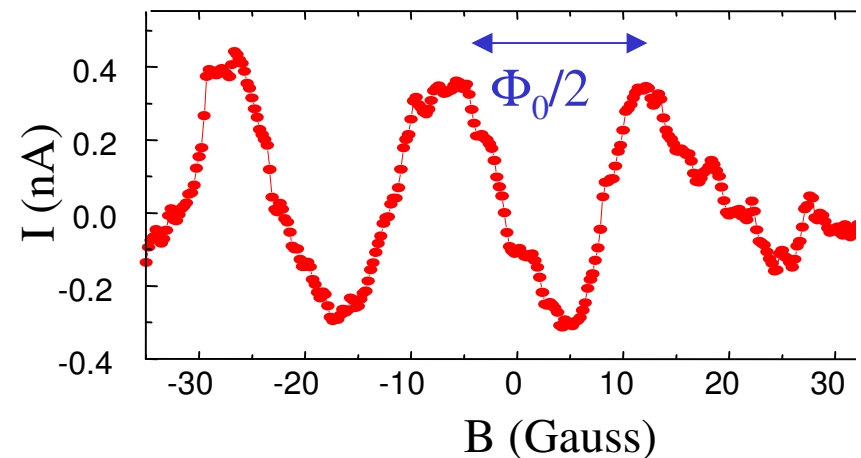
anneaux  
d'argent



T= 40mK

Susceptibilité magnétique :  $\chi_m = dI/d\phi$

- périodique en  $\Phi_0/2$
- diamagnétique à bas champ
- amplitude :  $I = 0.3 \text{ nA} = e/\tau_D$



# Mesoscopic rectification

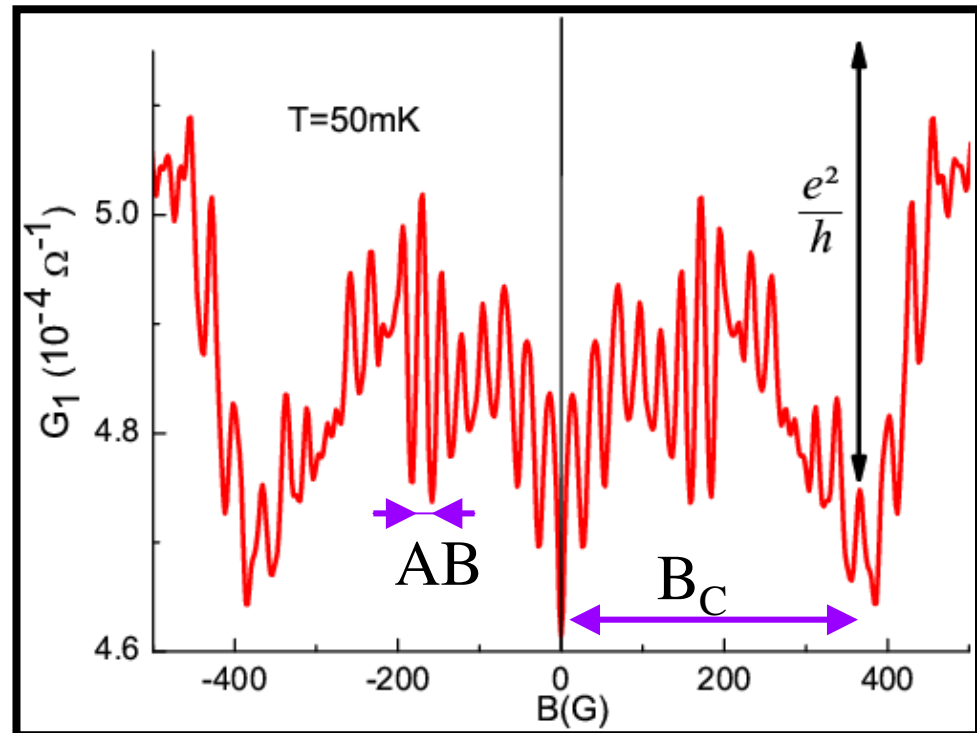
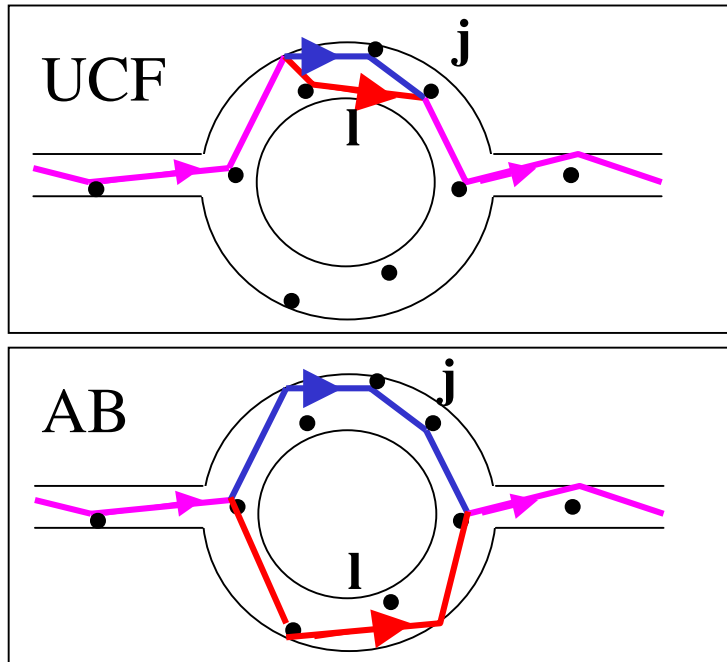
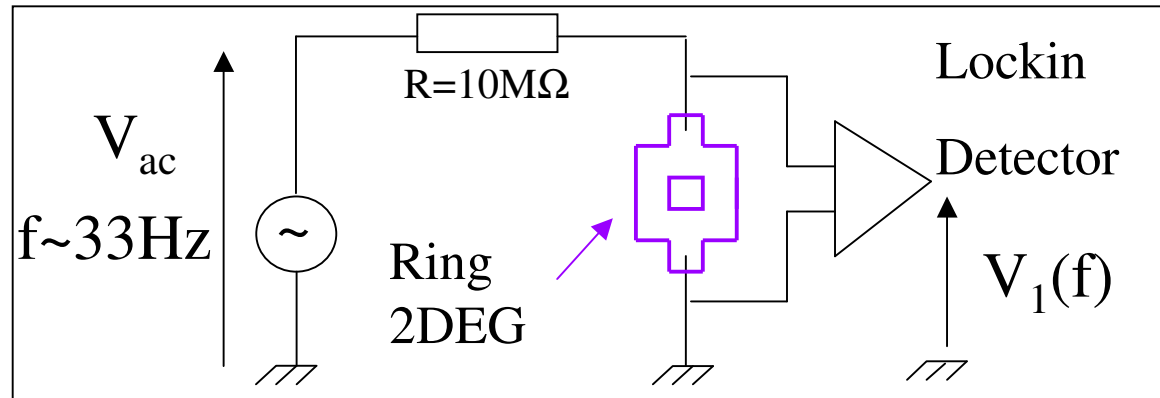
Why is non linear mesoscopic transport interesting?  
What kind of diodes are mesoscopic systems?

Field Asymmetry related to electron-electron interactions  
Amplitude of electron-electron interactions can be measured!

# Linear Conductance

$$I = G_1 V$$

2 wires  
measurement



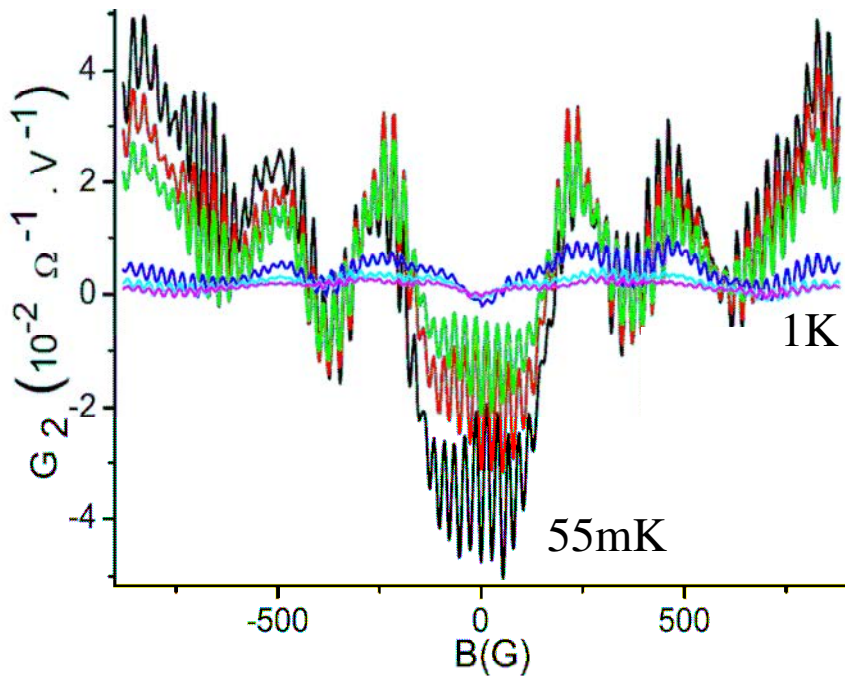
AB Oscillations and UCF even function of field

# Non Linear Conductance of second Order

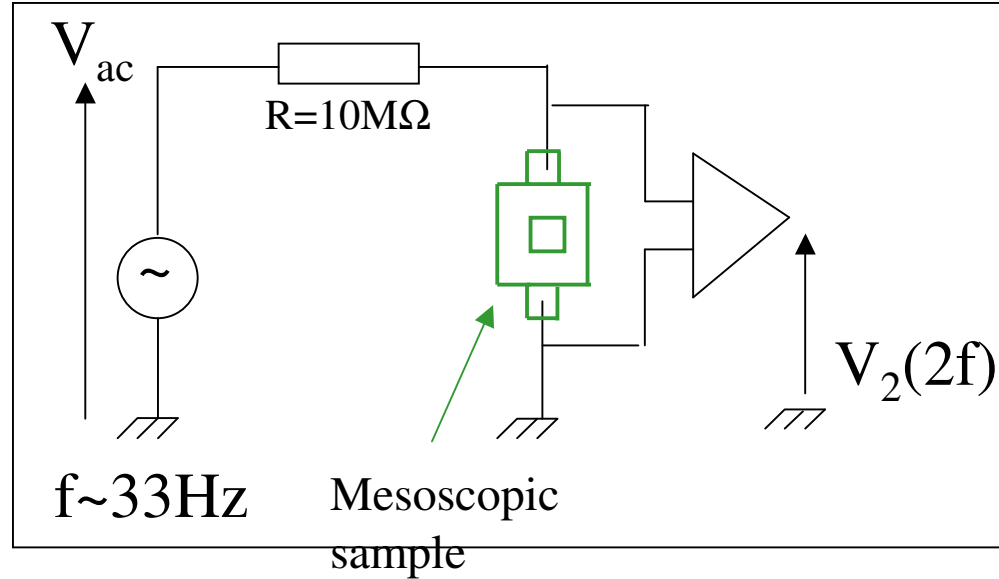
In a non centrosymmetric sample

$G_2$  is a mesoscopic quantity

$$\langle G_2 \rangle = 0$$



$$I = G_1 V + G_2 V^2$$



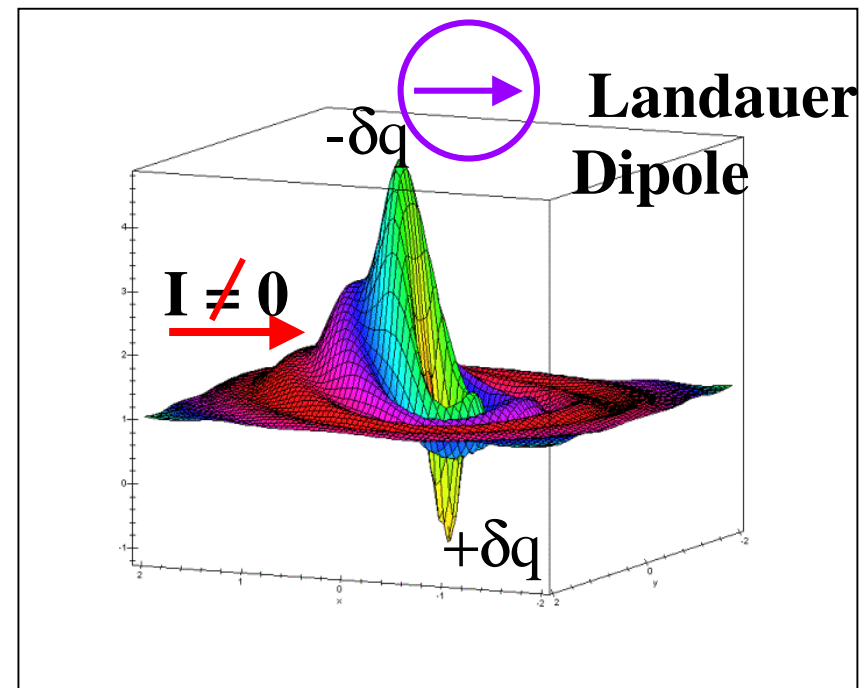
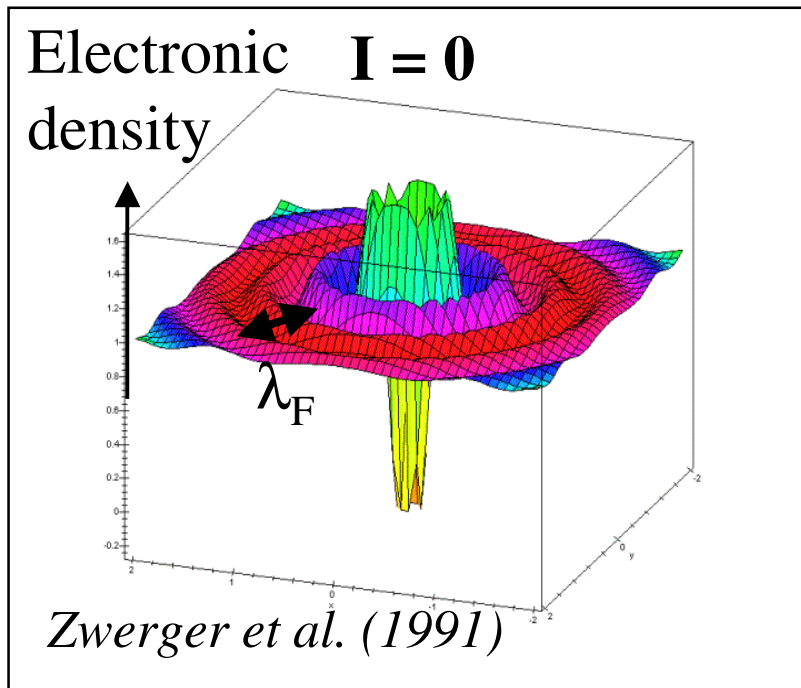
$G_2$  is not an even function of magnetic field

Observations on GaAS/GaAlAs ballistic systems *Angers 07*



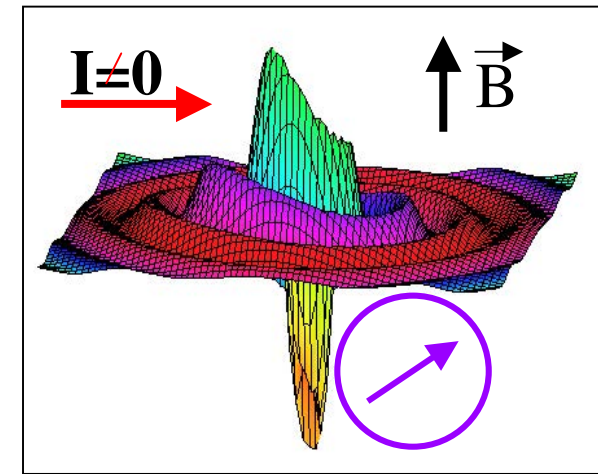
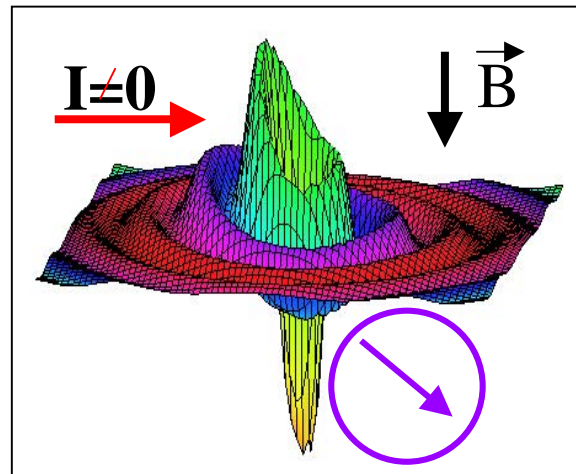
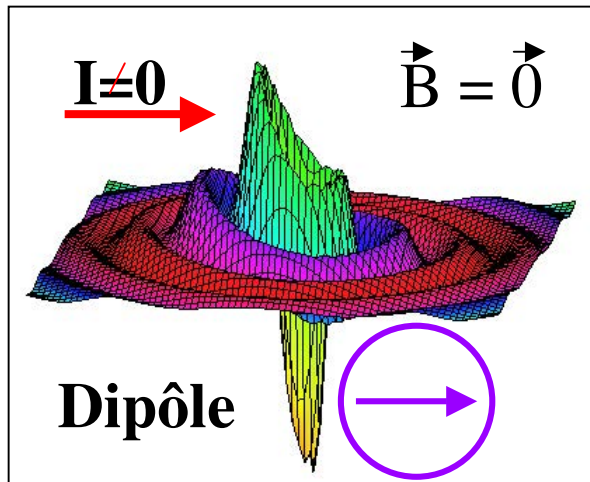
# Electron-electron interactions at the single impurity level

$$dn(I) \neq dn(-I)$$



# Electron-electron interactions at the single impurity level

$$dn(\vec{B}) \neq dn(-\vec{B})$$



$$H_{\text{int}} = \frac{\gamma_{\text{int}}}{\nu} \int d\vec{r} n^2(\vec{r})$$

$$dU_{\text{dés}}(\vec{r}) = \frac{\gamma_{\text{int}}}{2\nu} dn(\vec{r})$$

Bias induced Modification of the scattering potential has an odd component in  $\mathbf{I}$  and  $\mathbf{B}$

# Mesoscopic Effect

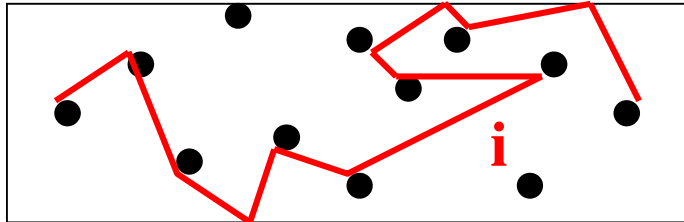
$$V(\vec{r})$$

Origin of

$$G_2$$

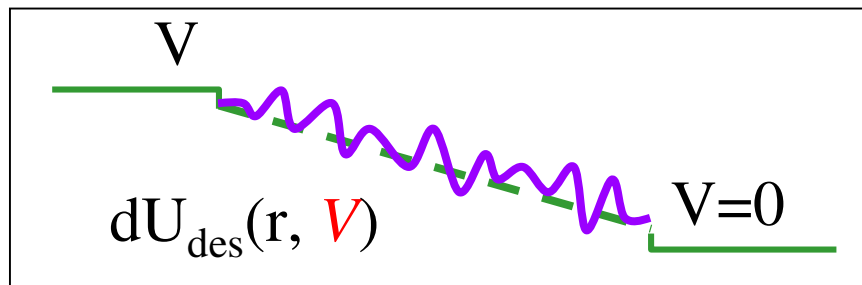
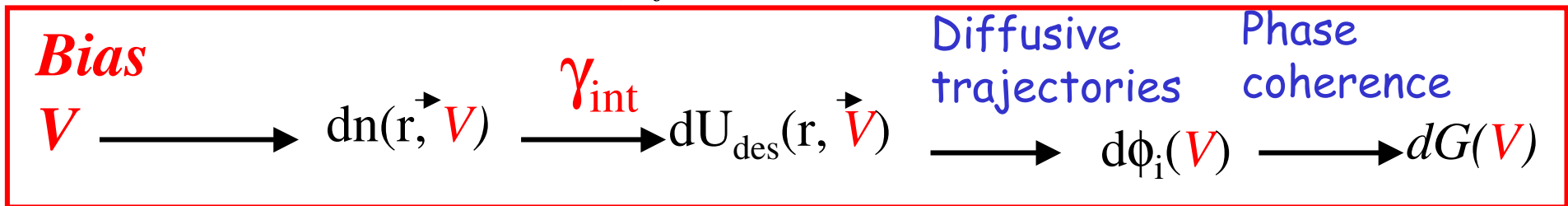
and

$$G_2^{AS}$$



$$d\phi_i = \int dU_{\text{des}}(\vec{r}(t), \mathbf{V}(\mathbf{r})) dt$$

Modification of the spatial charge distribution and potential landscape with bias  
(c.f. M. Büttiker, )



$$G_2 = \left( \frac{\partial dG[dU_{\text{dis}}, V, \gamma_{\text{int}}]}{\partial V} \right)_{V=0}$$

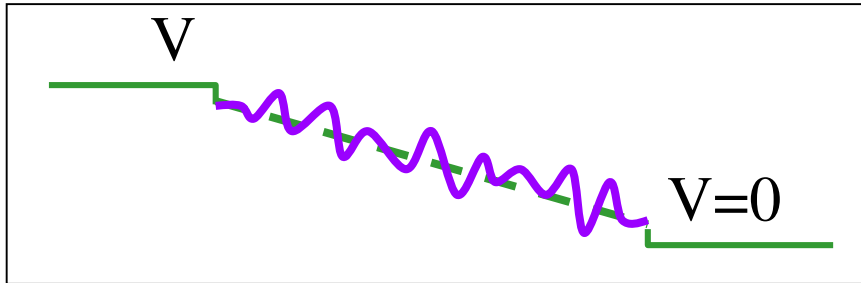
- Existence of a contribution **odd** in magnetic field in  $dn$  and  $G_2$

Importance of interactions

$$\delta G_2^S$$

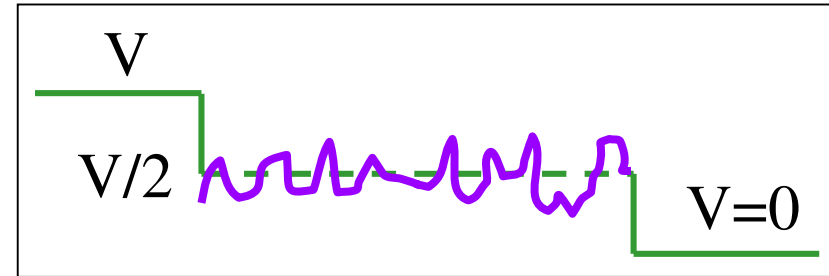
depends on the potential landscape

➤ Diffusive regime



$$\langle d\phi_i(V) \rangle = eV\tau_D/h$$

➤ Ballistic regime  
symmetrical contacts,  
strong interactions



No average voltage drop  
through the sample !

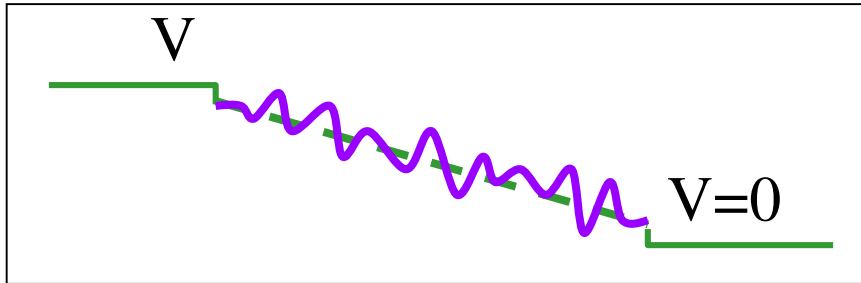
$$\langle d\phi_i(V) \rangle = 0$$

$$G_2 = dG(V)/dV = \sum_i (dG/d\phi_i)(d\phi_i(V)/dV)$$

$$\delta G_2^S$$

depends on the potential landscape

➤ Diffusive regime



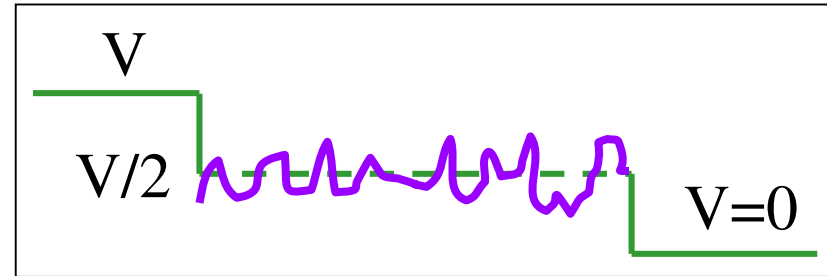
$$\delta G_2^S \cong \frac{e\tau_D}{\hbar} \delta G_1 + \frac{e\tau_D}{\hbar} \frac{\gamma_{\text{int}}}{g} (\delta G_1)^2$$

➔

$$\frac{\delta G_2^{AS}}{\delta G_2^S} \cong \frac{\gamma_{\text{int}}}{g} \delta G_1$$

➤ Ballistic regime

symmetrical contacts,  
strong interactions



No average voltage drop  
through the sample !

$$\delta G_2^S \cong 0 + \frac{e\tau_D}{\hbar} \frac{\gamma_{\text{int}}}{g} (\delta G_1)^2$$

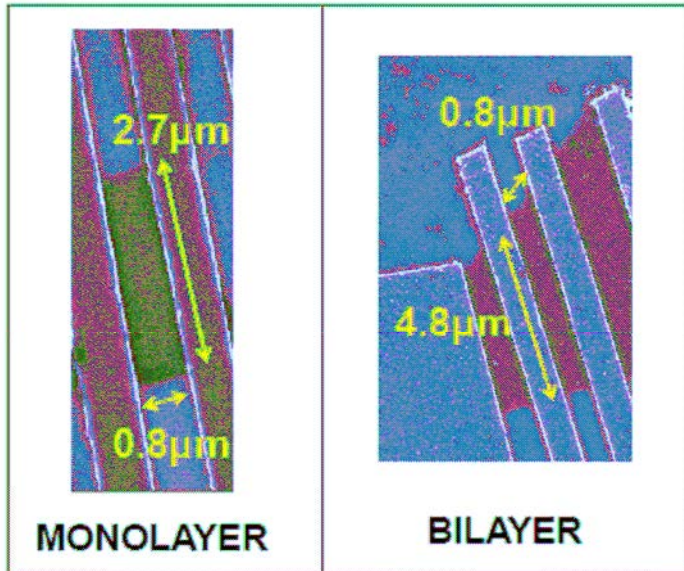
*Polianski et al.*

(2006 et 2007)

➔

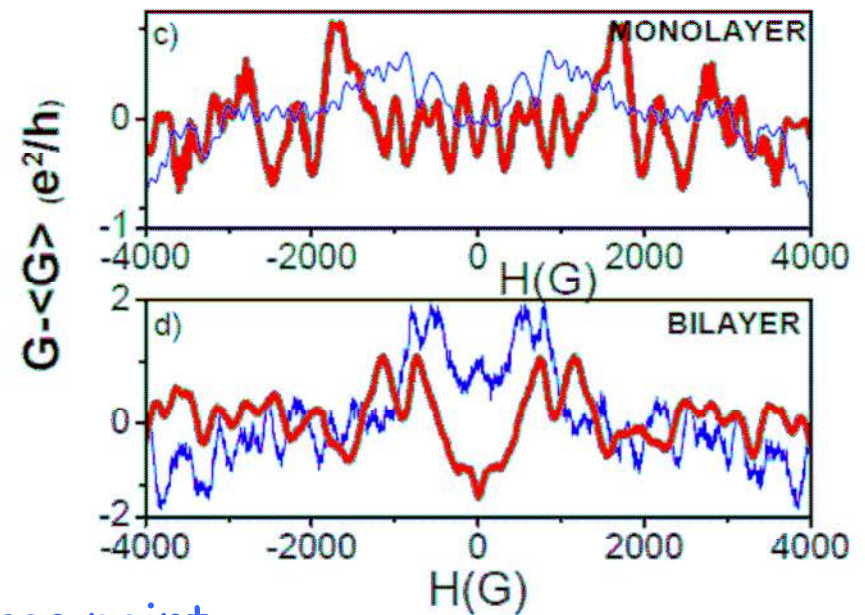
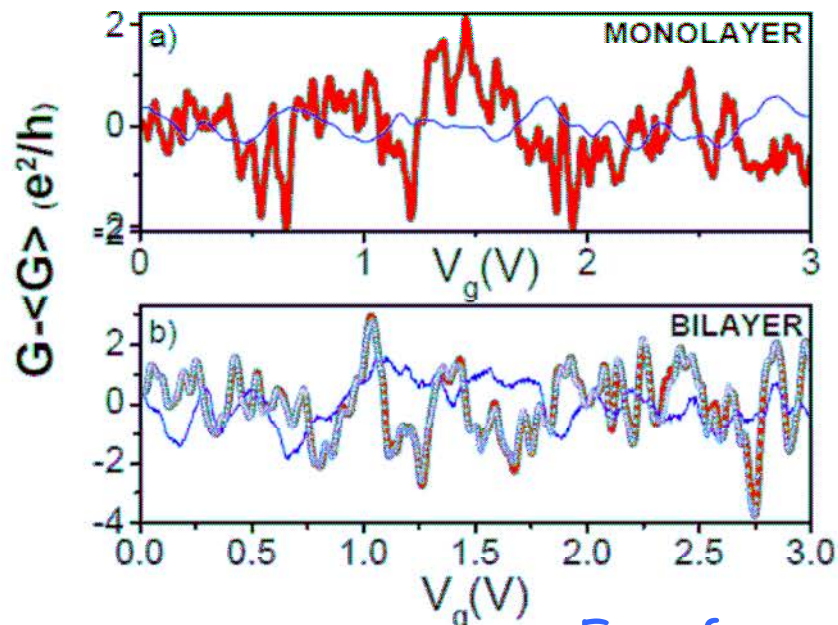
$$\frac{\delta G_2^{AS}}{\delta G_2^S} \cong 1$$

# Conductance fluctuations in graphene



Correlation fields and energies depend on doping level

*Ojeda et al. PRL 2010*



— Far from the Dirac point  
 — Close the Dirac Point

$G_2$  contains an antisymmetric component in magnetic field

Casimir Onsager Symmetry rules:  $G_1(B)=G_1(-B)$

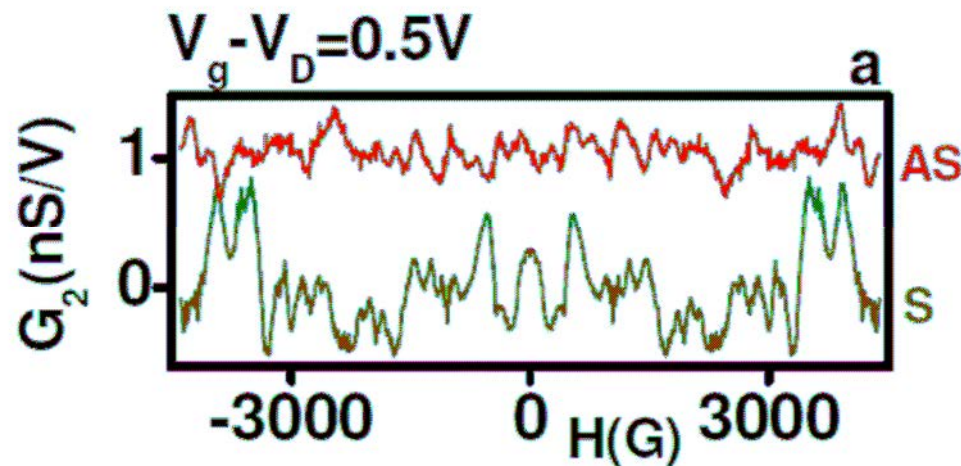
Do not apply for  $G_2(B)$

$$G_1^{AS} = 0 \quad G_2^{AS} \neq 0$$

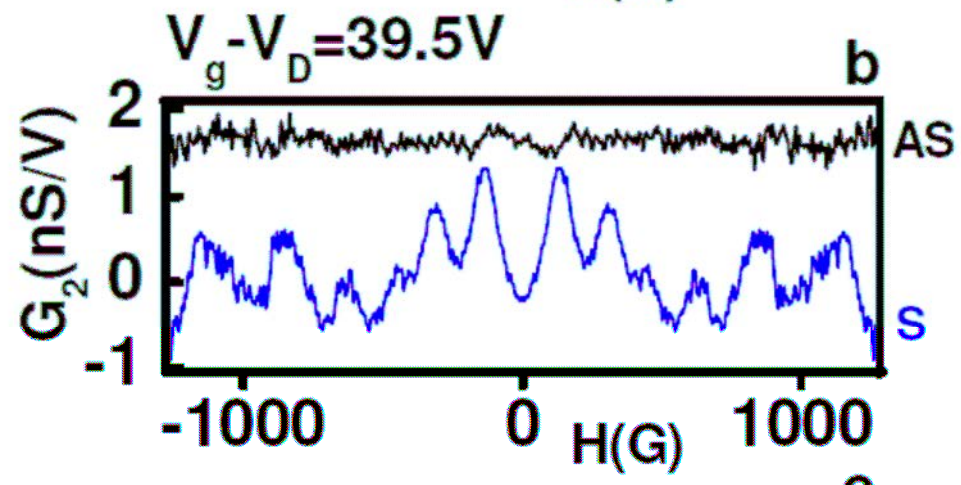
$$G_2^{S,AS} = (G_2(B) \pm G_2(-B))/2$$

GRAPHENE

T=50mK

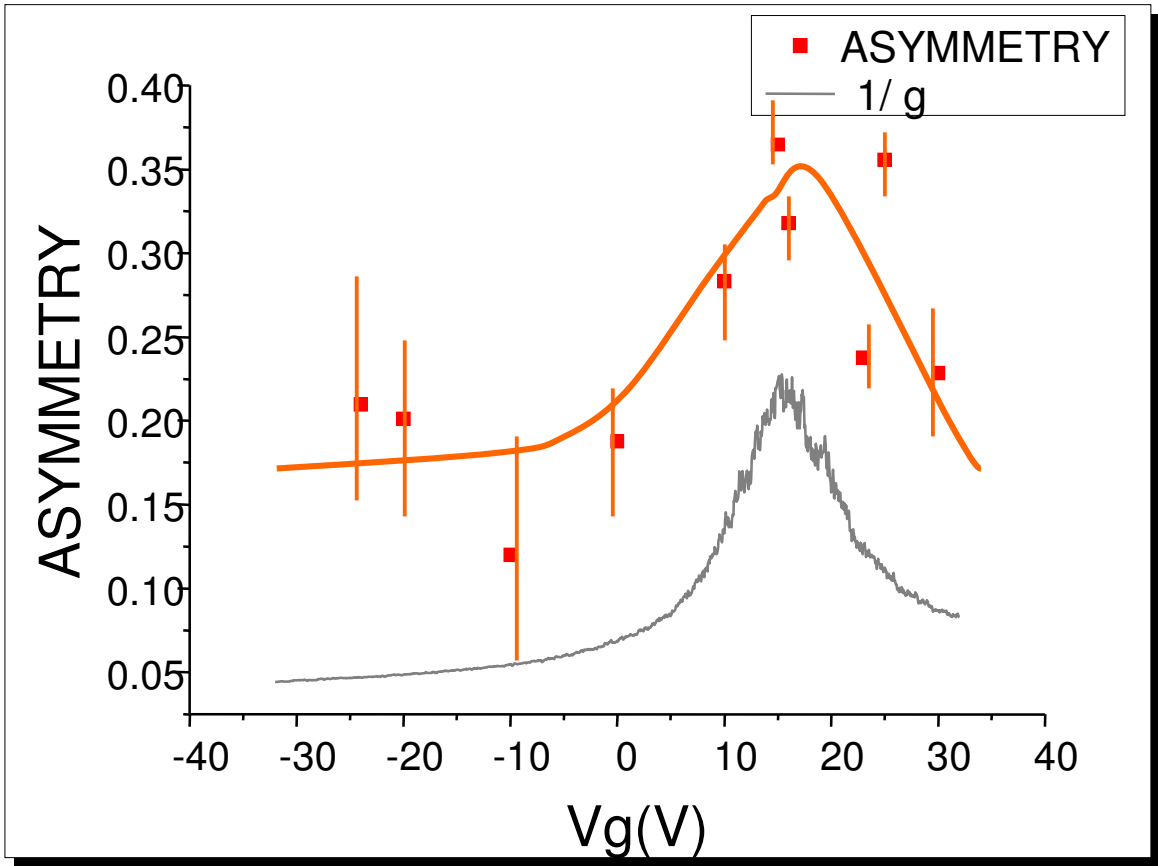


At neutrality point



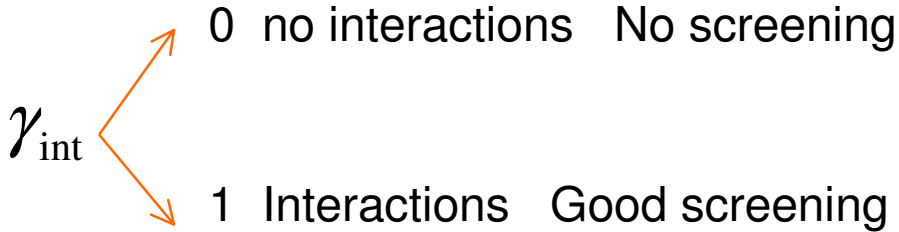
Far from neutrality point

# Conductance dependence of the field asymmetry



$$\frac{\delta G_{2}^{AS}}{\delta G_{2}^{S}} \approx \frac{\gamma_{int}}{g}$$

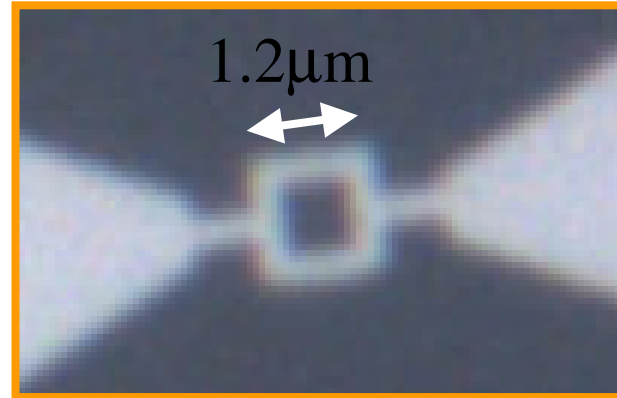
In qualitative agreement  
 With theoretical predictions  
 For diffusive systems





# Aharonov-Bohm Ring 2D e gas

**2 contacts**  
**No gate**



*LPS, Orsay Angers  
et al. 2006*

*with 2DEG  
U. Gennser LPN*

Other experiments :

*Zumbühl et al. (2006) quantum boxes  $g \sim 1$*

*Leturcq et al. (2006) small rings  $g < 1$*

Expected rectification asymmetry:

$$\propto \frac{1}{g}$$

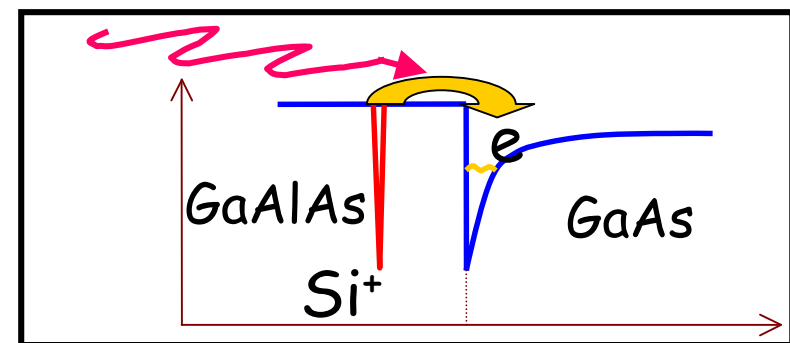
$$g = G / (e^2/h)$$

**Current pulses**  
**10 à 50 mA**

decrease of  $g$

**Illumination**

Increase of  $g$



➤ With 2 samples conductance is varied from:

**$g = 1$  to  $21$**

# Symmetry in magnetic field of $G_2$ and $G_1$

Casimir Onsager  
Symmetry rules:

$$G_1(B) = G_1(-B)$$

Do not apply for  $G_2(B)$

$$G^{S,AS} = \frac{G(B) \pm G(-B)}{2}$$

$$G_1^{AS} = 0$$

$$G_2^{AS} \neq 0$$

