

Signatures expérimentales des interactions électroniques dans le transport mésoscopique

Introduction : ordres de grandeur, rappels transport cohérent

Interactions et cohérence de phase

Désordre, dimensionnalité, environnement électromagnétique, géométrie
Comment déterminer le temps de cohérence de phase dans un système fini

Interactions et propriétés thermodynamiques

Correction à la densité d'états d'un système diffusif ; rôle de la dimensionalité
Système localisé : gap de Coulomb
Magnétisme orbital

Transport hors d'équilibre

Rectification mésoscopique et interactions

Que se passe t'il à la limite 1D ?

Instabilité du liquide de Fermi,

Exemple fils quantiques et nanotubes de carbone

Peut on parler de comportement liquide de Luttinger dans les conducteurs 1D

Blocage de Coulomb dynamique et environnement électromagnétique

Interactions dans les boîtes quantiques

Du blocage de Coulomb à l'effet Kondo

Contacts supraconducteurs : compétition entre effet Kondo et effet Josephson

Interactions et densité d'états

$$G_{\epsilon}^{R,A} = (\epsilon - H \pm i0)^{-1} \quad \delta(\epsilon - H) = \text{Im}(G_{\epsilon}^R)/2\pi = (G_{\epsilon}^R - G_{\epsilon}^A)/2\pi$$

Régime diffusif sans interactions: $\nu = \text{Tr}(\delta(\epsilon - H)) \sim N/E_F$
Pas de correction quantique à la DOS moyenne

En présence d'interactions:
Perturbation des niveaux électroniques à l'ordre 1:

$$\delta\epsilon_i^H = \sum_j f(\epsilon_j) \int U(r - r') \phi_j^*(r') \phi_j(r') dr' \phi_i^*(r) \phi_i(r) dr$$

Terme de Hartree

$$\delta\epsilon_i^F = \sum_j f(\epsilon_j) \int U(r - r') \phi_j^*(r') \phi_j(r) \phi_i^*(r) \phi_i(r') dr dr'$$

Terme de Fock (échange)

- Interactions et densité d'états

$$\Delta_\epsilon = \frac{1}{\nu_0} \sum_i \delta(\epsilon - \epsilon_i) \delta\epsilon_i \quad \frac{\delta\nu}{\nu_0} = -\partial\Delta_\epsilon/\partial\epsilon$$

Terme de Fock

$$\delta\epsilon_i^F = \sum_j f(\epsilon_j) \int U(r - r') \phi_j^*(r') \phi_j(r) \phi_i^*(r) \phi_i(r') dr dr'$$

$$\Delta_\epsilon^F = -\frac{1}{\nu_0} \int_{-\infty}^{+\infty} f(\epsilon - \omega) d\omega \sum_q U_q \text{Re} \frac{1}{i\omega + Dq^2}$$

Diffuson: contribution dominante

$$\delta\nu^F = -U \left[\frac{\epsilon}{D} \right]^{d/2} \Omega/\epsilon$$

Anomalie en: $\left\{ \begin{array}{ll} -\epsilon^{1/2} & \text{à 3D} \\ -\text{Log}(\epsilon) & \text{à 2D} \\ -\epsilon^{-1/2} & \text{à 1D} \end{array} \right.$

Longueur caractéristique

$$L_c = (\hbar D / \epsilon)^{1/2}$$

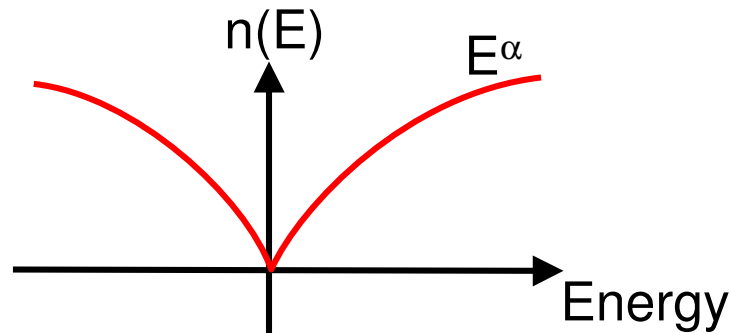
$\delta\nu^H < \delta\nu^F$ si écrantage modéré

A 1D system of interacting electrons

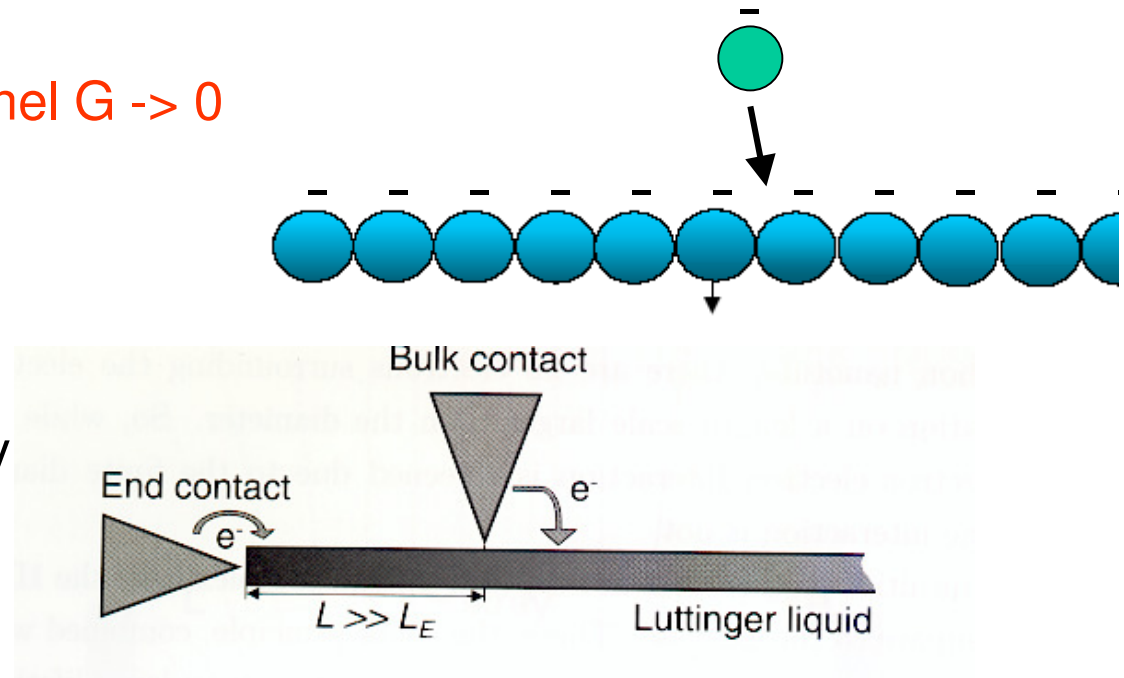
- At 3D, screening is efficient : excitations are quasi-particles, with small residual interactions (Fermi liquid).
- At 1D, interactions are much stronger: no quasi-particles, Luttinger liquid, with long wavelength charge and spin excitations.

Implications:

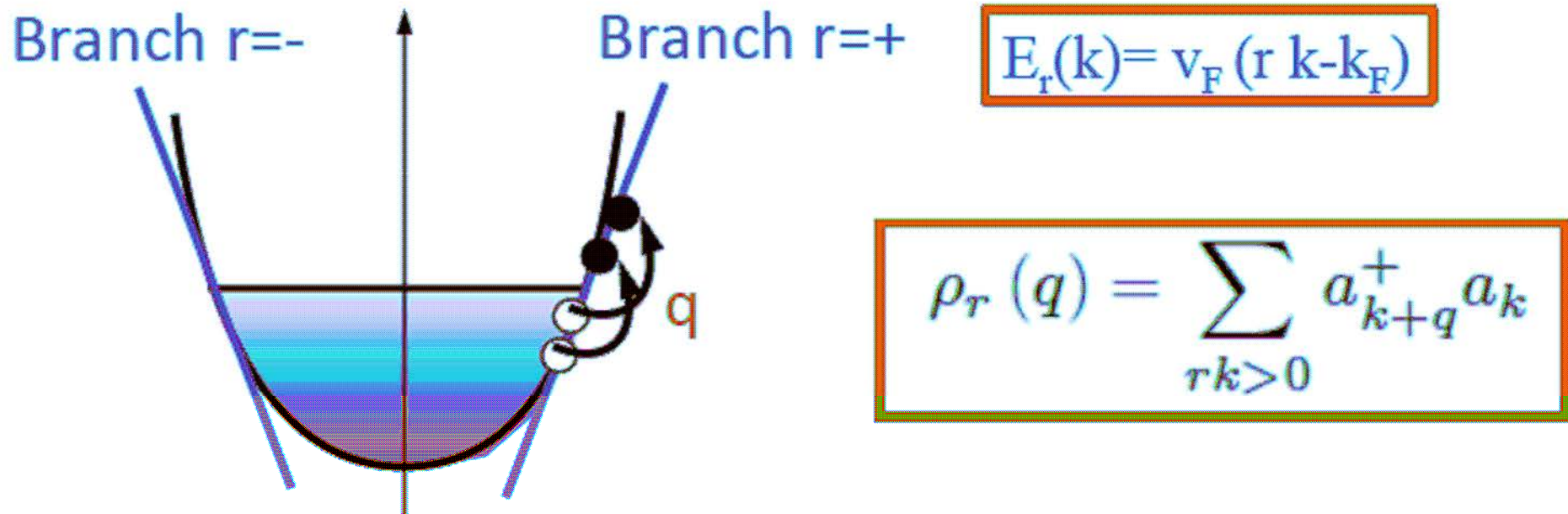
1D + repulsive interaction: tunnel $G \rightarrow 0$



$$G(T) \propto T^\alpha \text{ for } eV \ll k_B T$$



Transport électronique à une dimension (un seule mode de conduction!)



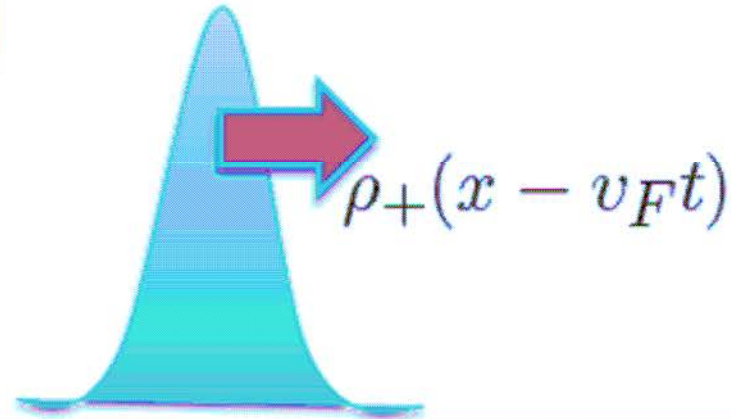
Linéarisation de la relation de dispersion et bosonisation

Long wave length excitations

$$\partial_t \rho_{\pm} = \pm v_F \partial_x \rho_{\pm}$$

$$\rho_+(x - v_F t)$$

$$\rho_-(x + v_F t)$$



Total charge density ($q \ll k_F$ component):

$$\rho = \rho_+ + \rho_-$$

Total current density ($q \ll k_F$ component):

$$j = v_F [\rho_+ - \rho_-]$$

Instabilités électroniques du conducteur unidimensionnel

$$H = \frac{\hbar v_F}{2} \int dx [\rho_+^2(x) + \rho_-^2(x)]$$

Interactions caractérisées par le paramètre g

$g = 1$ pas d'interaction

$g < 1$ interaction répulsive

E_c charging energy, Δ level spacing

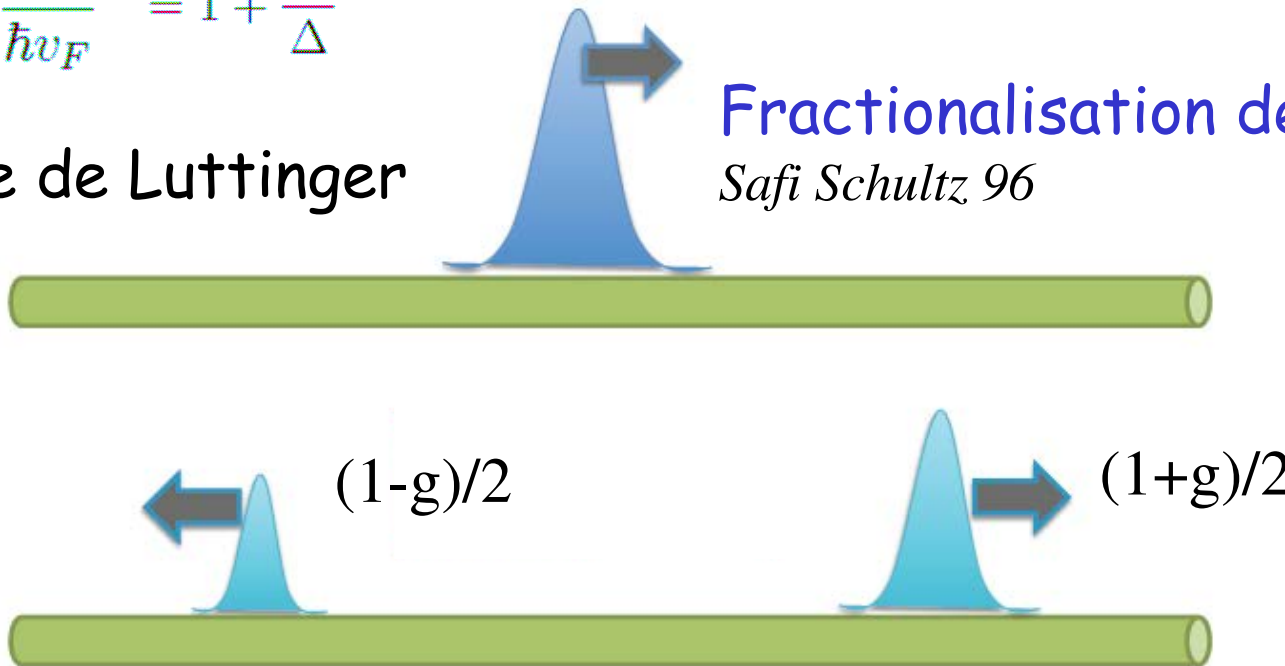
$$H_{int} = \int dx \frac{u}{2} \rho(x)^2$$

$$\frac{1}{g^2} = 1 + \frac{u}{\hbar v_F} = 1 + \frac{E_c}{\Delta}$$

Liquide de Luttinger

Fractionalisation de la charge

Safi Schultz 96



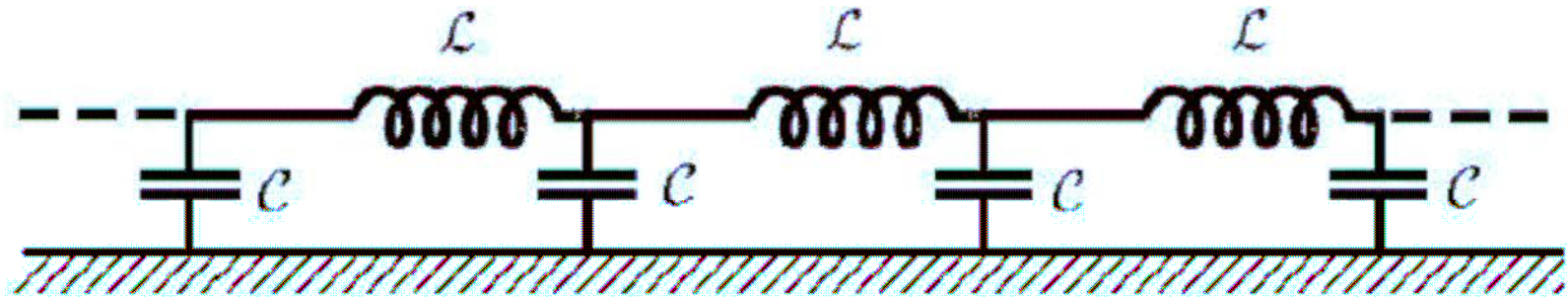
$$v_{eff} = v_F/g > v_F$$

For repulsive interactions

Modes propres en présence d'interactions

(pas d'effets de spin)

$$H = \frac{\hbar v_F}{2g} \int dx \frac{u}{2} [\tilde{\rho}_+^2(x) + \tilde{\rho}_-^2(x)] \quad \tilde{\rho}_\pm = \frac{1 \pm g}{2} \rho_+ + \frac{1 \mp g}{2} \rho_-$$



Analogie avec modes de transmission sur une ligne LC

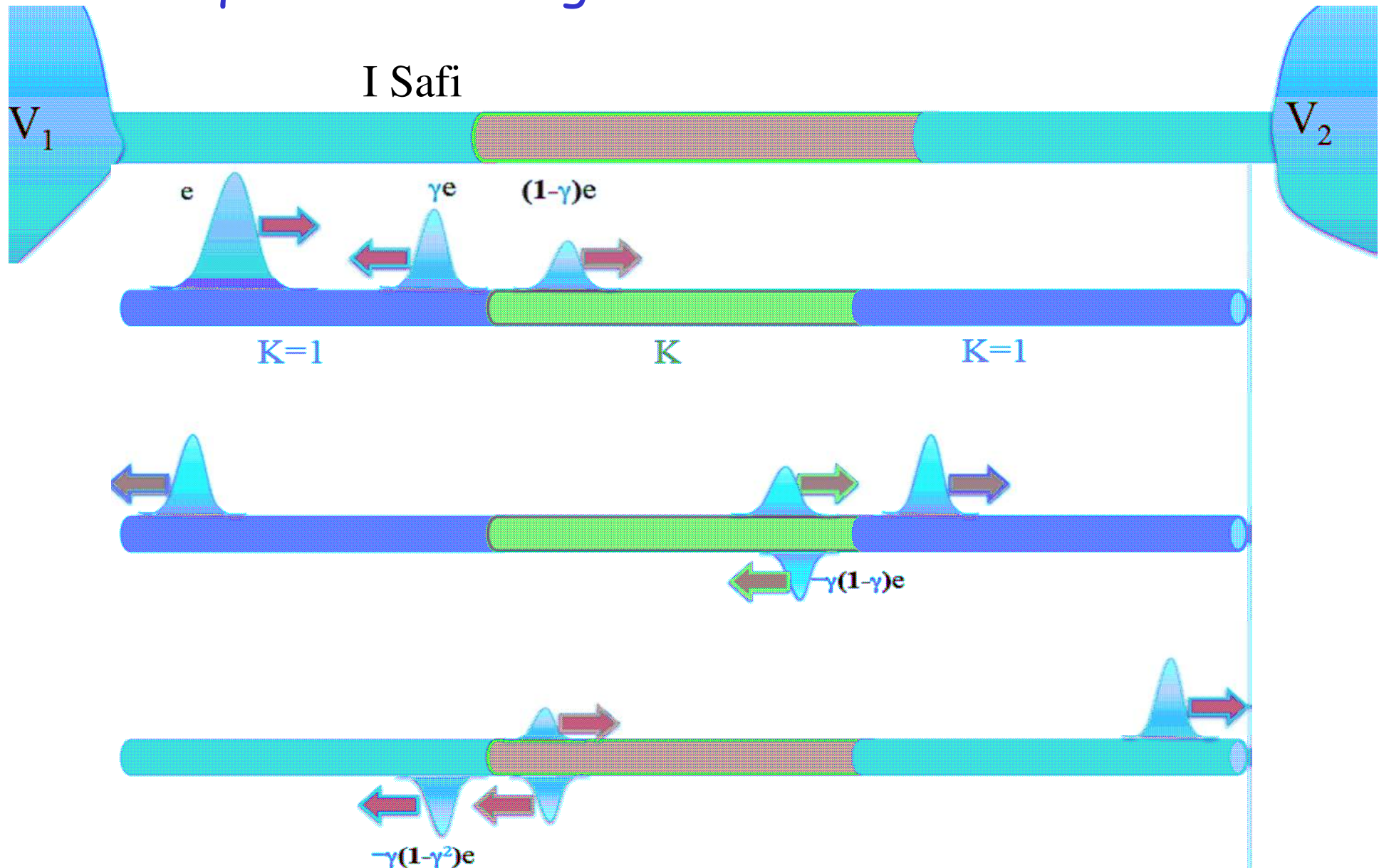
$$C = 2g (e^2/h) / v_F \quad L = (h/2e^2) / g v_F$$

Conductance du fil infini G renormalisée: $2g e^2/h$

Mesurable? En général NON!

(cas particulier effet Hall quantique fractionnaire)

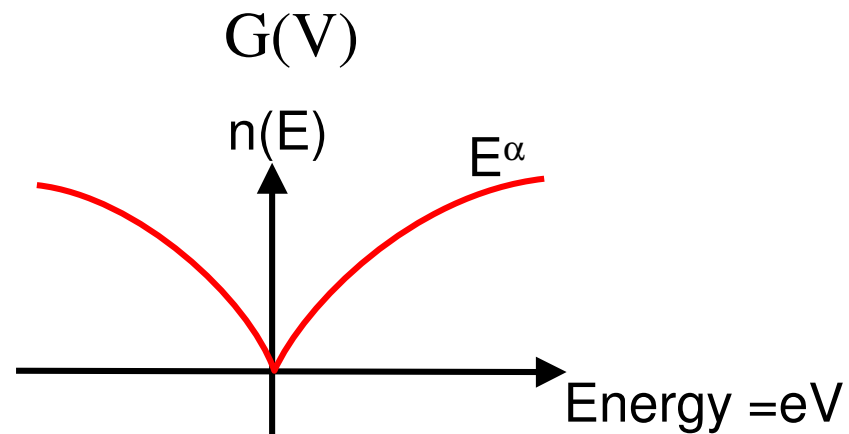
Liquide de Luttinger connecté à des réservoirs



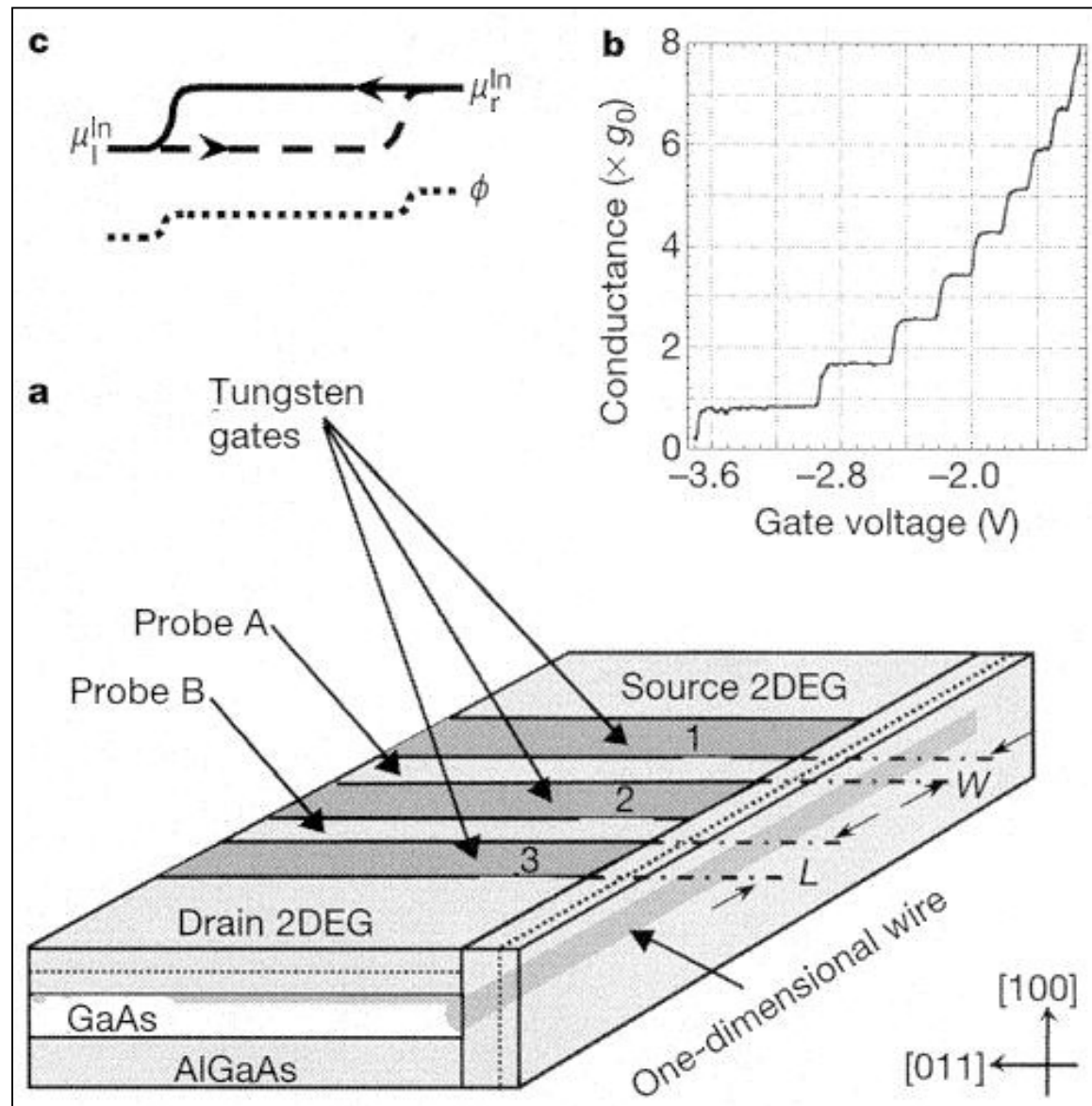
Réflexions cohérentes multiples aux réservoirs $G=2e^2/h!$

Conductance tunnel

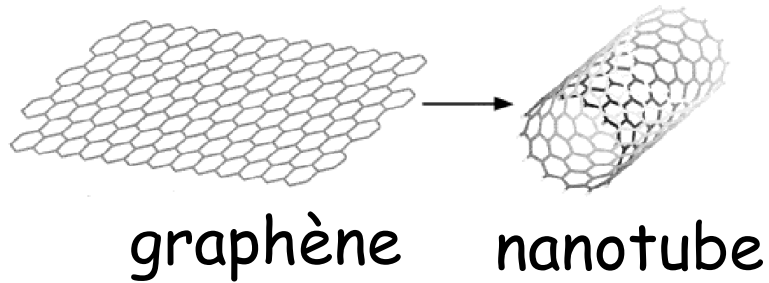
$$n(E) = E^\alpha \quad \text{avec} \quad \alpha = \frac{1}{2}\left(g + \frac{1}{g}\right) - 1$$



$$G(T) \propto T^\alpha \quad \text{for} \quad eV \ll k_B T$$



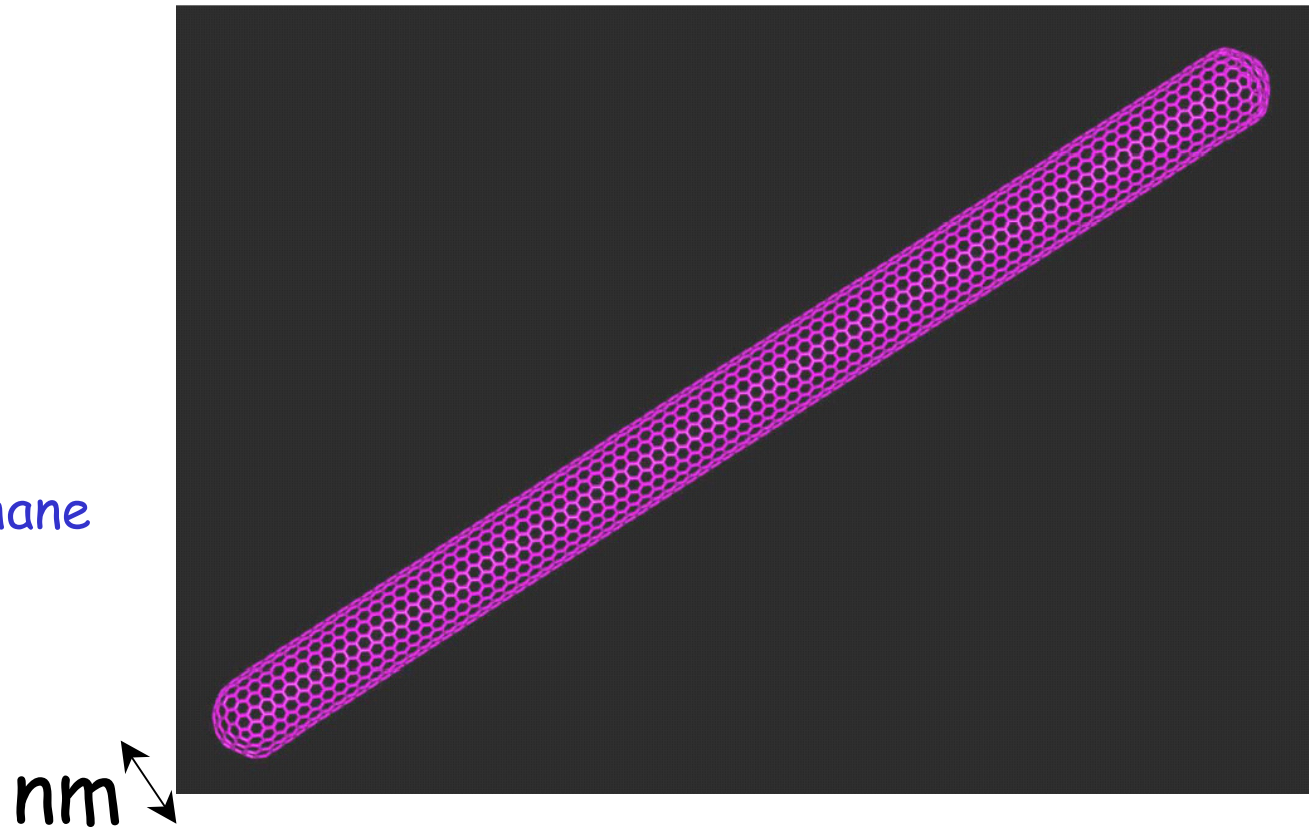
Nanotubes de Carbone



Découverts par Ijima (1991)

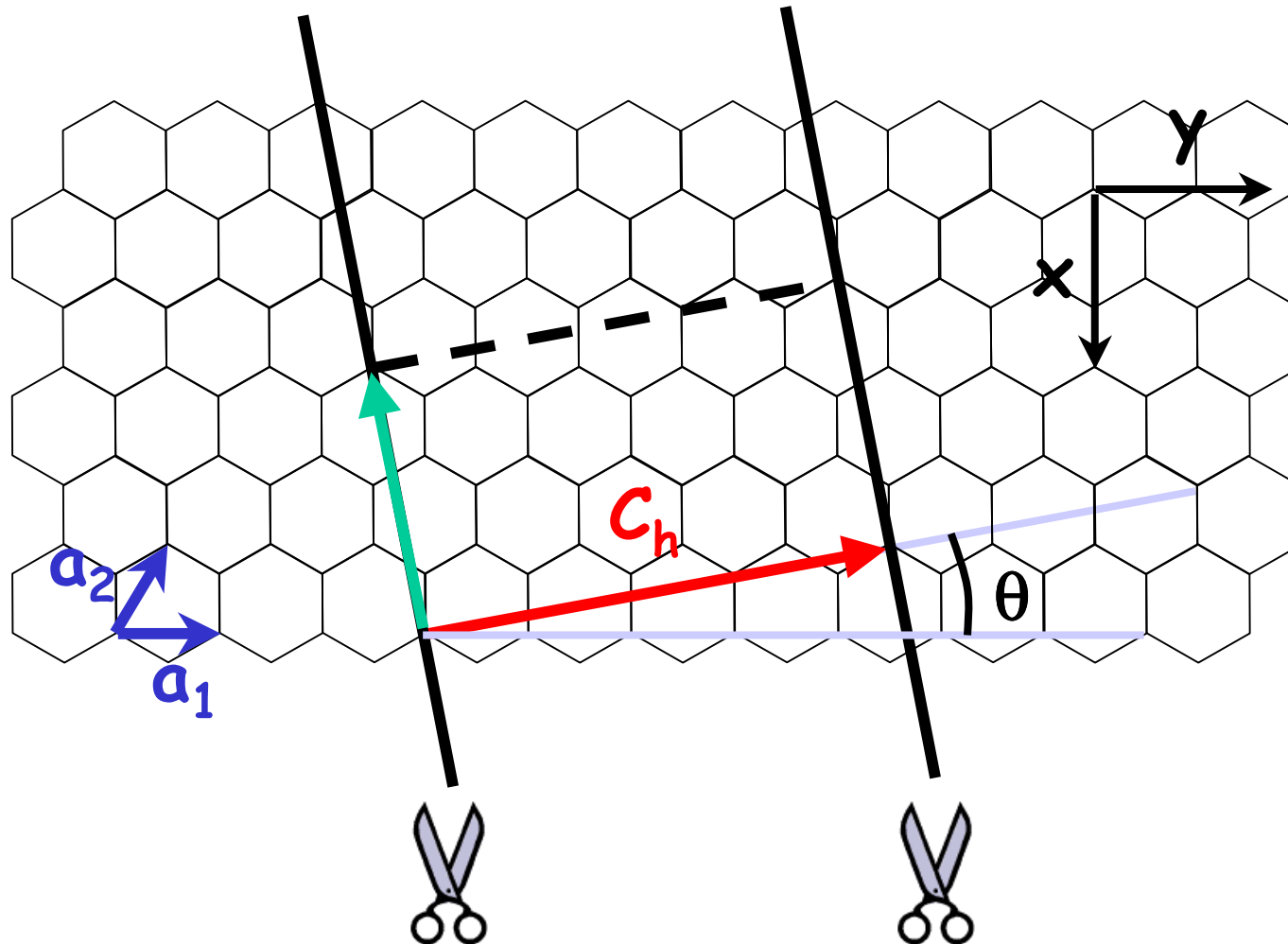
Fabrication:

- Arc électrique
- Ablation Laser
- Décomposition méthane



Des molécules macroscopiques rigides!

Helicité

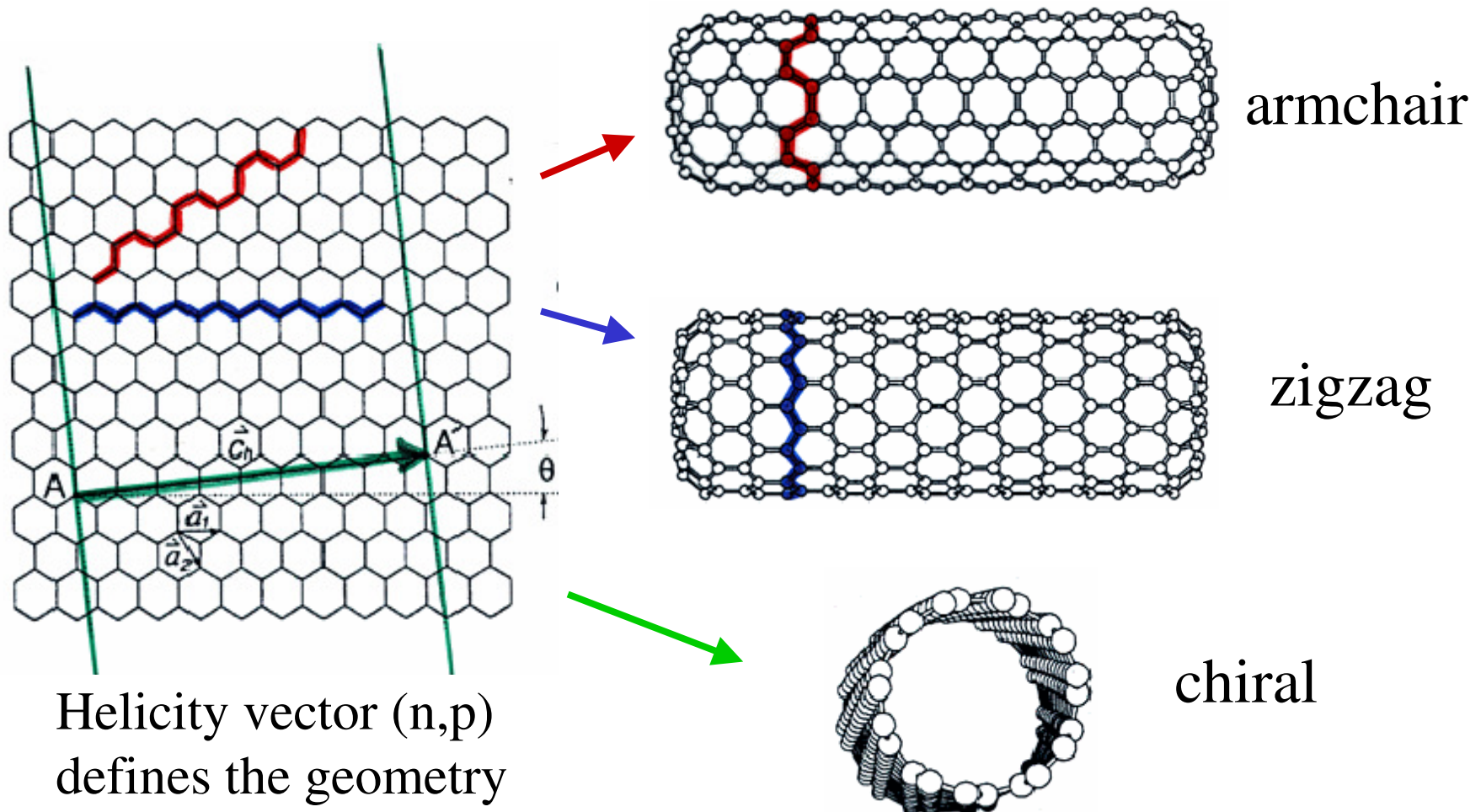


Caractérisée par le vecteur : $C_h = n a_1 + m a_2$

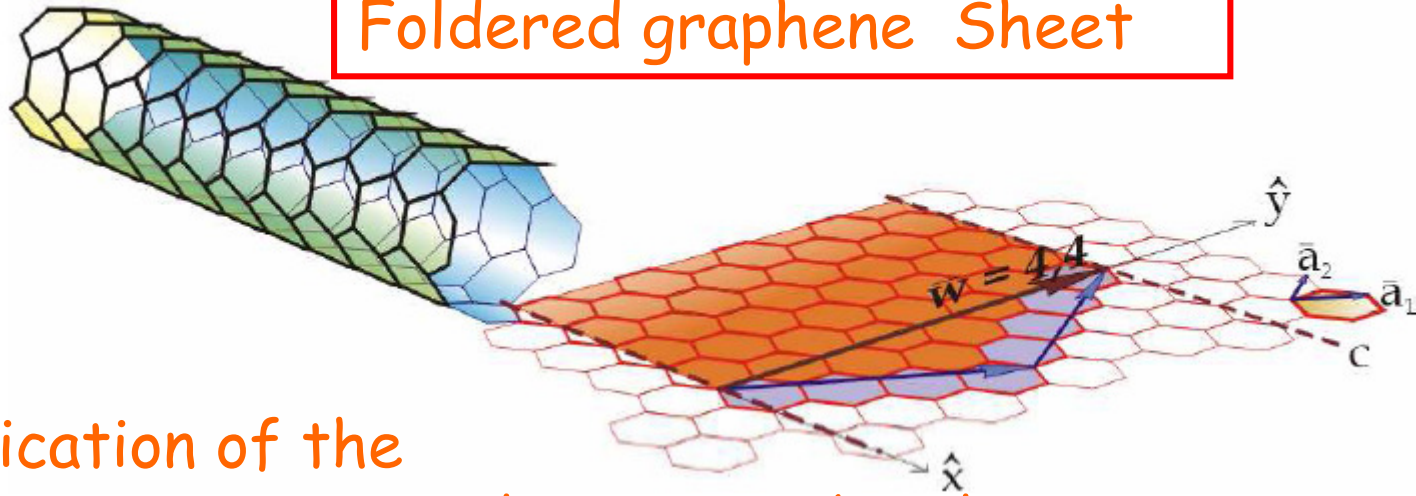


tube (n, m)

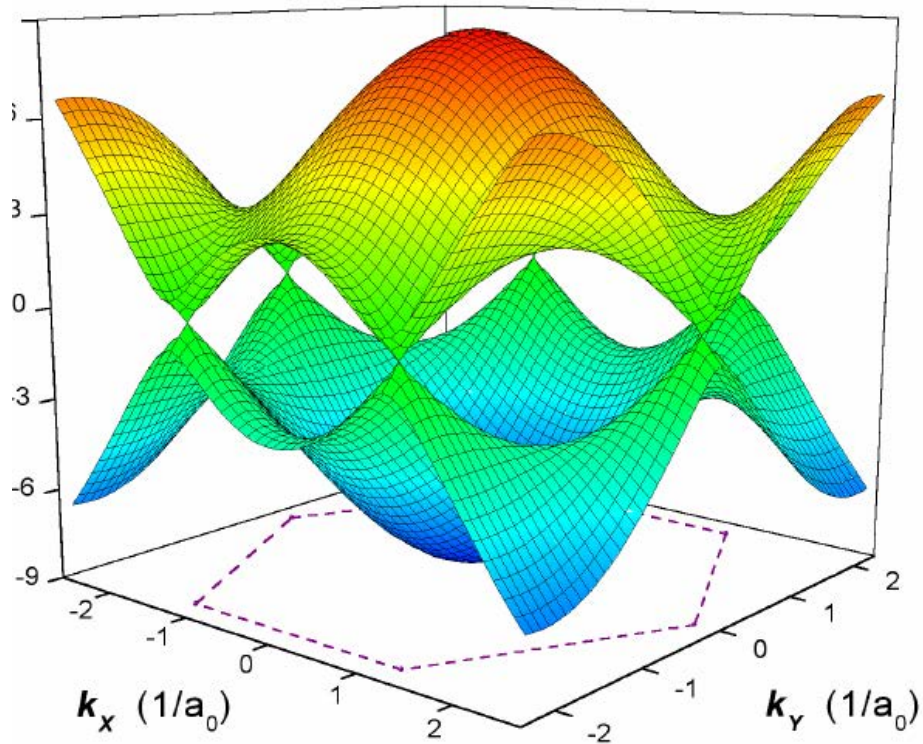
Folding graphene into tubes



Foldered graphene Sheet

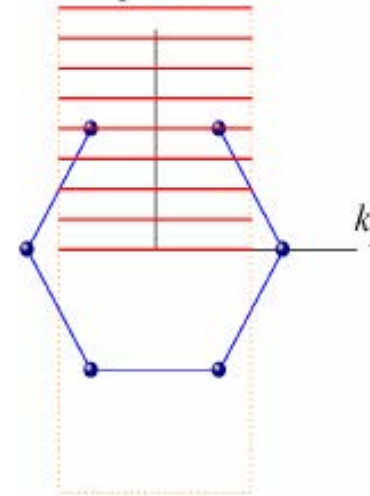


Quantification of the transverse wave vector determines band structure

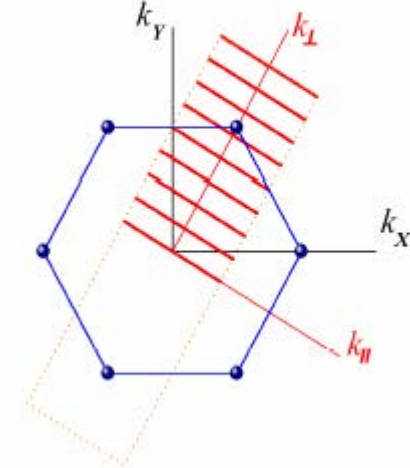


SWNT 4,4

$$k_y = k_{\perp}$$



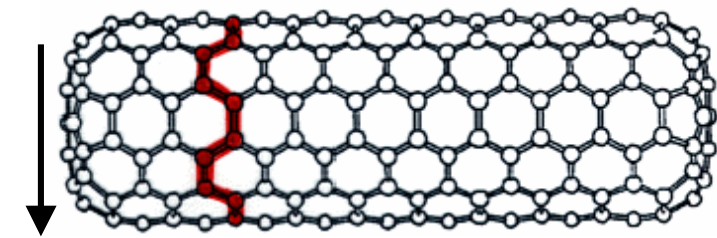
SWNT 8,0



Band structure of a carbon nanotube

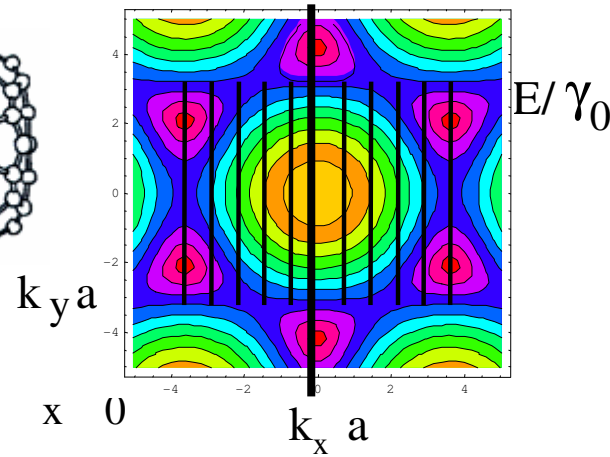
Each band corresponds to a different k_{perp}

“Armchair ”

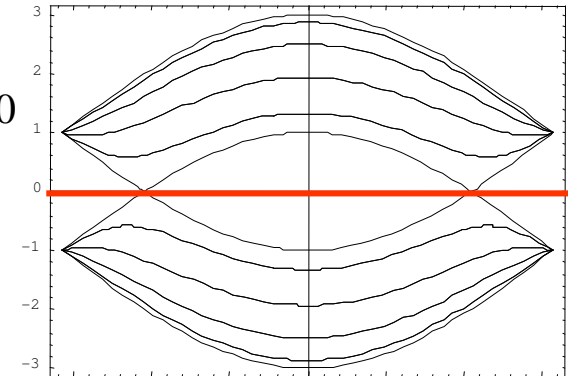


x

$$k_x \cdot \text{circumference} = 2\pi n$$

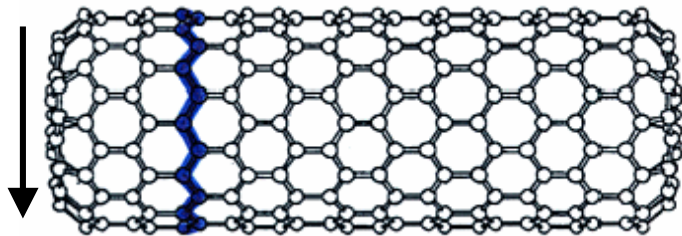


Metallic 1/3



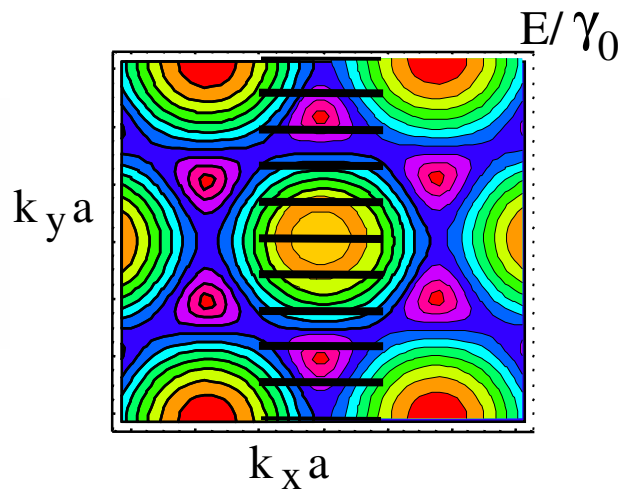
$k_y a$

Zig-zag

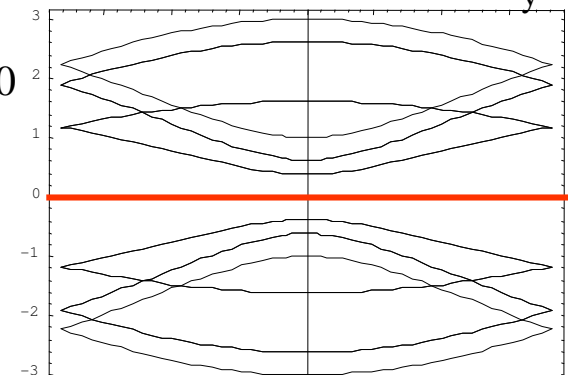


y

$$k_y \cdot \text{circumference} = 2\pi n$$



E/γ_0



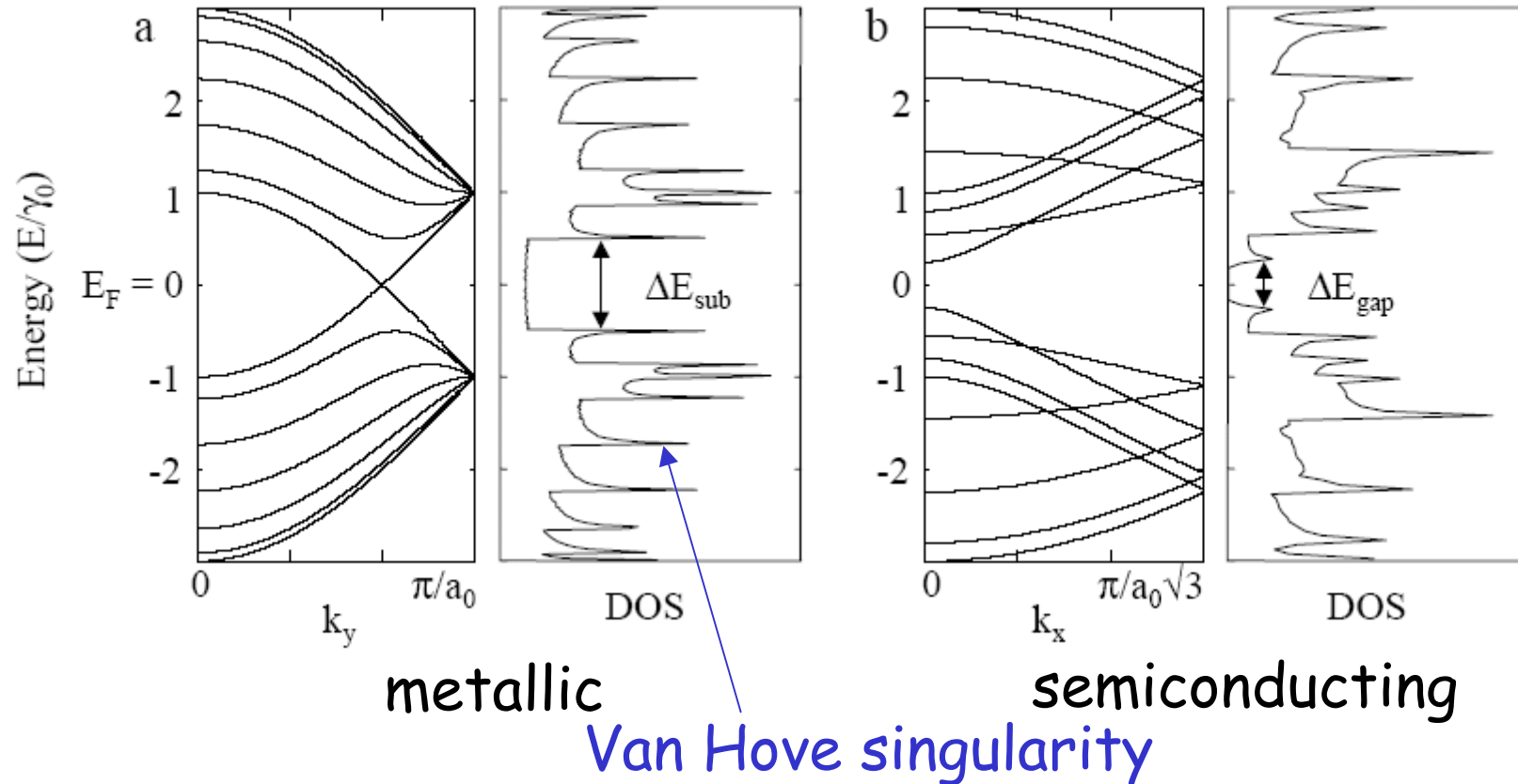
$k_x a$

Semiconducting
2/3

Density of states

Armchair tube (6,6)

Zigzag tube (7,0)

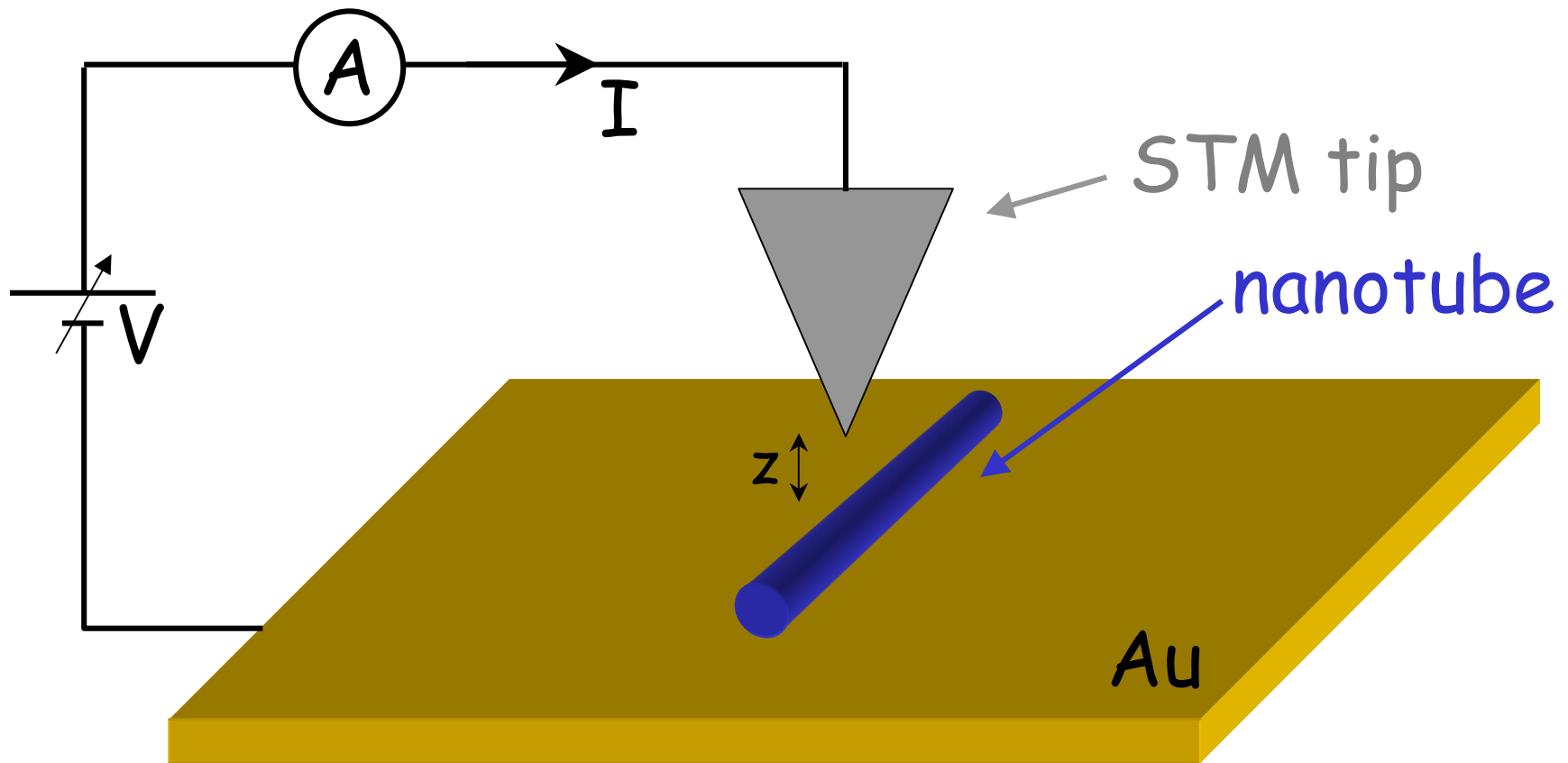


Metallic tube if : $n - m = 3q$ with q integer

Small opening at $\frac{1}{2}$ filling due :

- disorder
- deformation of curved C-C : 10 meV (zig-zag & chiral)

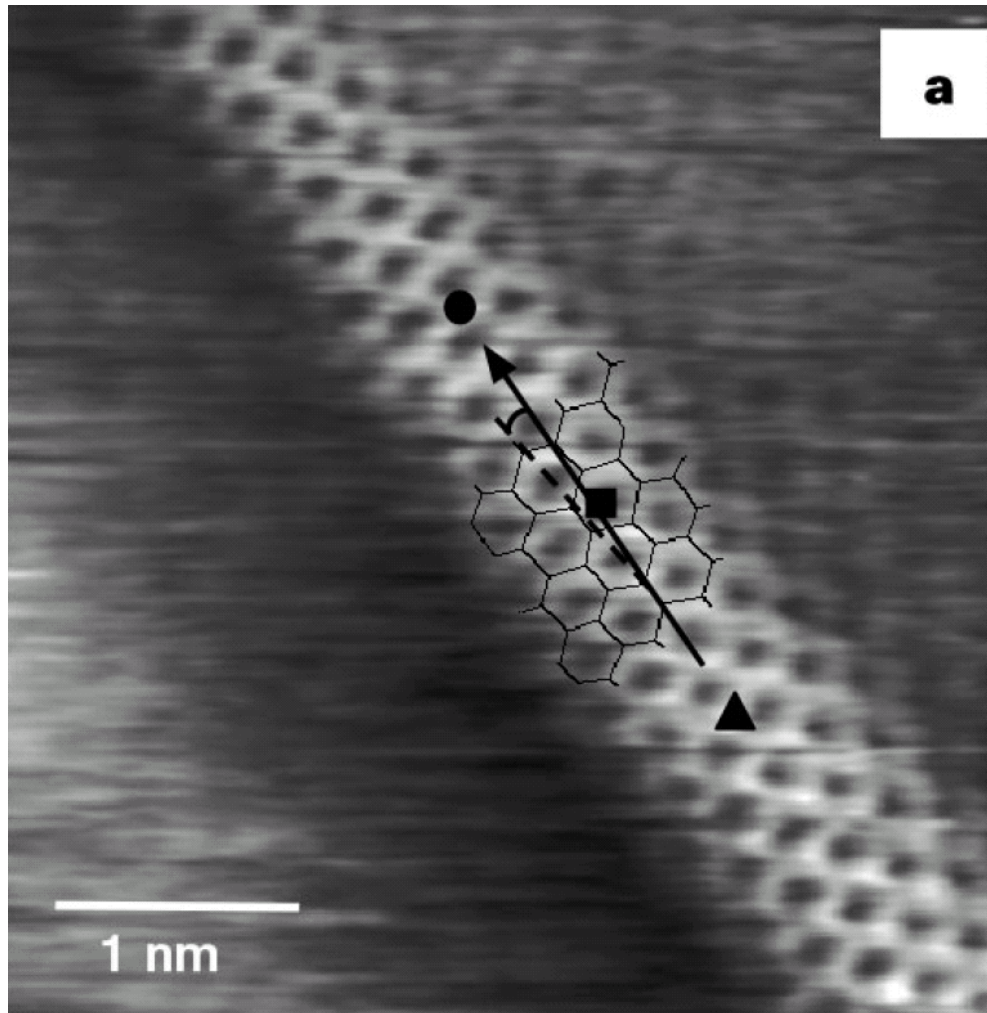
STM : probe for structure and LDOS



$$\frac{dI}{dV}(V, r) \propto \sum_{|eV - \epsilon_j| < \delta} |\psi_j(r)|^2 \sim \rho_S(E_F - eV, r) \quad \delta = \text{energy resolution}$$

Two modes : **Topography** : $I = \text{constant}$, z is measured
Spectroscopy : z is fixed, $I(V)$ is measured

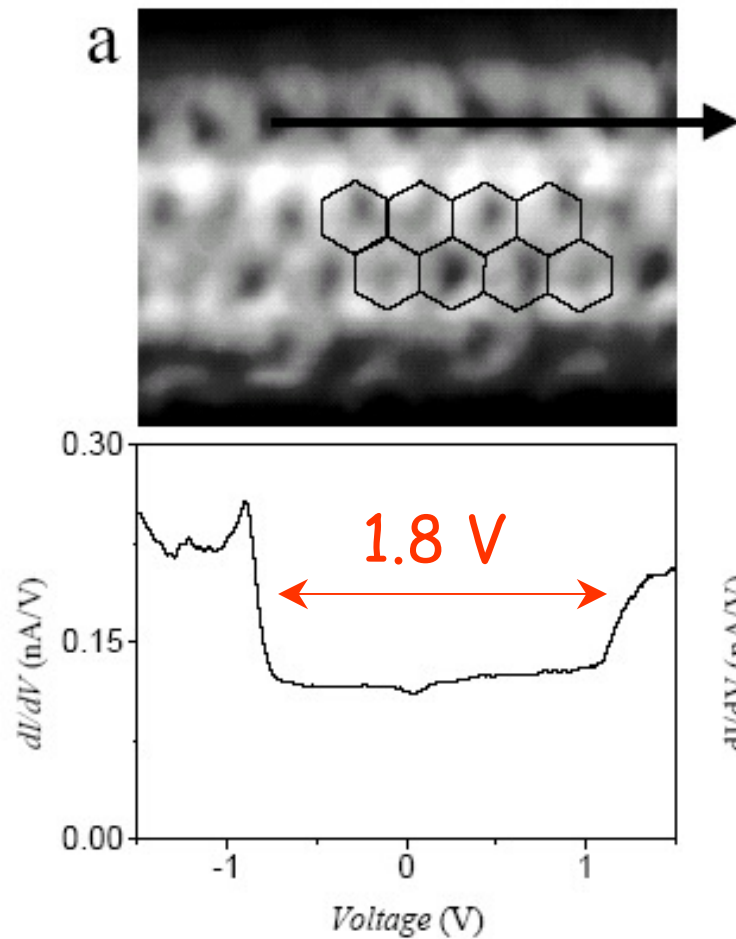
Topography of carbon nanotubes



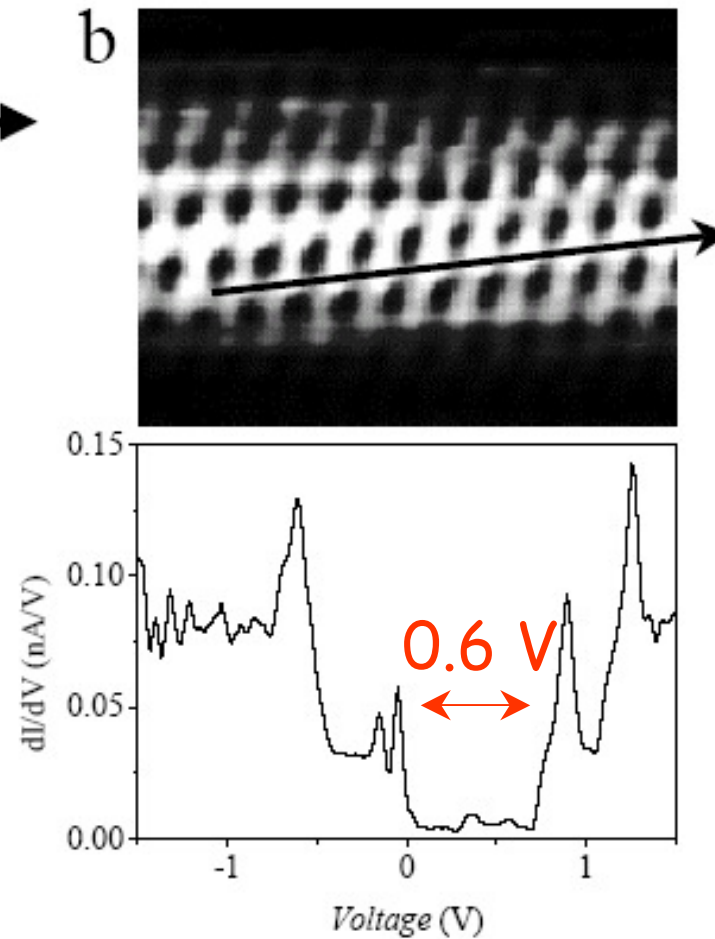
Odom et al., Nature (1998)

➔ Determination of the chirality

Density of states measured by STM



Armchair tube
(metallic)



Chiral tube
(semiconducting)

Wildöer *et al.*,
Nature (1998)

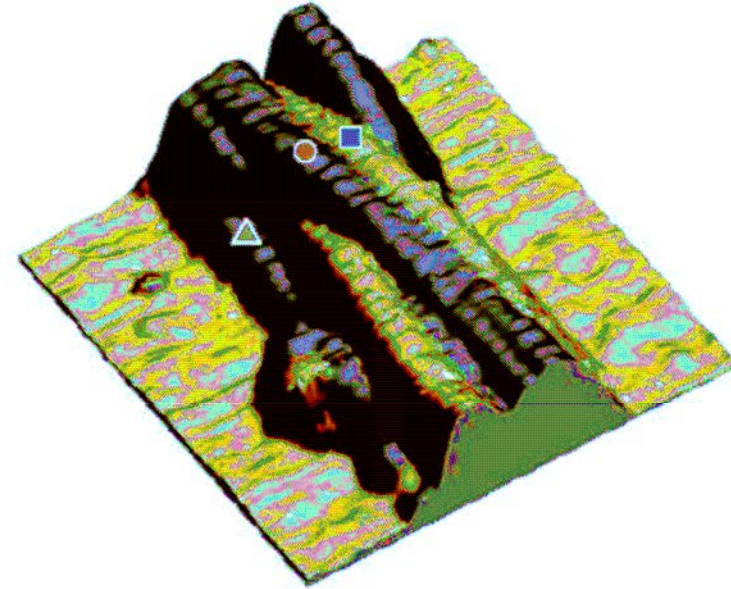


Relation between structure and electronic properties

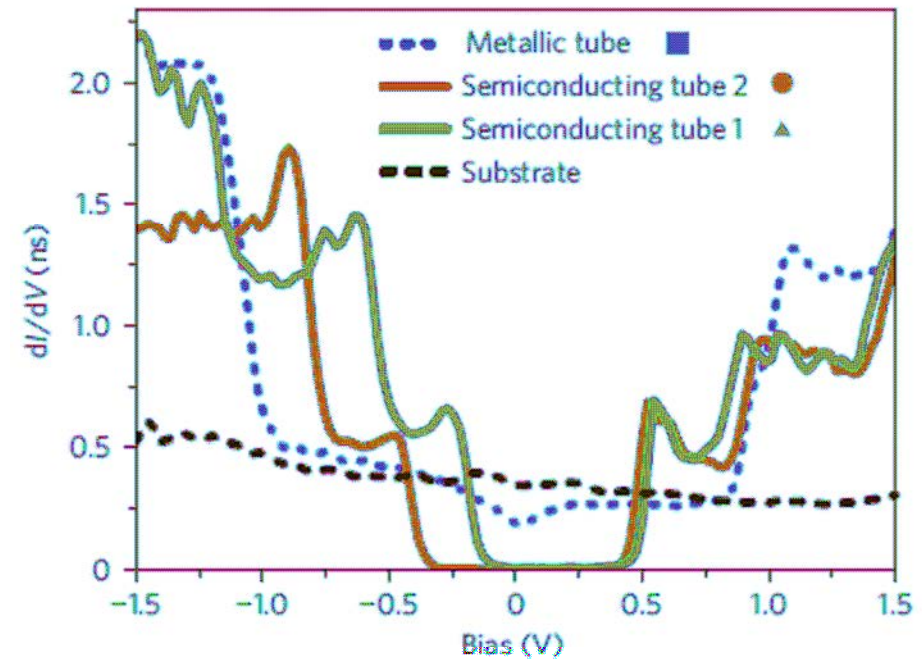
Observation de nanotubes dans un faisceau

Lin et al Nature Materials 2010

Substrat Au

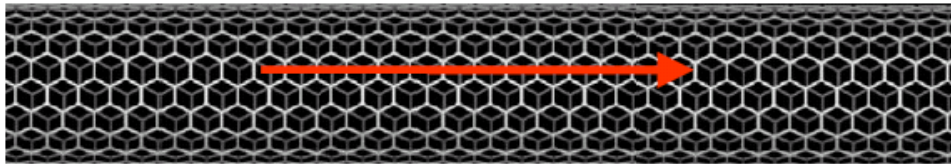


Spectroscopie STM
Gap modifié par écrantage
du substrat



1D imaging of wavefunctions

- Short tubes (30 nm) to enhance finite size effect
- low-bias



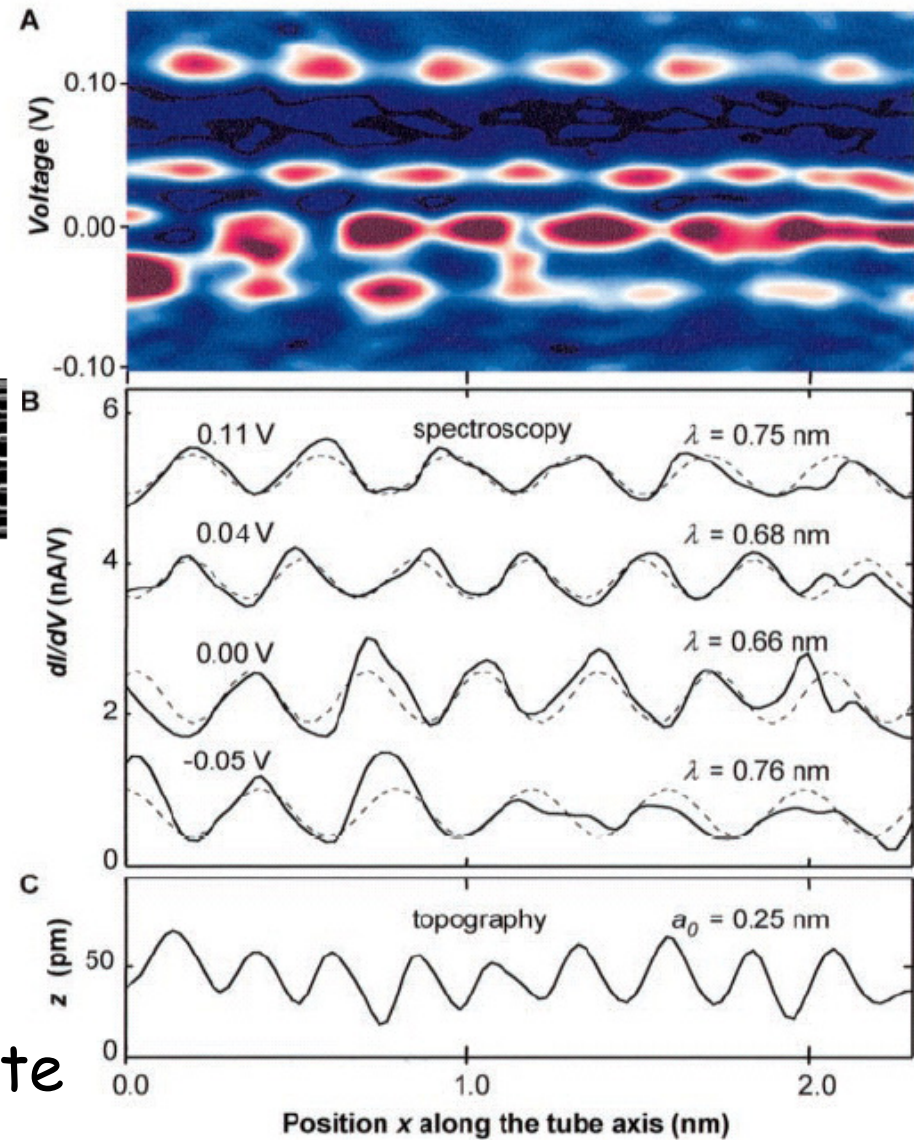
Standing waves in the nanotube

$$|\psi(x)|^2 = A \sin^2(2\pi x/\lambda_F + \varphi)$$

For armchair nanotube :

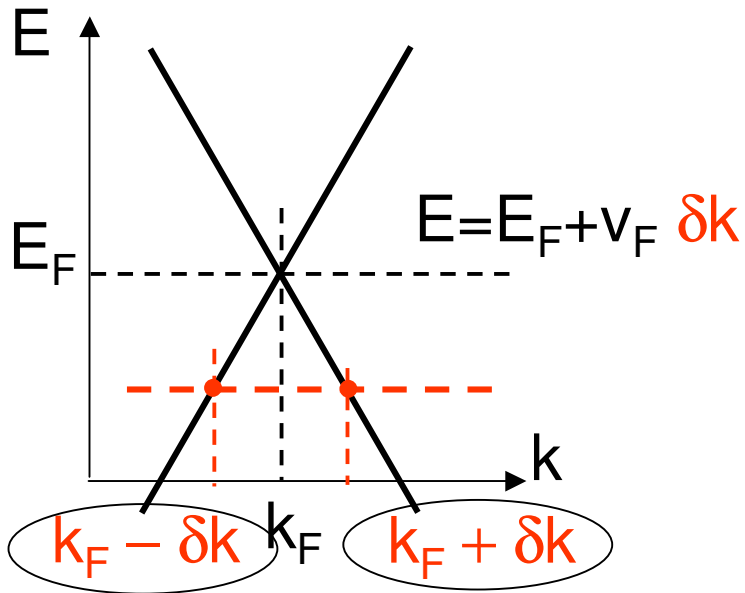
$$\lambda_F = 3 a_0 \sim 0.74 \text{ nm}$$

+ E_F shift due to Au substrate

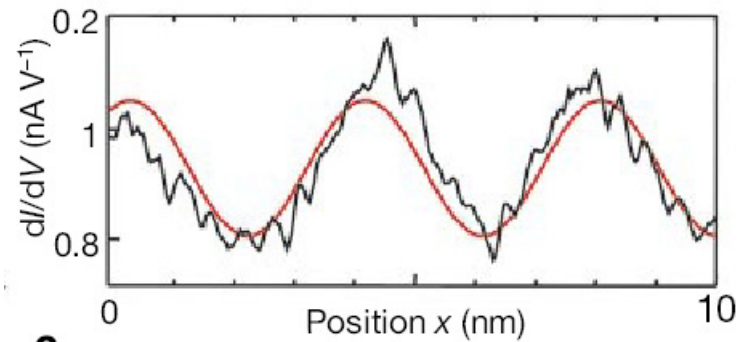


Venema *et al.*, Science (1999)

Interference of wave functions

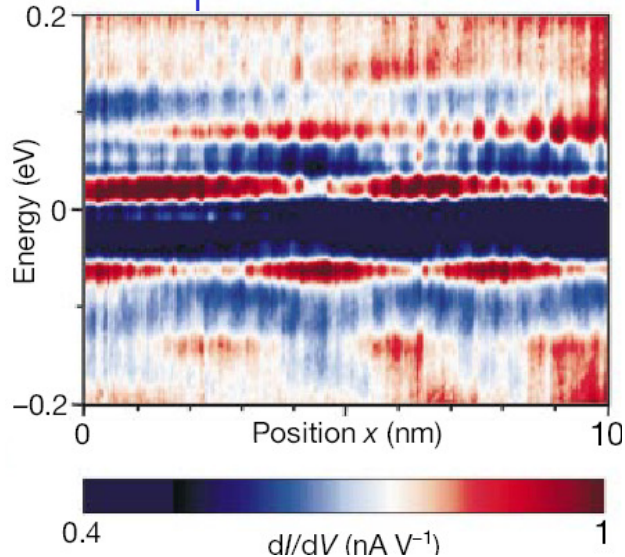


Lemay *et al.*, Nature (2001)

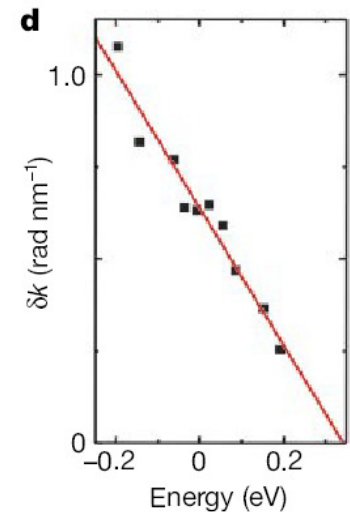


$$|\psi(x)|^2 = A \cos(2\delta k x + \phi)$$

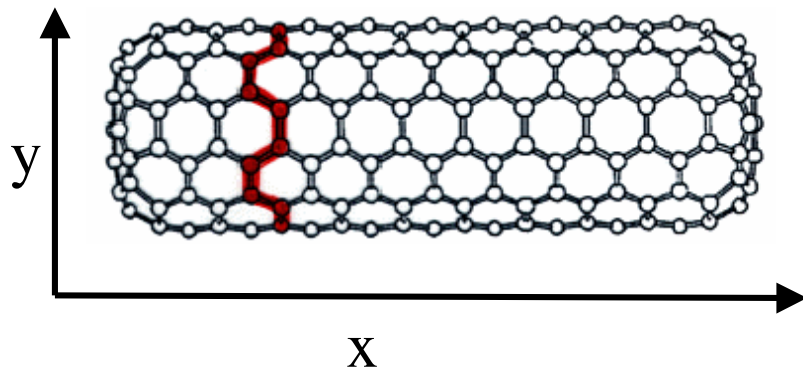
oscillation at k_F and modulation at δk



Band structure

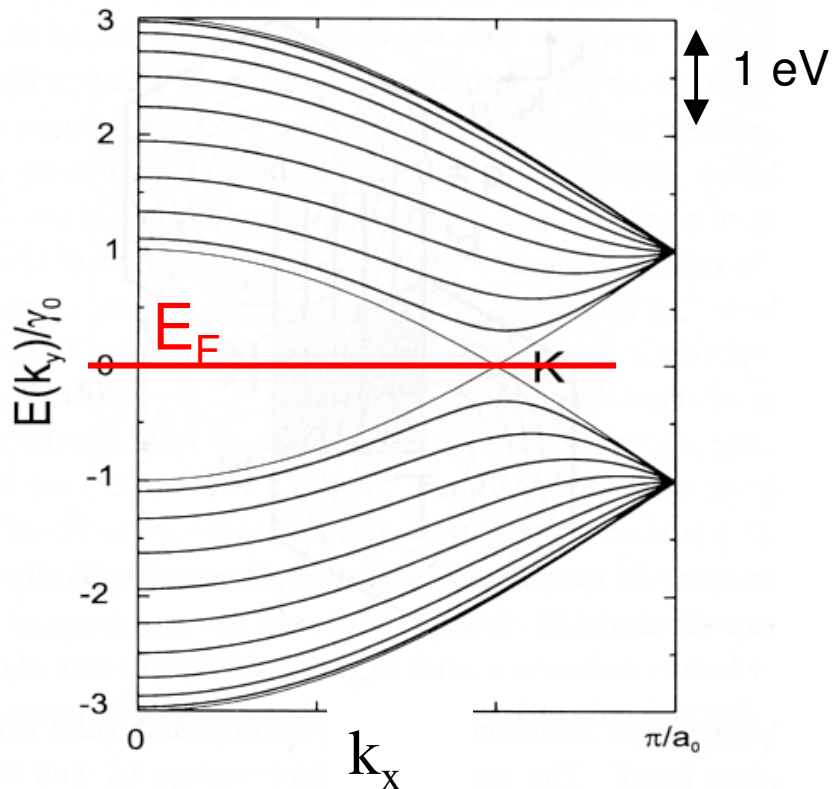


Nanotube « Armchair »



Metallic Carbon Nanotube
Ideal 1D conductor

Band Structure $k_y = 2n\pi / L_y$
 $N_y = L_y / \lambda_F$



At most 2 conduction modes

- $v_F \sim 10^6$ m/s

Long elastic mean free path:

$l_e \sim 0.3$ to $1 \mu\text{m}$

→ Resistance minimum

$R_{\min} = R_0 / 2 = h / 4e^2 = 6.5 \text{ k}\Omega / \text{tube}$

Normal contacts

No quantum diffusive regime:

Localisation length $\sim l_e$

Tunnel contacts Quantum Dot:

Level spacing : $h v_F / L \sim 1 \text{ meV} \sim 10 \text{ K!}$

SWNT on tunnel contacts: $T \gg E_c$

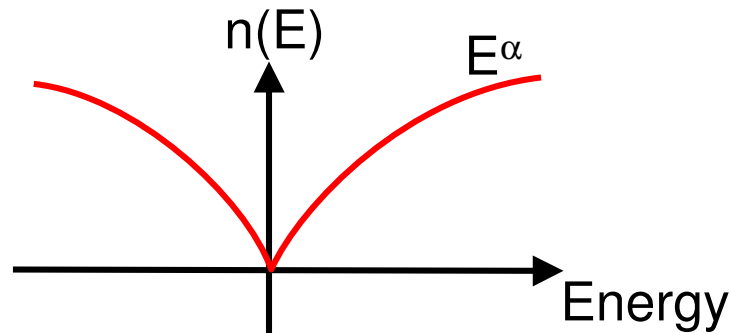
Luttinger liquid behavior?

A 1D system of interacting electrons

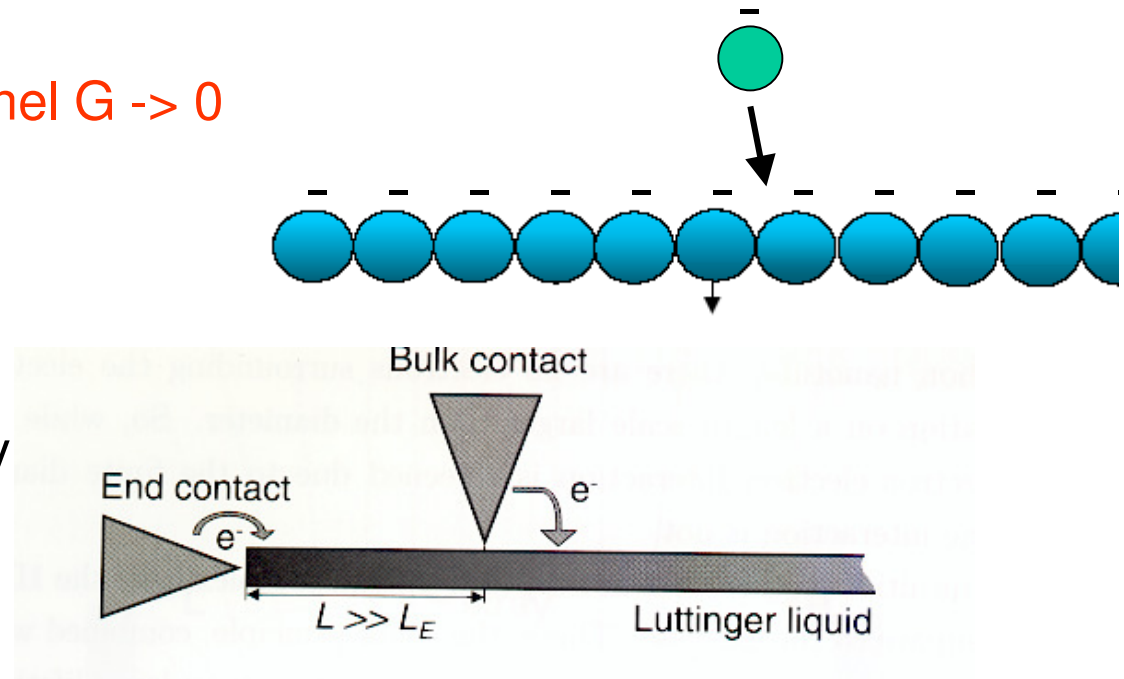
- At 3D, screening is efficient : excitations are quasi-particles, with small residual interactions (Fermi liquid).
- At 1D, interactions are much stronger: no quasi-particles, Luttinger liquid, with long wavelength charge and spin excitations.

Implications:

1D + repulsive interaction: tunnel $G \rightarrow 0$



$$G(T) \propto T^\alpha \text{ for } eV \ll k_B T$$



Interaction strength characterized by the **LL parameter g**

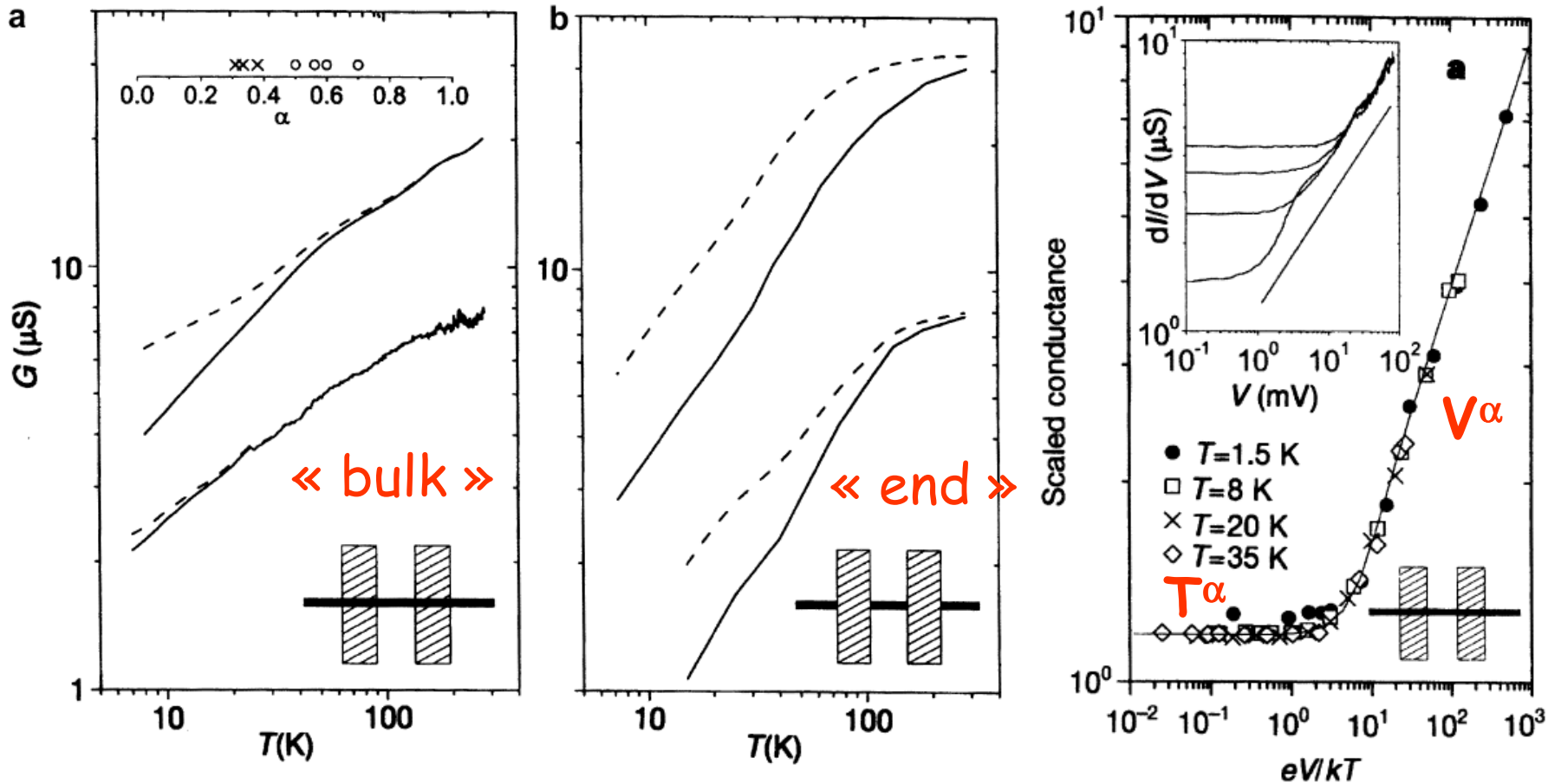
$g = 1$ no interaction

$g < 1$ repulsive interaction

E_c charging energy, Δ level spacing

$$g = (1 + 2E_c / \Delta)^{1/2}$$

Tunneling in SW carbon nanotubes



$$g = (1 + 2E_c/\Delta)^{-1/2} \sim 0.28 \quad \Rightarrow$$

$$\alpha_{\text{bulk}}(\text{theory}) = (1/g + g - 2)/8 = 0.24$$

$$\alpha_{\text{end}}(\text{theory}) = (1/g - 1)/4 = 0.65 > 2 \alpha_{\text{bulk}}$$

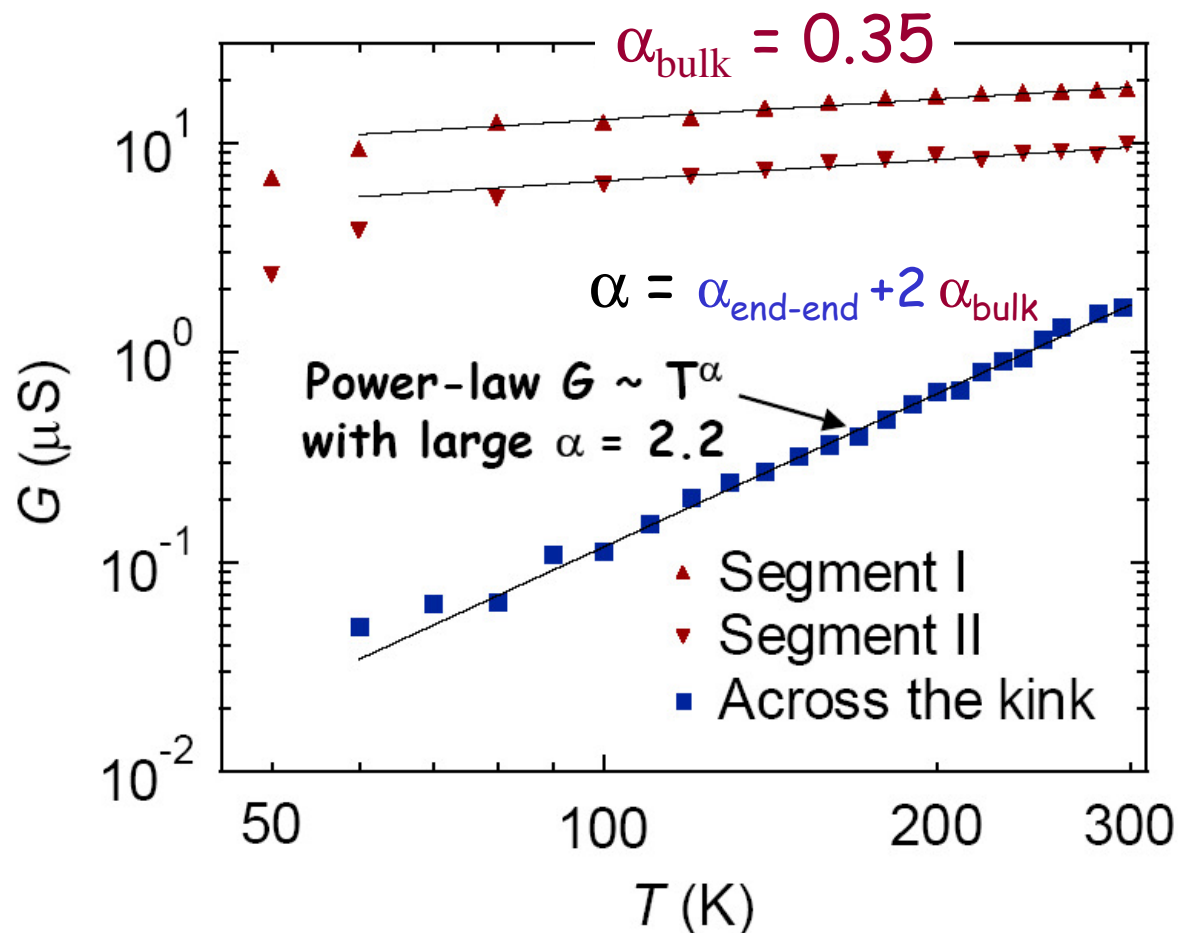
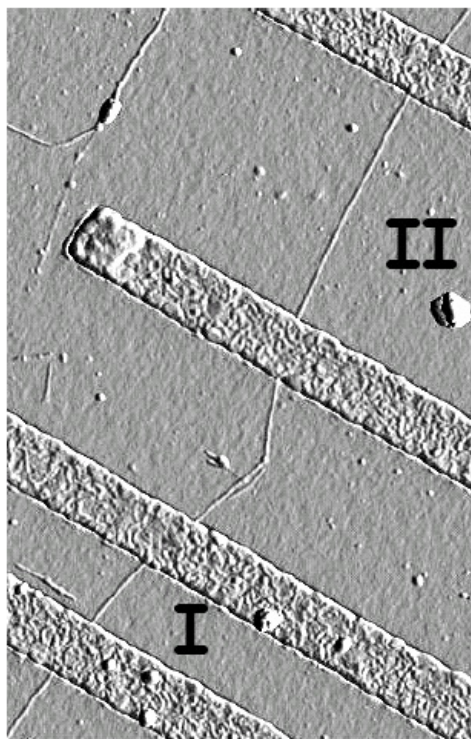
$T < 10$ K : Coulomb Blockade

$T > 10$ K : scaling behaviour

Bochrath *et al.*, Nature (1999)

Bulk versus end tunneling

Metal-metal junction



$$\alpha_{\text{bulk}} = (1/g + g - 2)/8$$

$$\alpha_{\text{end-end}} = 2\alpha_{\text{end}} = (1/g - 1)/2$$

with $g = \text{LL parameter}$

→ $g = 0.22$

→ $g = 0.18$

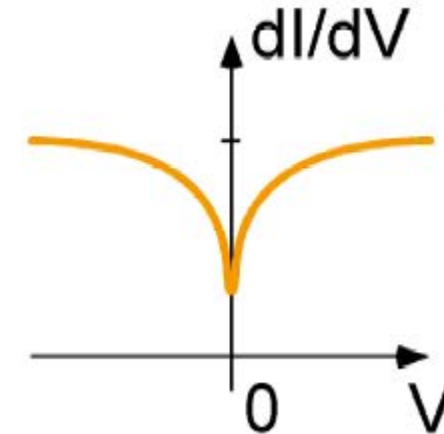
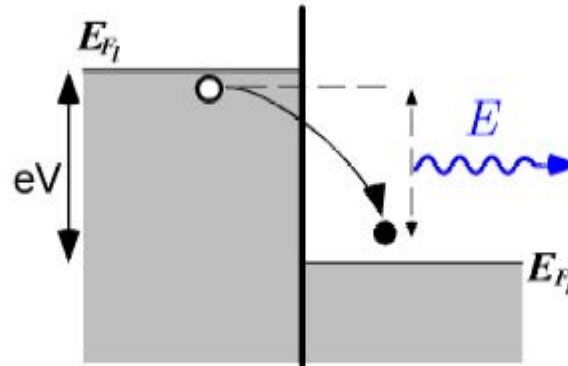
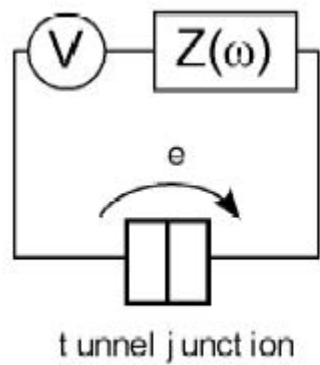
Yao *et al.*, Nature (1999)

Environmental Coulomb Blockade

Ingold, Nazarov 1992

Power laws not specific of Luttinger Liquid physics...

R. Cron
thesis 2001



Charge transfer excites electromagnetic modes
Of the electromagnetic environment $Z(\omega)$

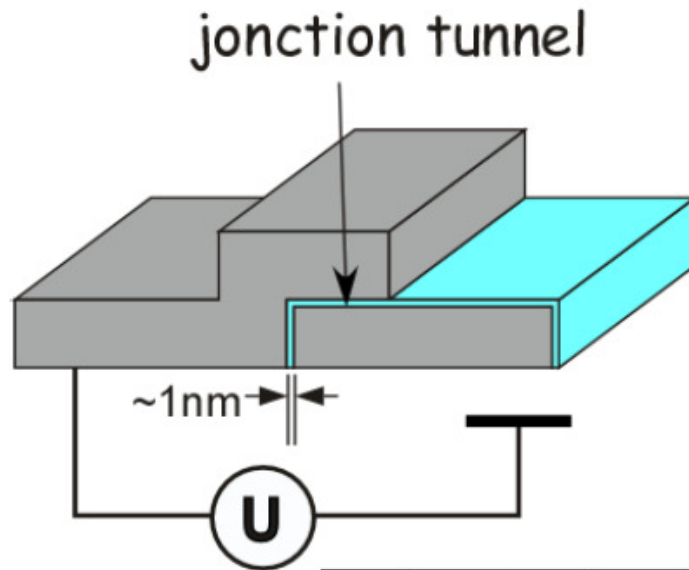
$$I = G_t \iint^{eV} P(E_1 - E_2) dE_1 dE_2 \quad eV < E_c, \quad dI/dV \sim V^{2/g}$$

$$g \sim Z(0)/R_q$$

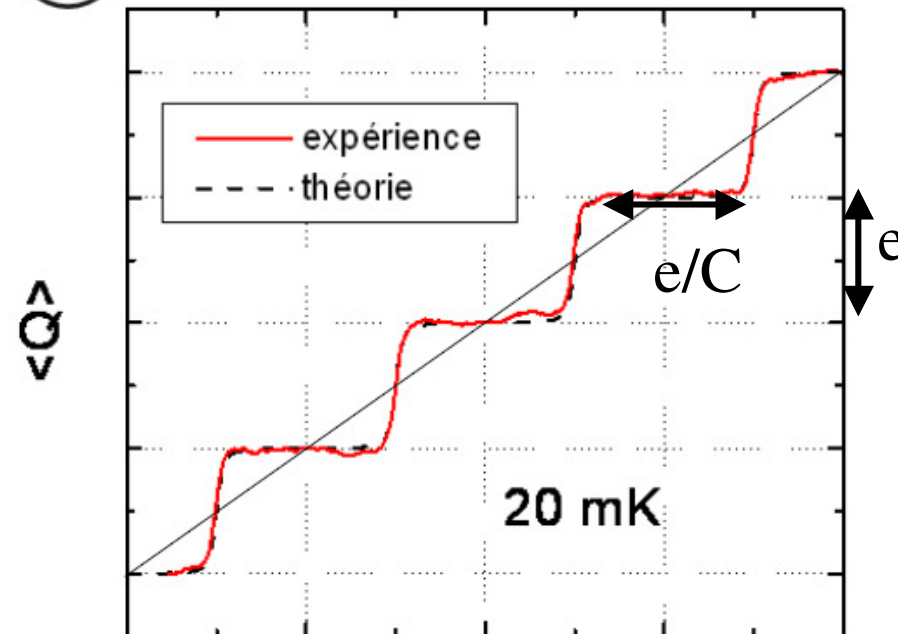
Not in contradiction with the LL picture:

Luttinger Liquids can be modelised by a LC transmission line...

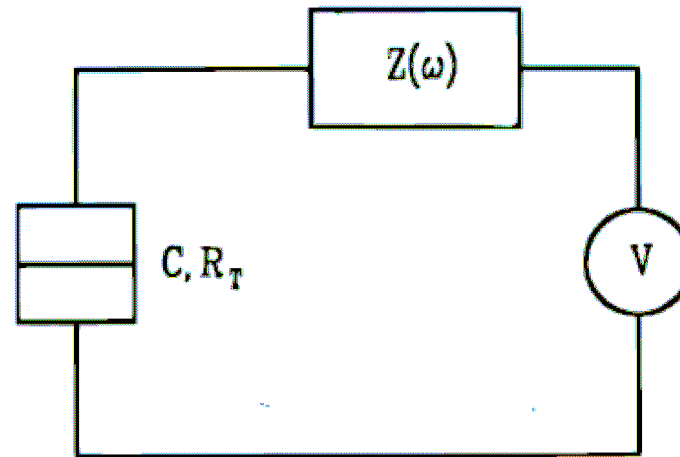
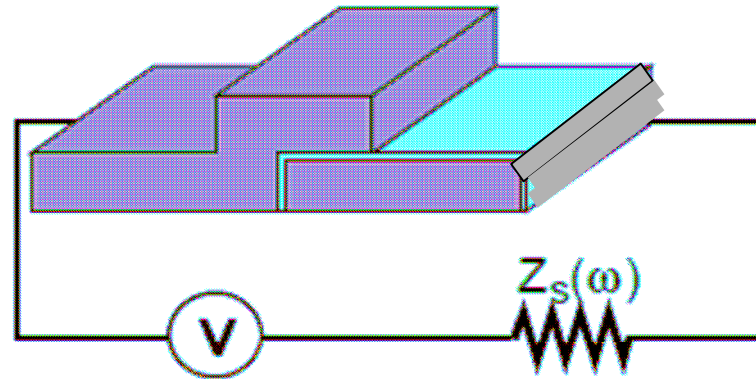
Blocage de Coulomb (Statique)



Aire $\sim (100 \text{ nm})^2$
Capa $\sim 1 \text{ fF}$
 $E_C = e^2/2C \sim 1 \text{ k}_B\text{K}$
 $R_T \gg R_K = h/e^2 \approx 26 \text{ k}\Omega$

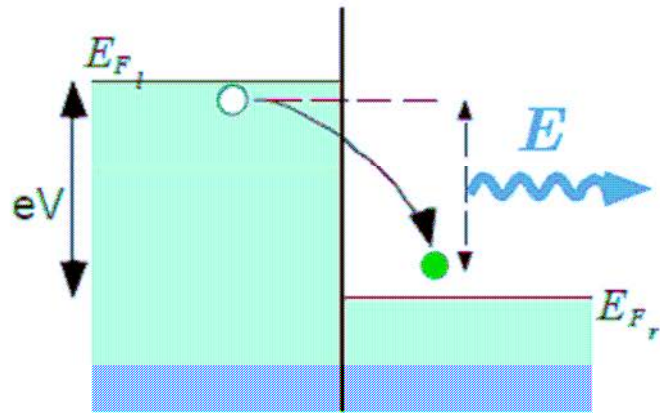


Blocage de Coulomb dynamique



Caractéristique courant tension

Charge Tunneling Rates in Ultrasmall Junctions
Ingold Nazarov cond-mat/0508728

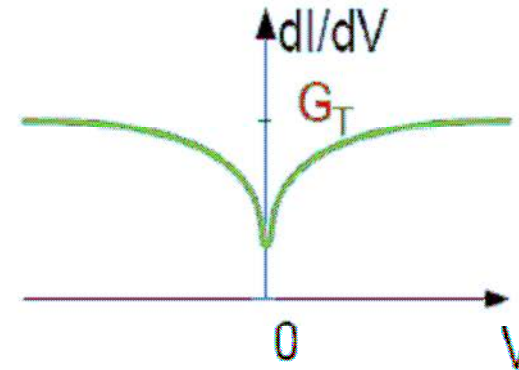


effet tunnel inélastique

Excitation des modes électromagnétiques de l'environnement R

Point de départ: règle d'or de Fermi $\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle f | \tilde{H}_T | i \rangle|^2 \delta(E_i - E_f)$

$$I(V) = e(\vec{\Gamma}(V) - \overleftarrow{\Gamma}(V))$$



"Hamiltonien" tunnel

Doit coupler les deux électrodes et l'environnement

Hamiltonien tunnel
(traité en perturbation) $H_T = T + T^\dagger$

$$T = e^{i\varphi} \sum_{\ell,r} t_{\ell r} c_\ell^\dagger c_r$$

décale la charge transférée de e

$$\varphi(t) = eV_{dc}t/\hbar + \tilde{\varphi}(t)$$

Charge et phase sont des variables

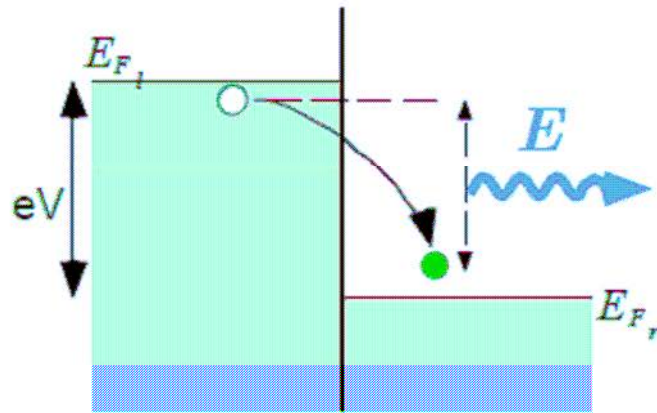
Phase
électromagnétique

$$[\varphi, Q_T] = ie$$

$$e^{-i\varphi} Q_T e^{i\varphi} = Q_T + e$$

Caractéristique courant tension

Charge Tunneling Rates in Ultrasmall Junctions
Ingold Nazarov cond-mat/0508728



effet tunnel inélastique

Excitation des modes électromagnétiques de l'environnement R

Point de départ: règle d'or de Fermi $\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle f | \tilde{H}_T | i \rangle|^2 \delta(E_i - E_f)$

$$I(V) = e(\bar{\Gamma}(V) - \tilde{\Gamma}(V))$$

$$\bar{\Gamma}(V) = \frac{1}{e^2 R_T} \int_{-\infty}^{+\infty} dE dE' f(E) [1 - f(E')] \sum_{R, R'} |\langle R | e^{-i\tilde{\varphi}} | R' \rangle|^2 P_\beta(R) \delta(E + eV + E_R - E' - E'_R)$$

$$\tilde{\varphi}(t) = \varphi(t) - \frac{e}{\hbar} V t$$

$$\delta(E + eV + E_R - E' - E'_R) = \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} dt \exp\left(\frac{i}{\hbar}(E + eV + E_R - E' - E'_R)t\right)$$

Caractéristique courant tension

$$\vec{\Gamma}(V) = \frac{1}{e^2 R_T} \int_{-\infty}^{+\infty} dE dE' \int_{-\infty}^{+\infty} \frac{dt}{2\pi\hbar} \exp\left(\frac{i}{\hbar}(E - E' + eV)t\right) f(E)[1 - f(E')] \times \sum_{R,R'} P_\beta(R) \langle R | e^{i\tilde{\varphi}(t)} | R' \rangle \langle R' | e^{-i\tilde{\varphi}(0)} | R \rangle.$$

Théorème
fluctuation
dissipation

$$J(t) = \langle \tilde{\varphi}(t) \tilde{\varphi}(0) \rangle = 2 \int_{-\infty}^{+\infty} \frac{d\omega}{\omega} \frac{\text{Re} Z_t(\omega)}{R_K} \frac{e^{-i\omega t}}{1 - e^{-\beta\hbar\omega}}.$$

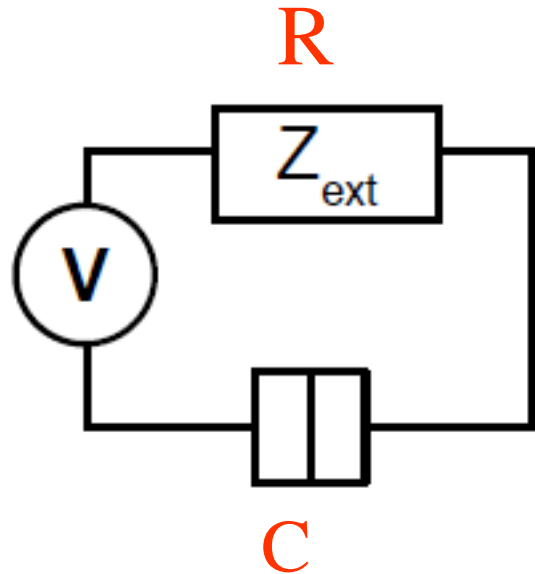
$$P(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} dt \exp\left[J(t) + \frac{i}{\hbar}Et\right].$$

$$\vec{\Gamma}(V) = \frac{1}{e^2 R_T} \int_{-\infty}^{+\infty} dE dE' f(E)[1 - f(E' + eV)] P(E - E')$$

$$I(V) = \frac{1}{eR_T} (1 - e^{-\beta eV}) \int_{-\infty}^{+\infty} dE \frac{E}{1 - e^{-\beta E}} P(eV - E).$$

Limite $T=0$ $I(V) = \frac{1}{eR_T} \int_0^{eV} dE (eV - E) P(E).$

Environnement Ohmique



$$\frac{\text{Re}Z_t(\omega)}{R_K} = \frac{1}{g} \frac{1}{1 + (\omega/\omega_R)^2} \quad g = \frac{R_K}{R}$$

$$E \ll E_c$$

$$\frac{dP(E)}{dE} = \left(\frac{2}{g} - 1\right) \frac{P(E)}{E} \quad P(E) \sim E^{(2/g-1)}$$

$$P(E) = \frac{\exp(-2\gamma/g)}{\Gamma(2/g)} \frac{1}{E} \left[\frac{\pi E}{g E_c}\right]^{2/g}$$

$$I(V) = \frac{\exp(-2\gamma/g)}{\Gamma(2 + 2/g)} \frac{V}{R_T} \left[\frac{\pi e|V|}{g E_c}\right]^{2/g}$$

$$E \gg E_c$$

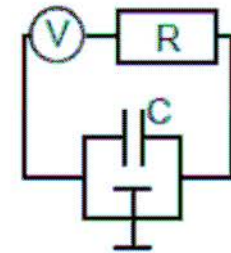
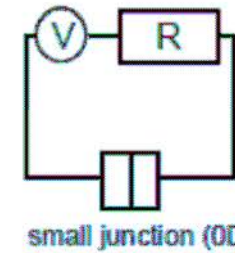
$$P(E) = \frac{2g E_c^2}{\pi^2 E^3} \quad \text{for } E \rightarrow \infty.$$

$$I(V) = \frac{1}{R_T} \left[V - \frac{e}{2C} + \frac{g e^2}{\pi^2 4C^2} \frac{1}{V} \right] \quad \text{for } V \rightarrow \infty.$$

Propriétés de dI/dV

Cas où $Z=R//C$

$$\operatorname{Re} Z(\omega) = \frac{R}{1 + (RC\omega)^2}$$



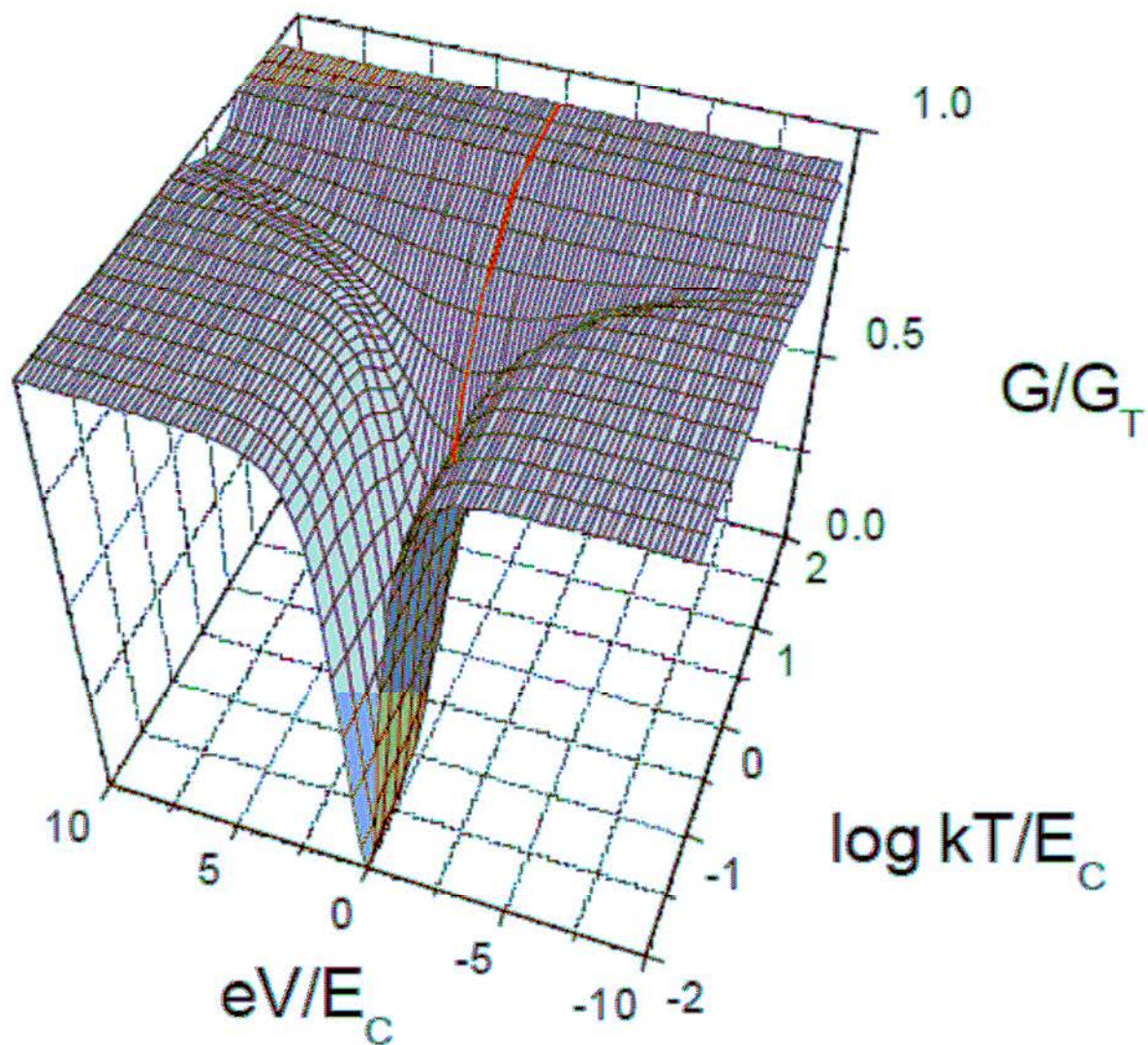
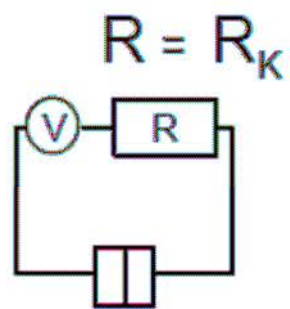
échelles d'énergies :

- eV énergie disponible de la source
- $\frac{e^2}{2C} = E_C$ énergie de charge du condensateur
- $\frac{\hbar}{RC} = E_{RC}$ énergie liée au temps RC
(fréquence de coupure de l'environnement)
- kT énergie des fluctuations thermique

$$\frac{E_C}{E_{RC}} = \pi R \frac{e^2}{h} = \pi \frac{R}{R_K}$$

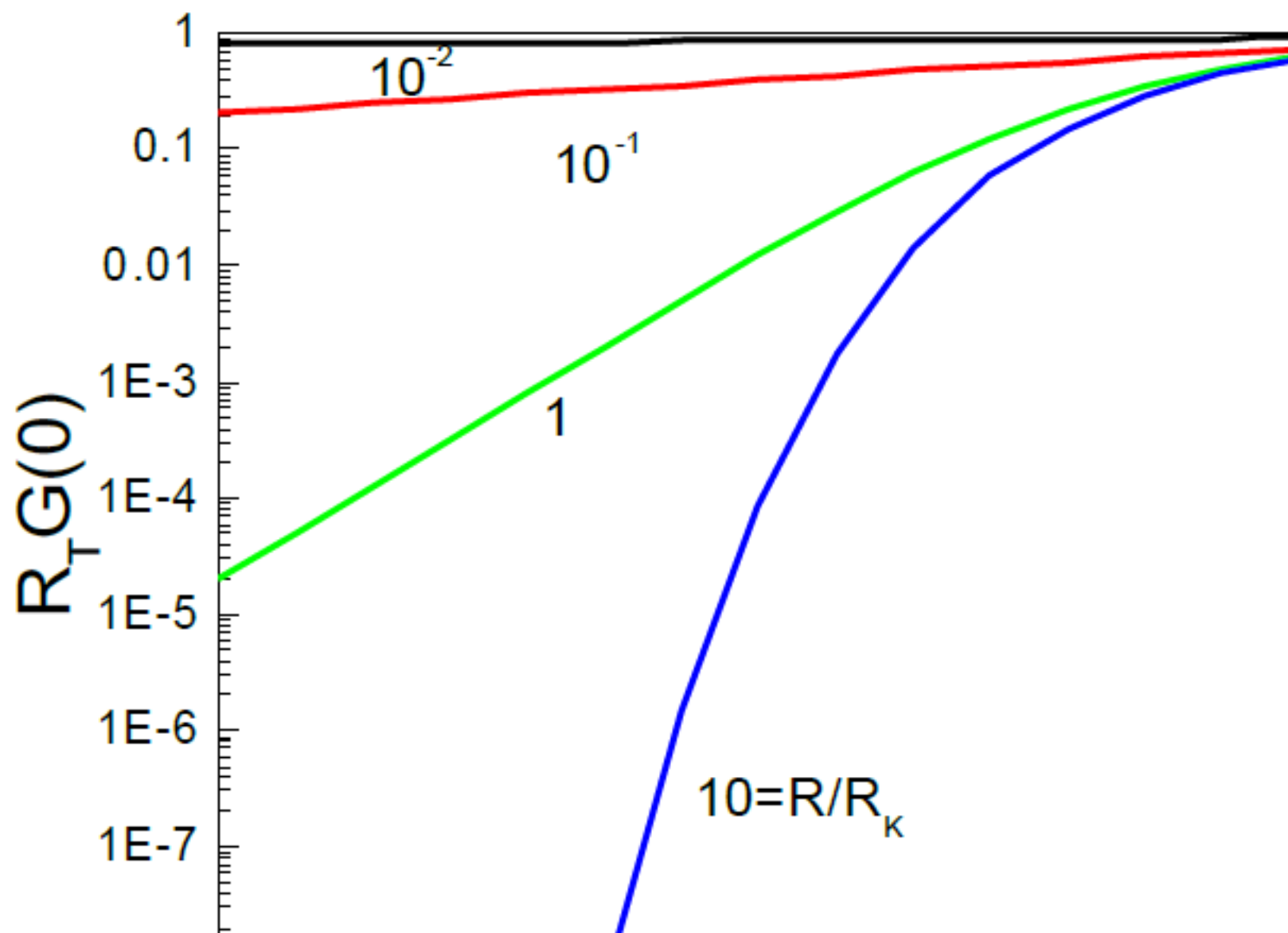
Situations différentes selon $R \ll R_K$ or $R \gg R_K$

Propriétés de dI/dV

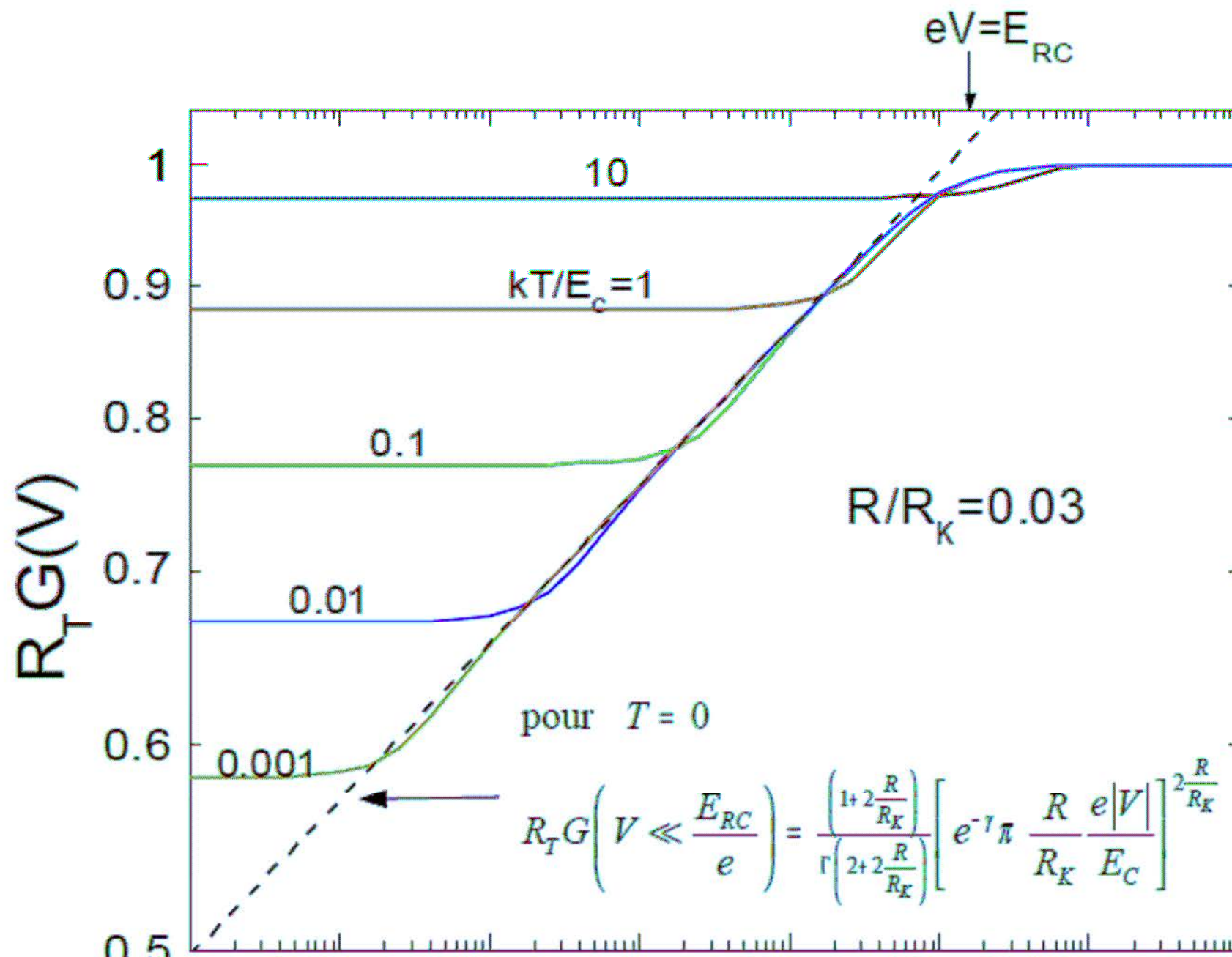


Profondeur du pic à tension nulle : loi de puissance

pour $kT \ll \text{Min}(E_C, E_{RC})$ $G(0) \propto T^{\frac{2R}{R_K}}$

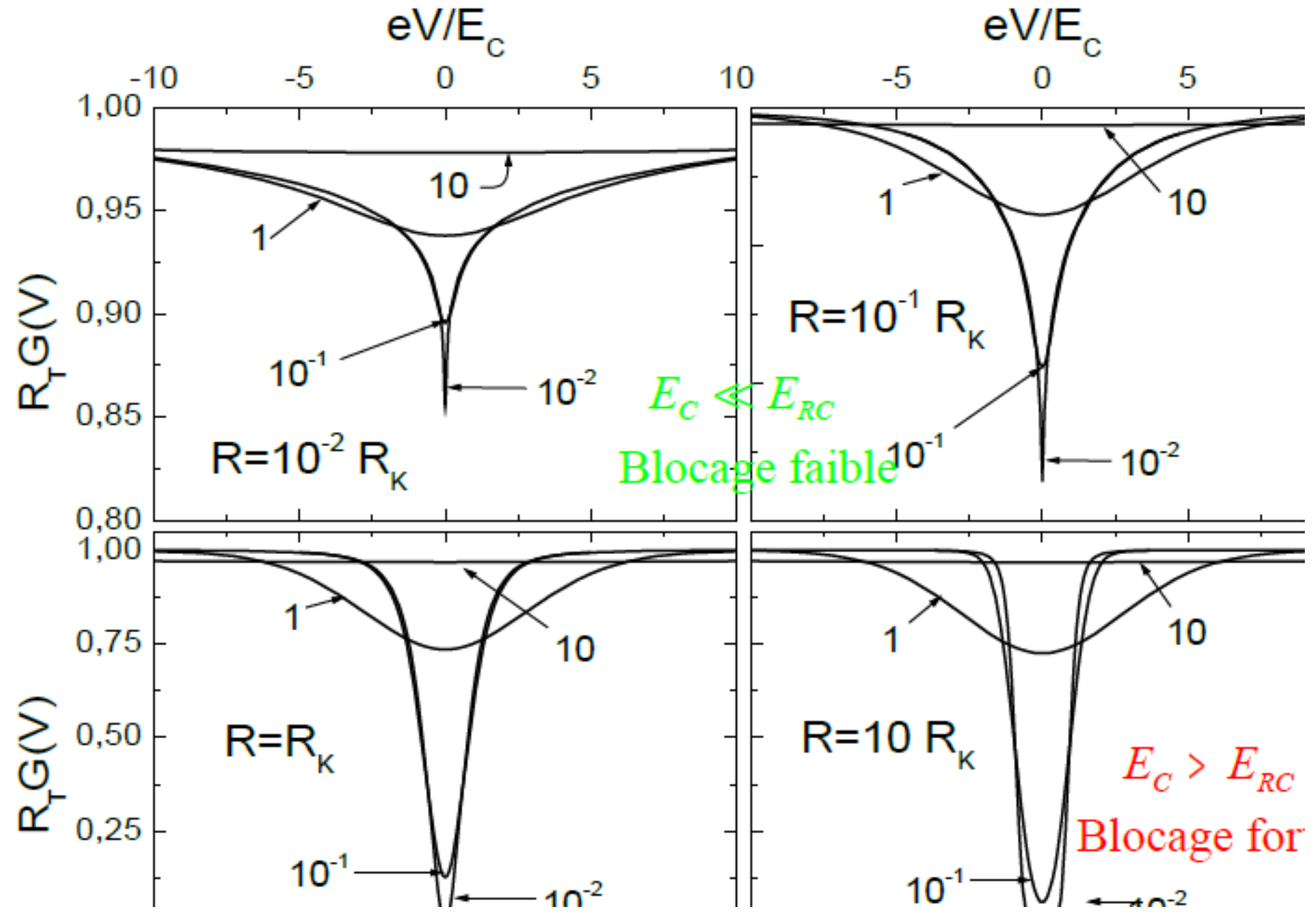


"loi de puissance en tension" pour $kT < eV < E_{RC}$

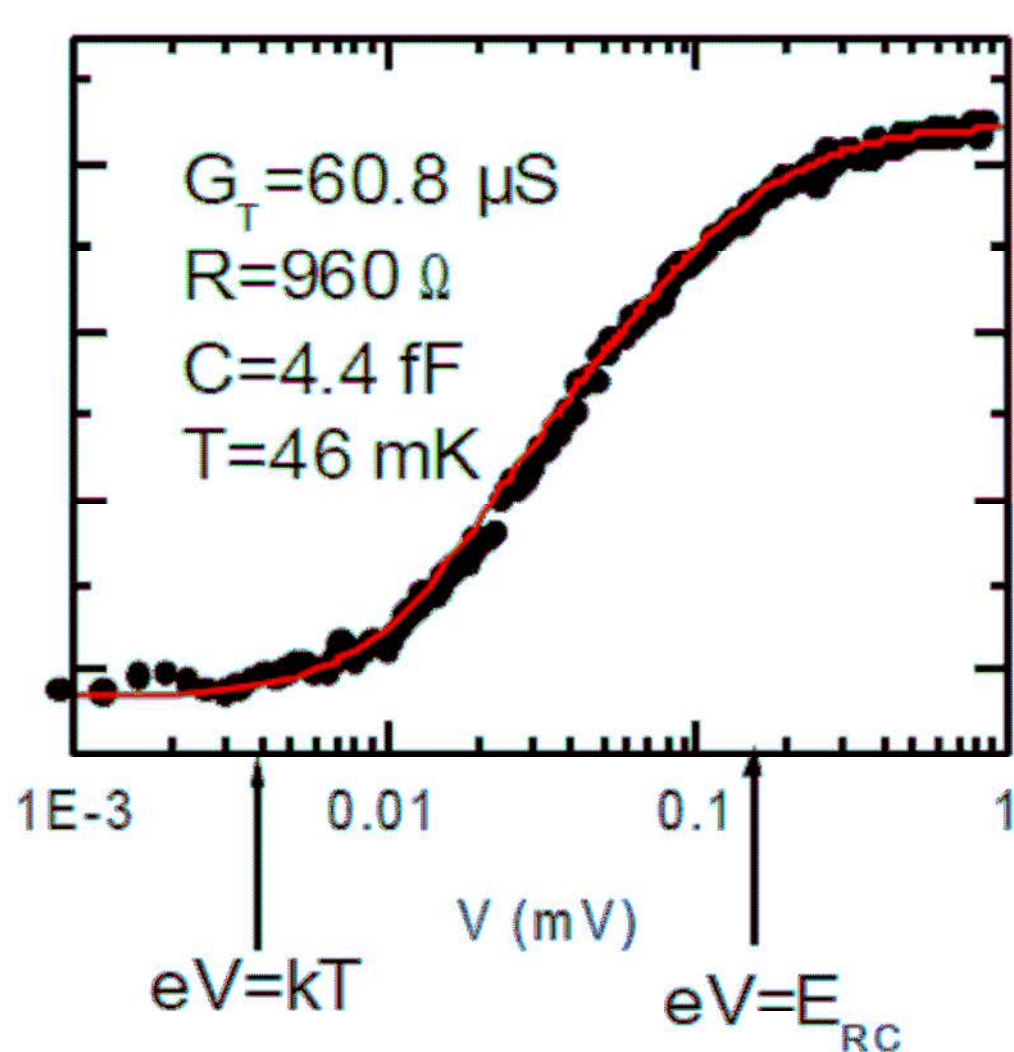


Propriétés de dI/dV

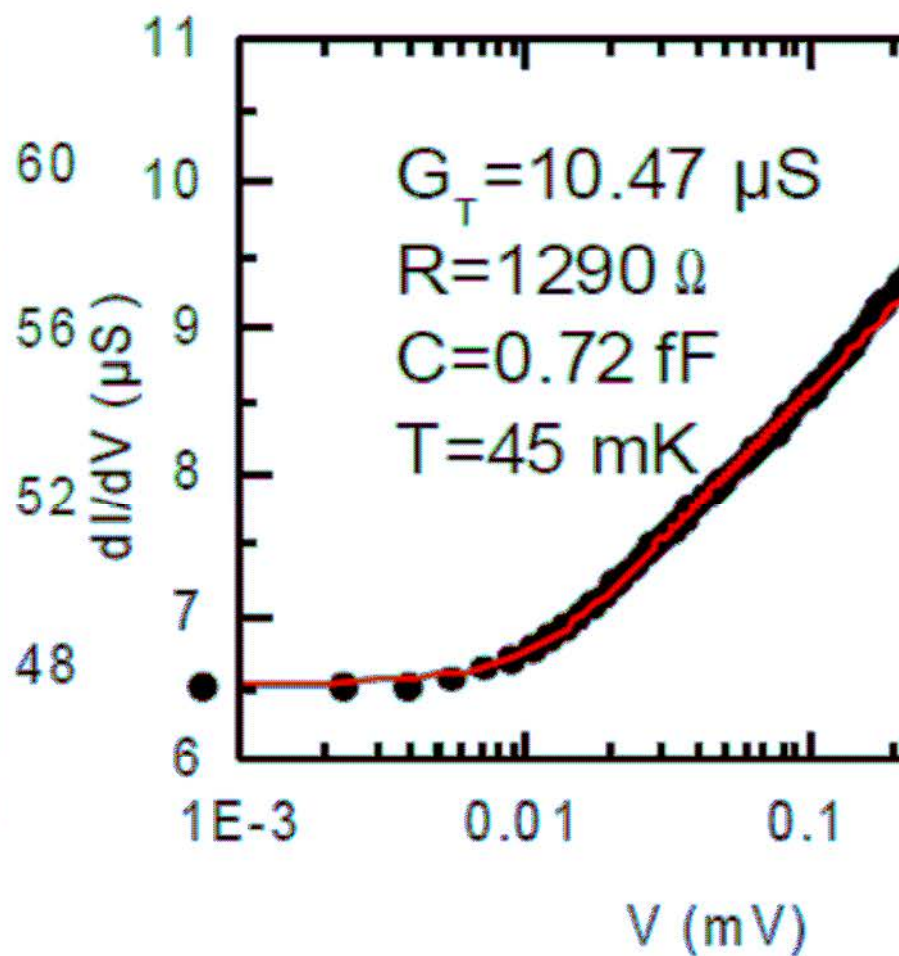
P.Joyez Cargèse 2008



dI/dV faible blocage :
 "loi de puissance en tension" exemple expérimental

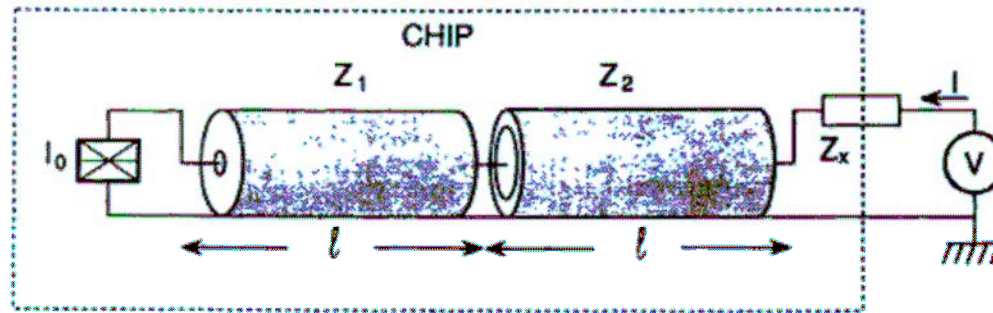


observations expérimentales



Cleland '90
 Phys. Rev. Lett. 64, 104

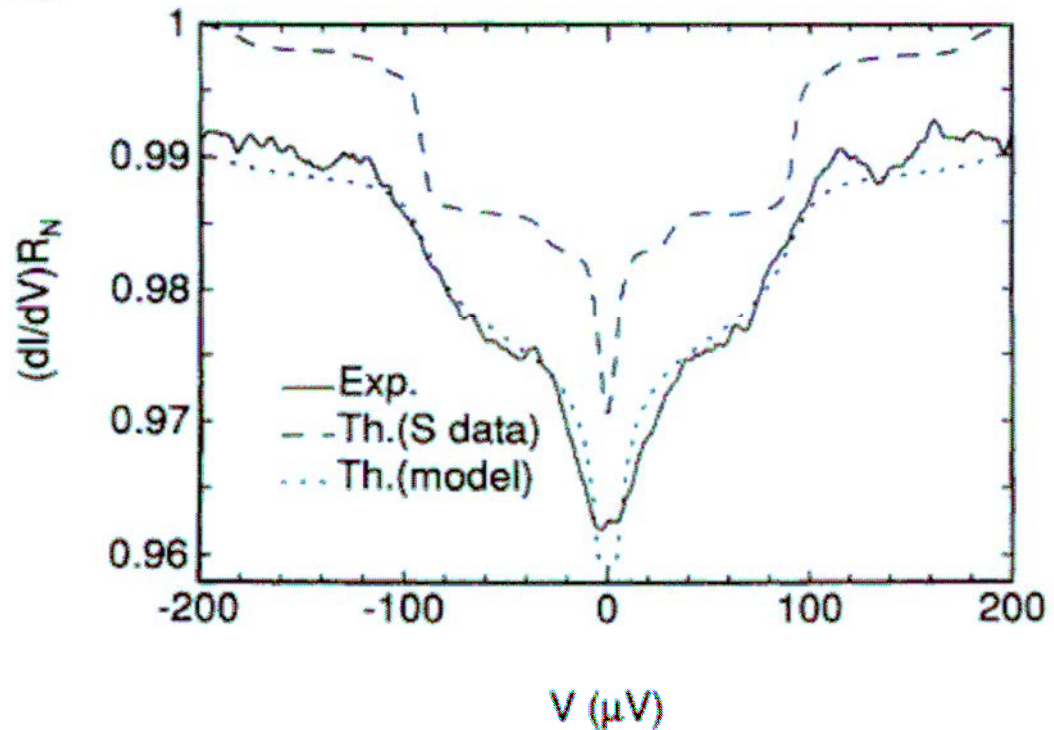
Environnement = résonateur électromagnétique



$$P(E) = e^{-\rho} \sum_{k=0}^{\infty} \frac{\rho^k}{k!} \delta(E - k\hbar\omega_s)$$

$$\rho = \frac{E_c}{\hbar\omega_s}$$

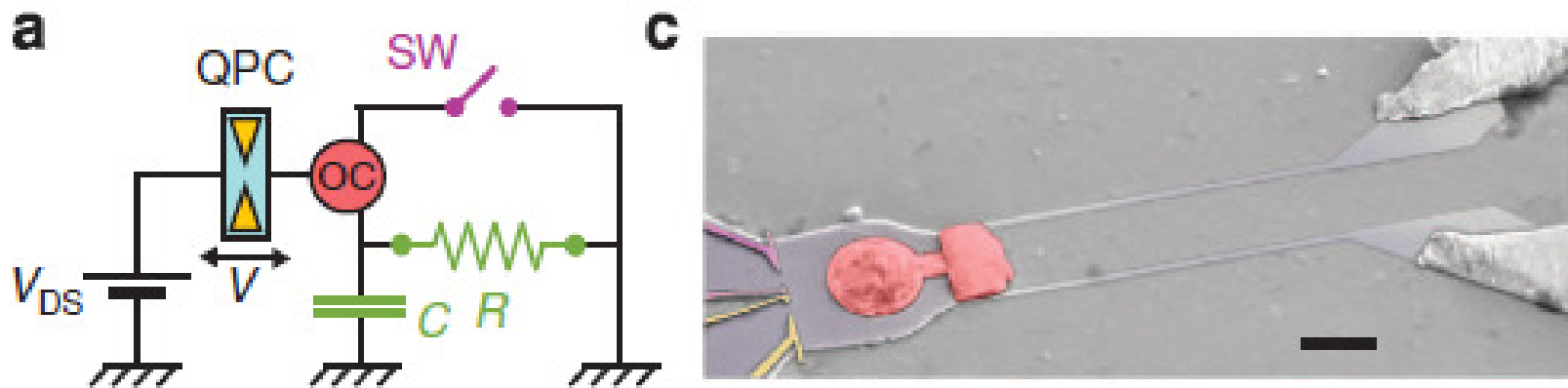
Holtz et al.
PRL 94



De la jonction tunnel au fil quantique

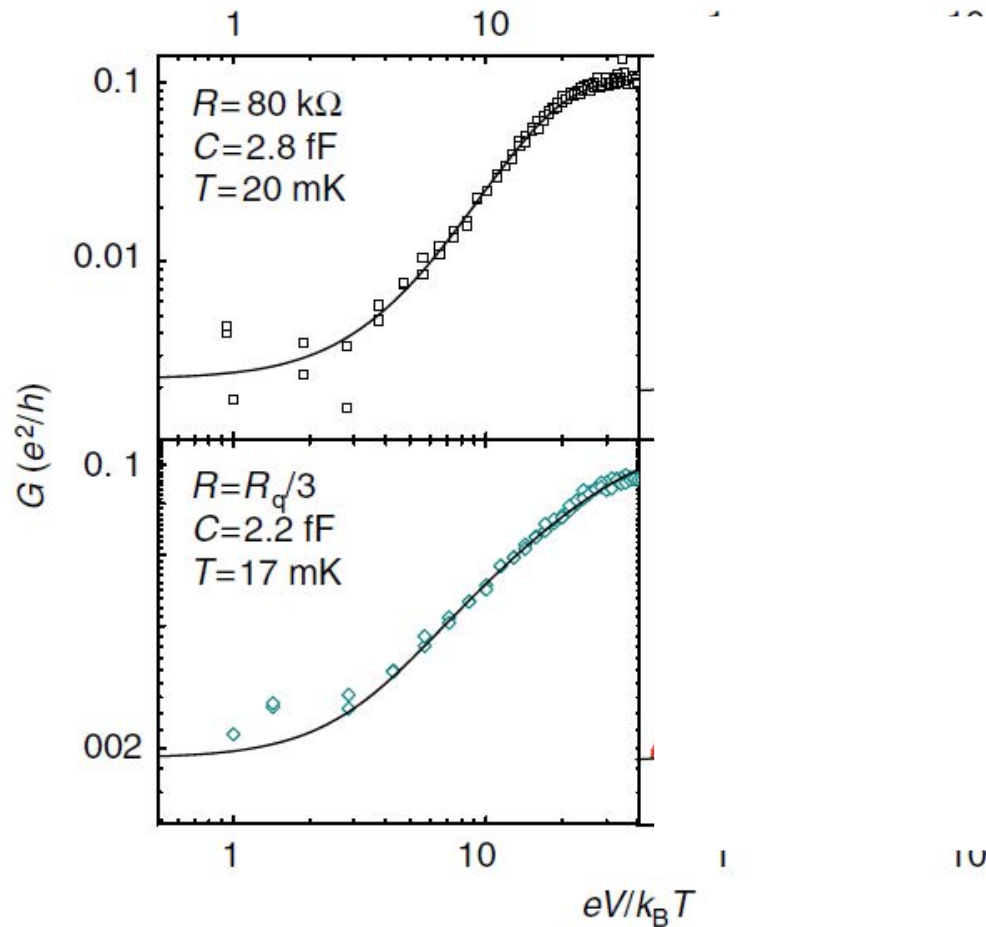
Environnement contrôlable

Jezoin et al 2012, 201



Canaux de bord effet Hall quantique
Conductance élevée $\sim e^2/h$

De la jonction tunnel au fil quantique

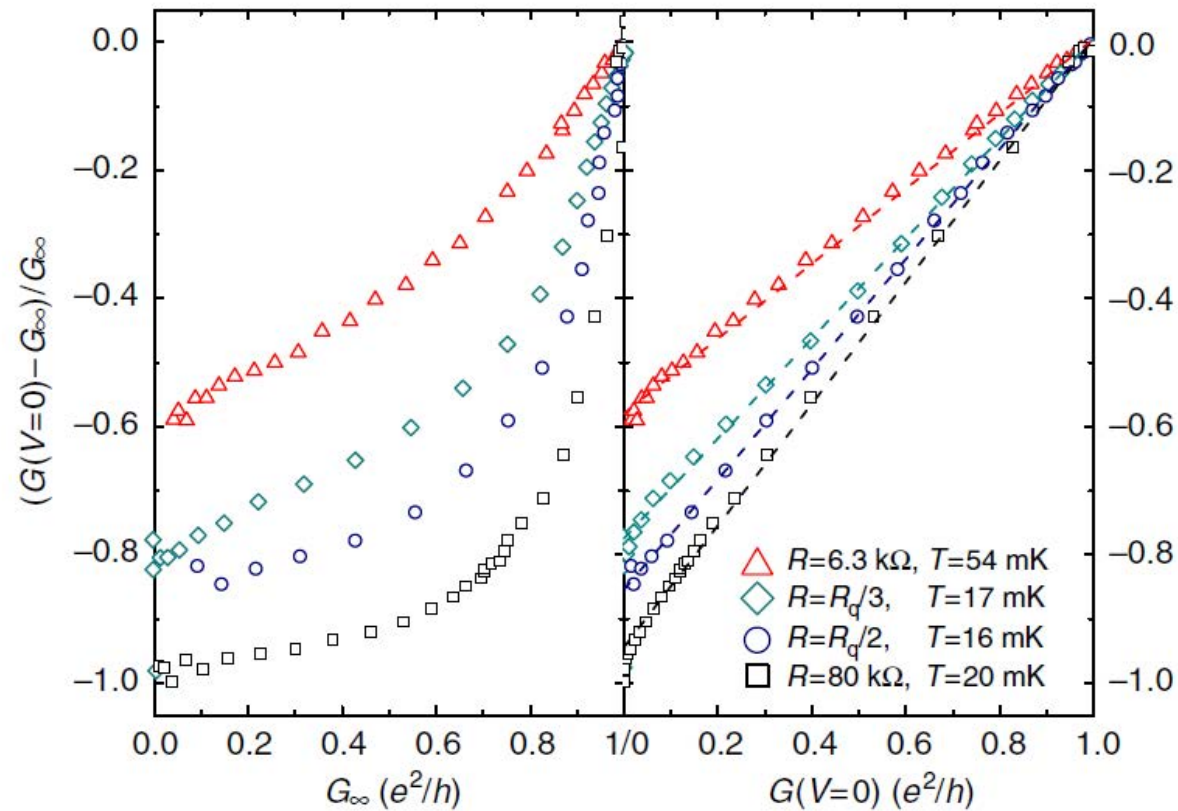


Canaux de bord effet Hall
quantique
Conductance élevée $\sim e^2/h$

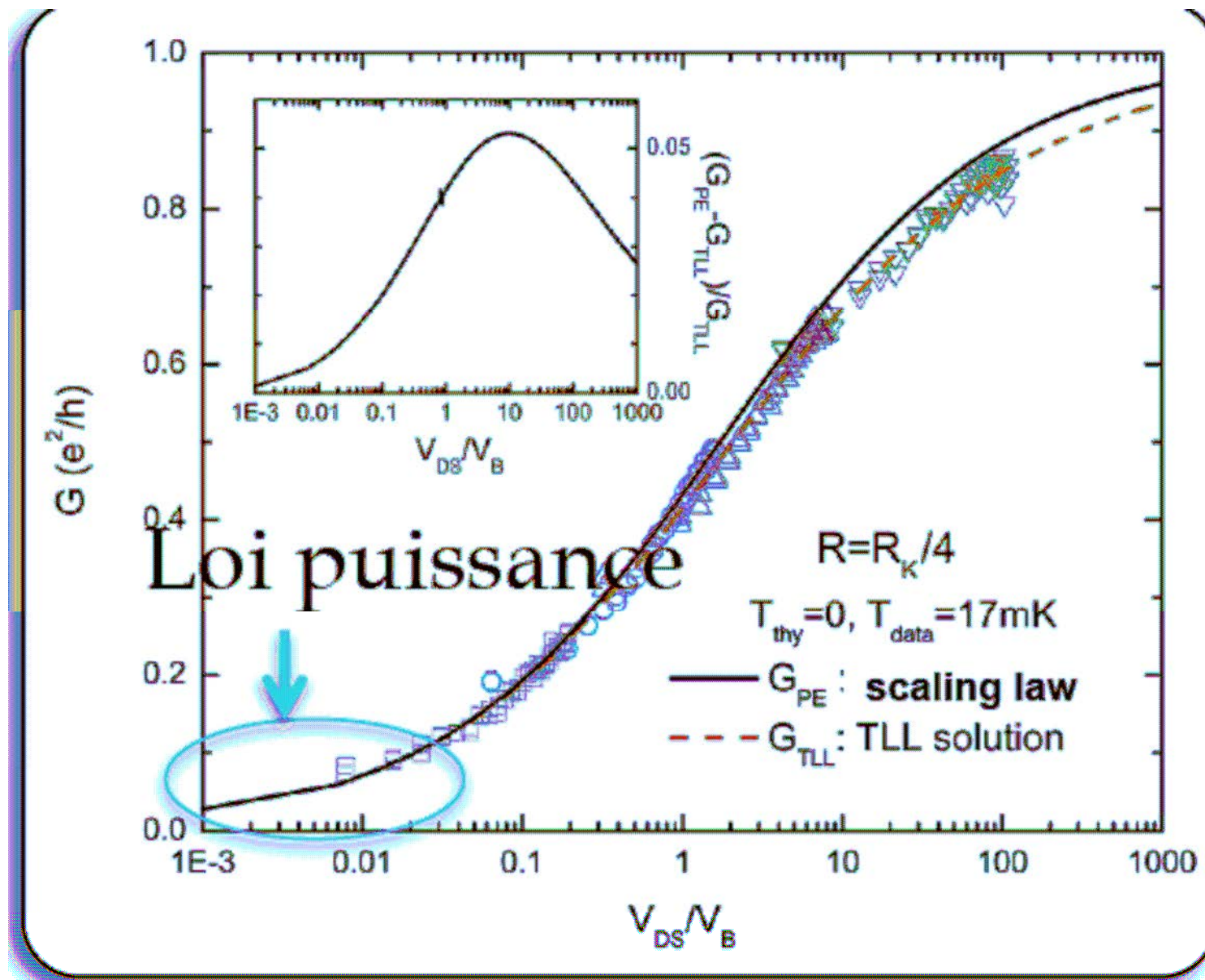
Environnement
contrôlable

De la jonction tunnel au fil quantique

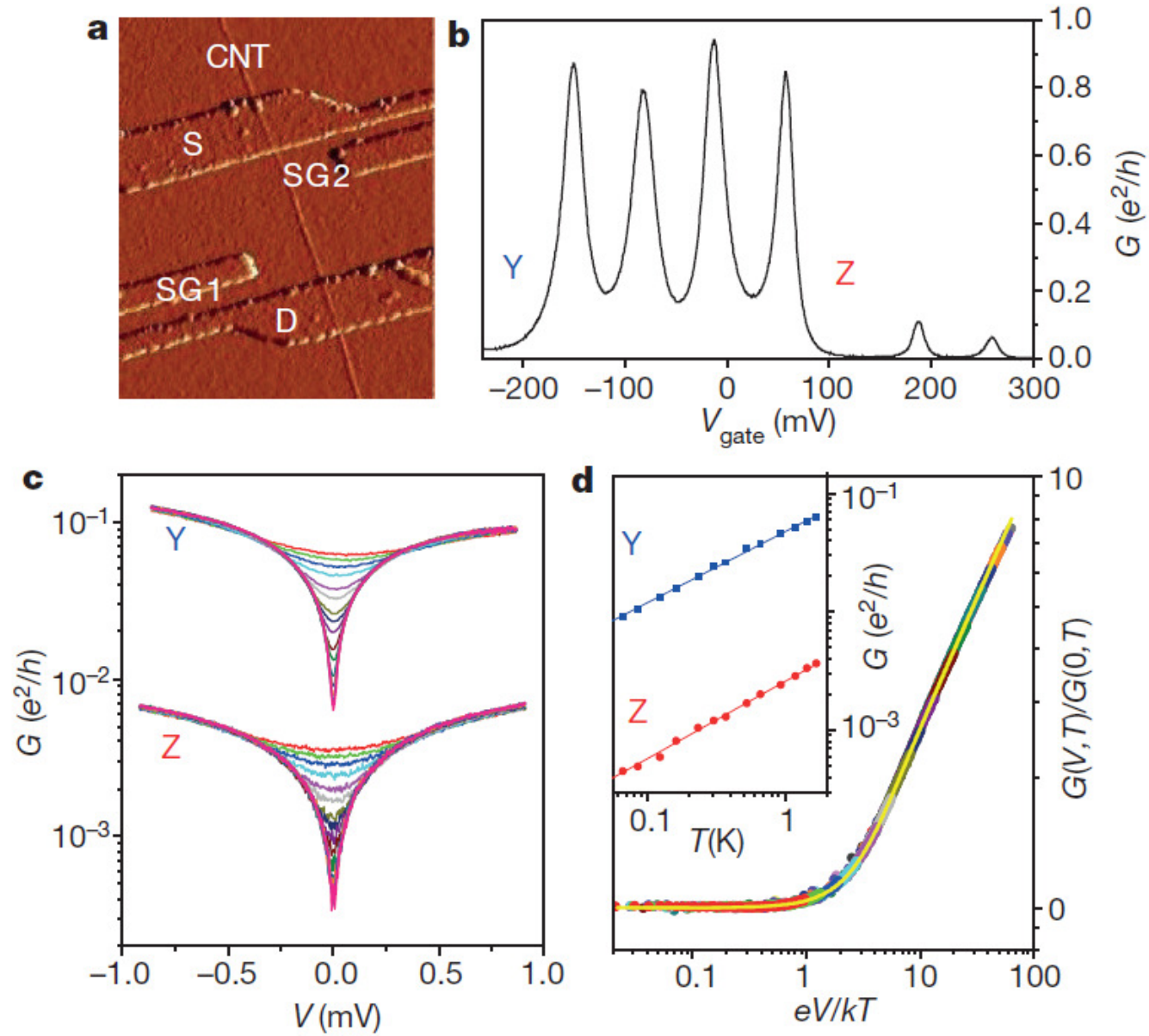
Environnement et jonction contrôlables

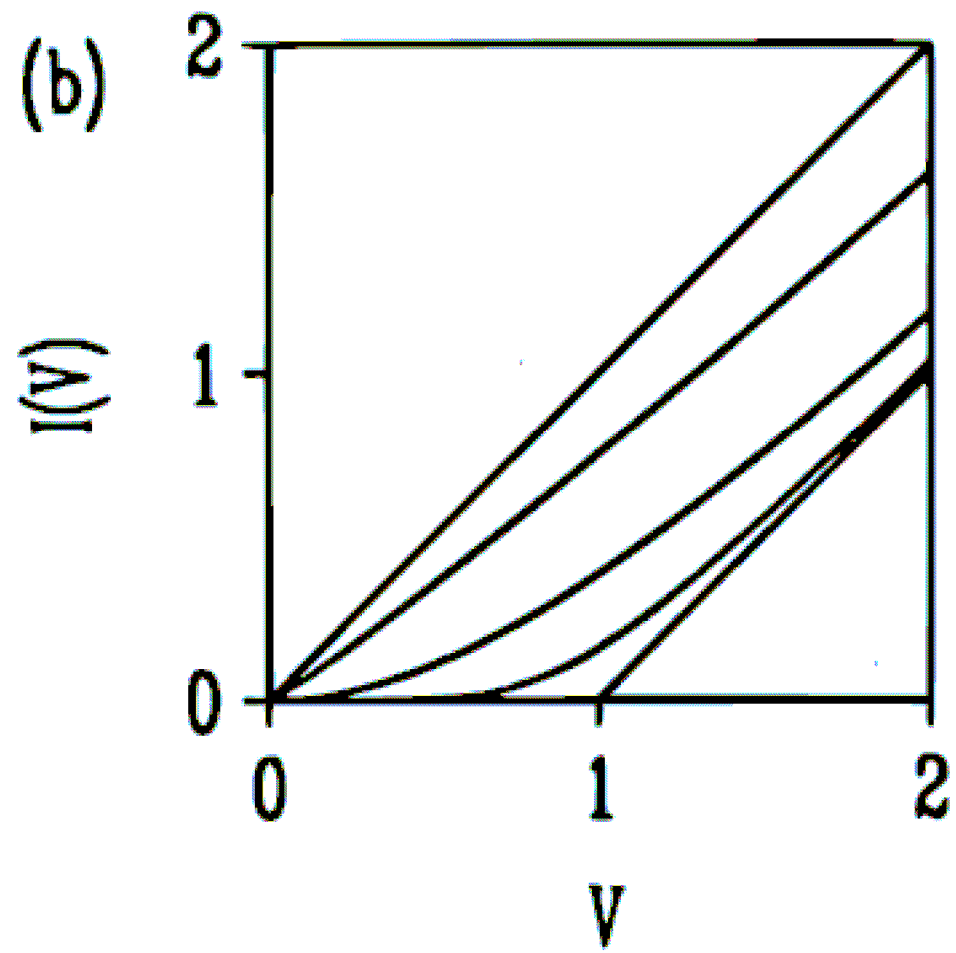


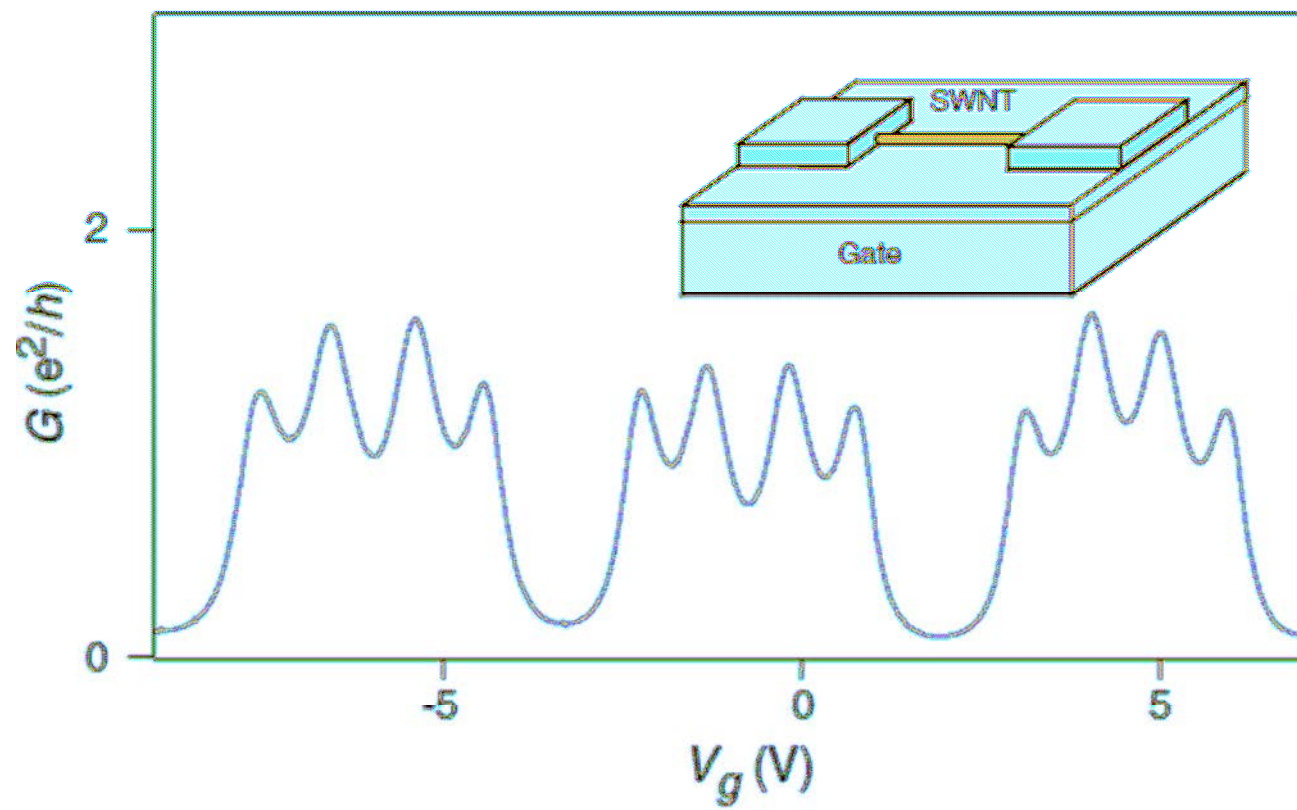
Correspondance entre les problèmes de blocage de Coulomb dynamique et liquide de Luttinger!



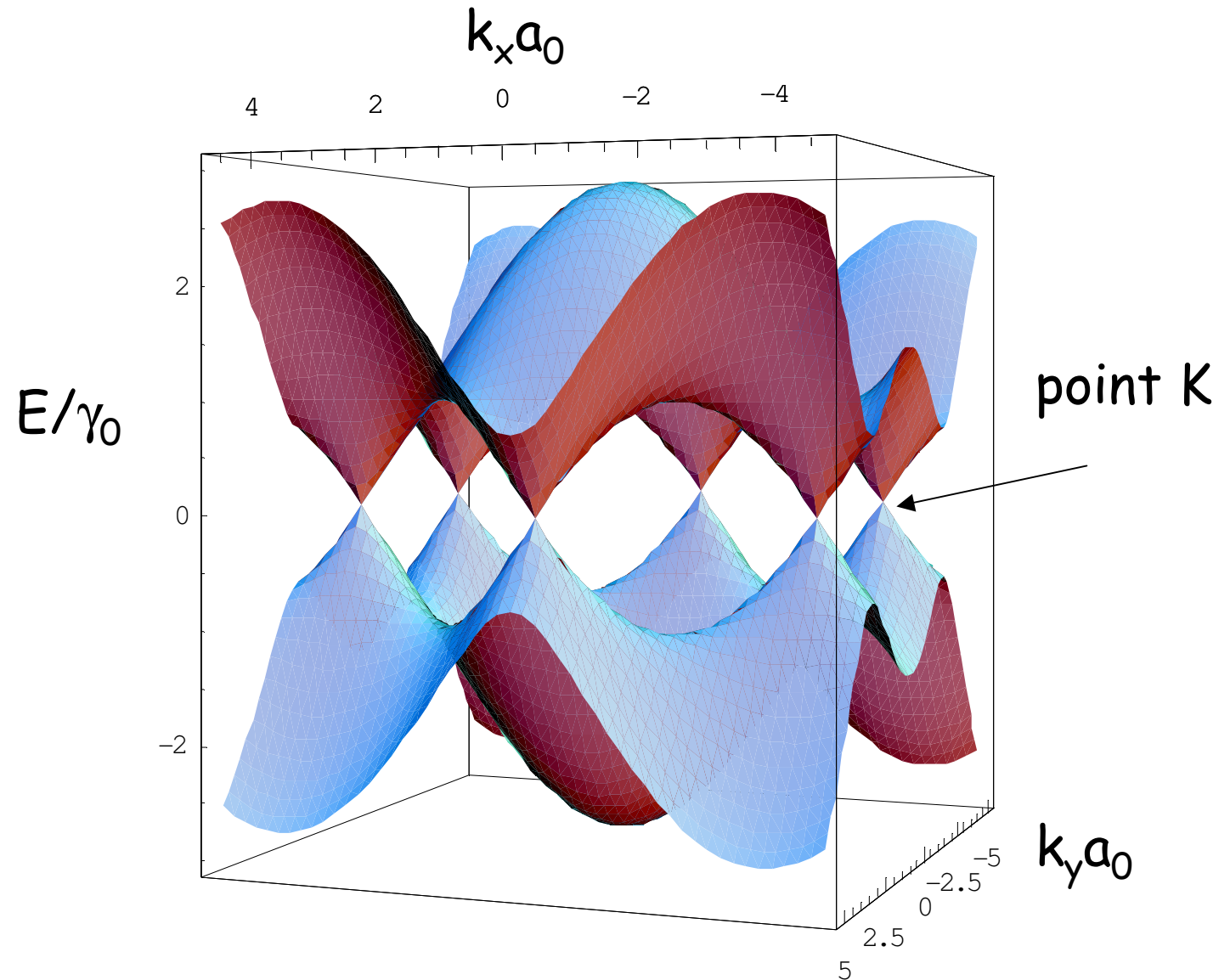
Mebrathu et al.
2012





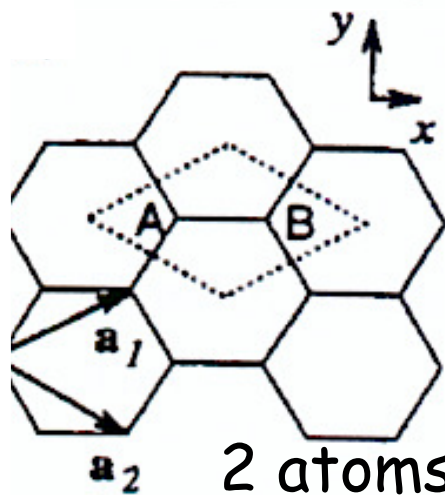
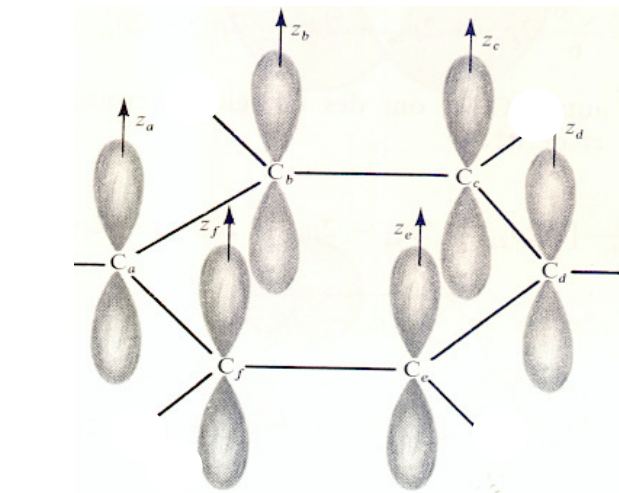


Structure de bande du graphène



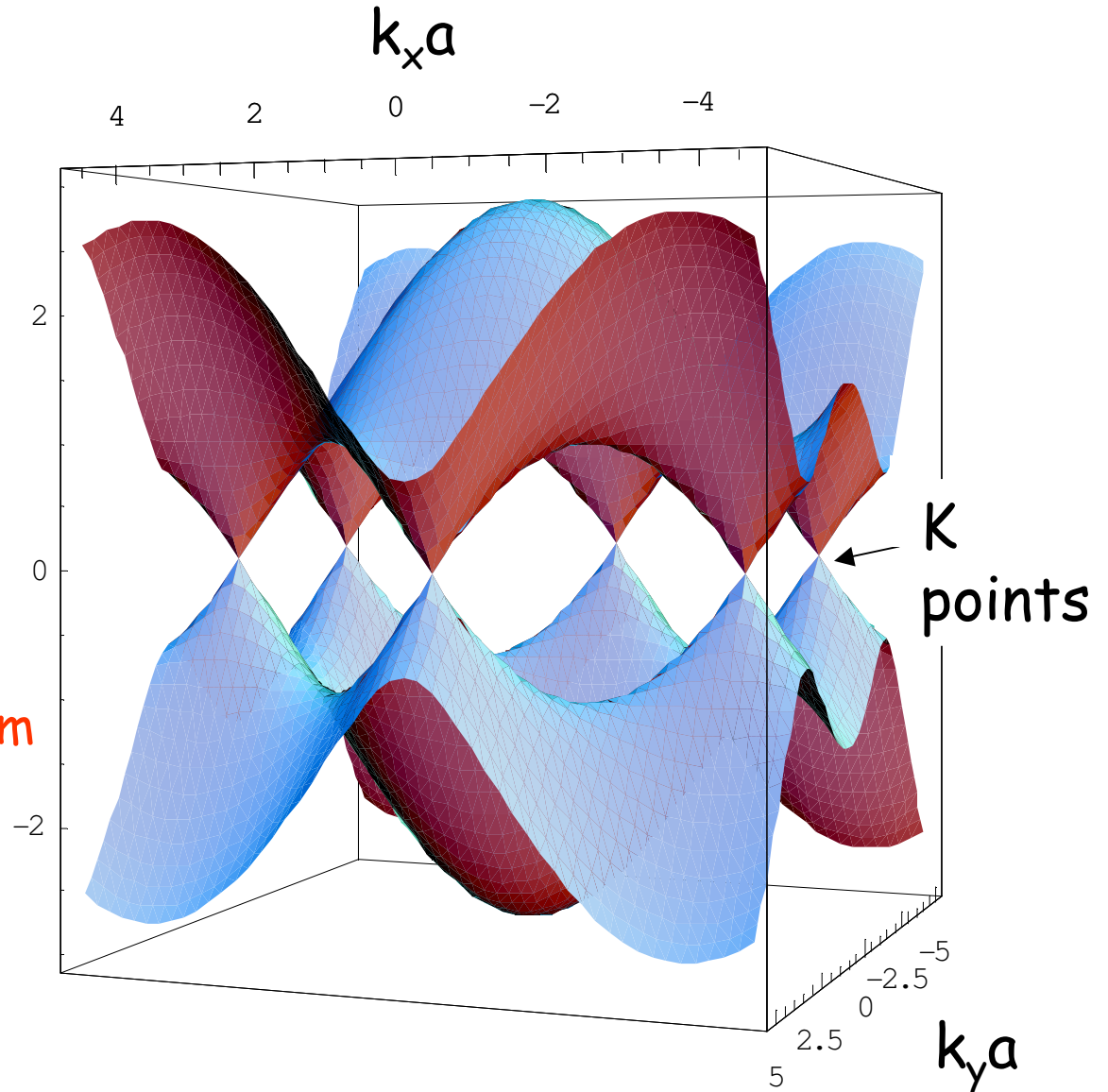
Liaison C-C
 $\gamma_0 = 2.5$ eV,
 $a_0 = 0.246$ nm

Band structure of graphene



E/γ
 C-C bond:
 $a = 0.246 \text{ nm}$
 $\gamma = 2.5 \text{ eV}$,

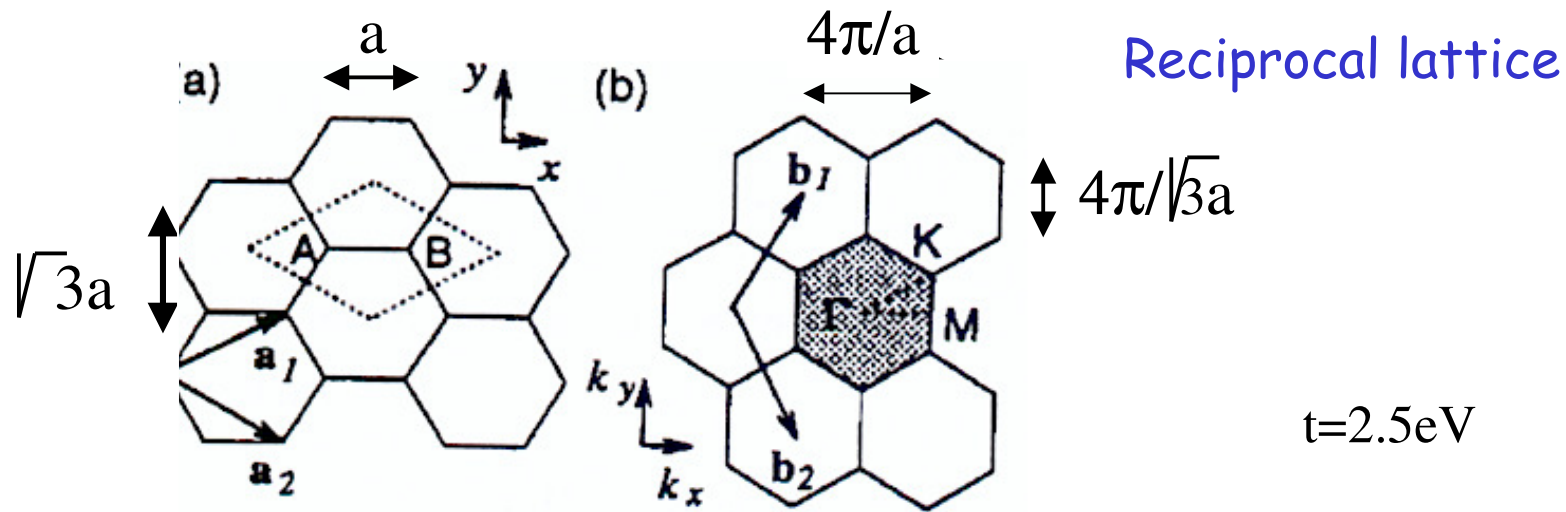
2 atoms /unit cell



$$E_{+,-}(k) = \pm \gamma (3 + 2\cos(k(a_1 - a_2)) + 4\cos(k(a_1 + a_2)/2) \cos(k(a_1 - a_2)/2))^{1/2}$$

Electronic Structure of a graphene sheet

Hexagonal lattice: 2 atoms per cell



$$H = \sum_{ij} t c_{(i,j),A}^\dagger c_{(i,j),B} + t c_{(i+1,j),A}^\dagger c_{(i,j),B} + t c_{(i,j),A}^\dagger c_{(i,j+1),B} + C.C$$

$$\Psi_{p,k} = \sum \exp(\mathbf{i}k \mathbf{R}_{i,j}) [\chi(\mathbf{r} - \mathbf{R}_A) + \phi_{pk} \chi(\mathbf{r} - \mathbf{R}_B)]$$

$$H_k = \begin{bmatrix} 0 & h_k \\ h_k^* & 0 \end{bmatrix}$$

$$h_k = \exp(\mathbf{i}k \mathbf{R}_{BA}) [1 + \exp(-\mathbf{i}k \mathbf{a}_1) + \exp(-\mathbf{i}k \mathbf{a}_2)]$$

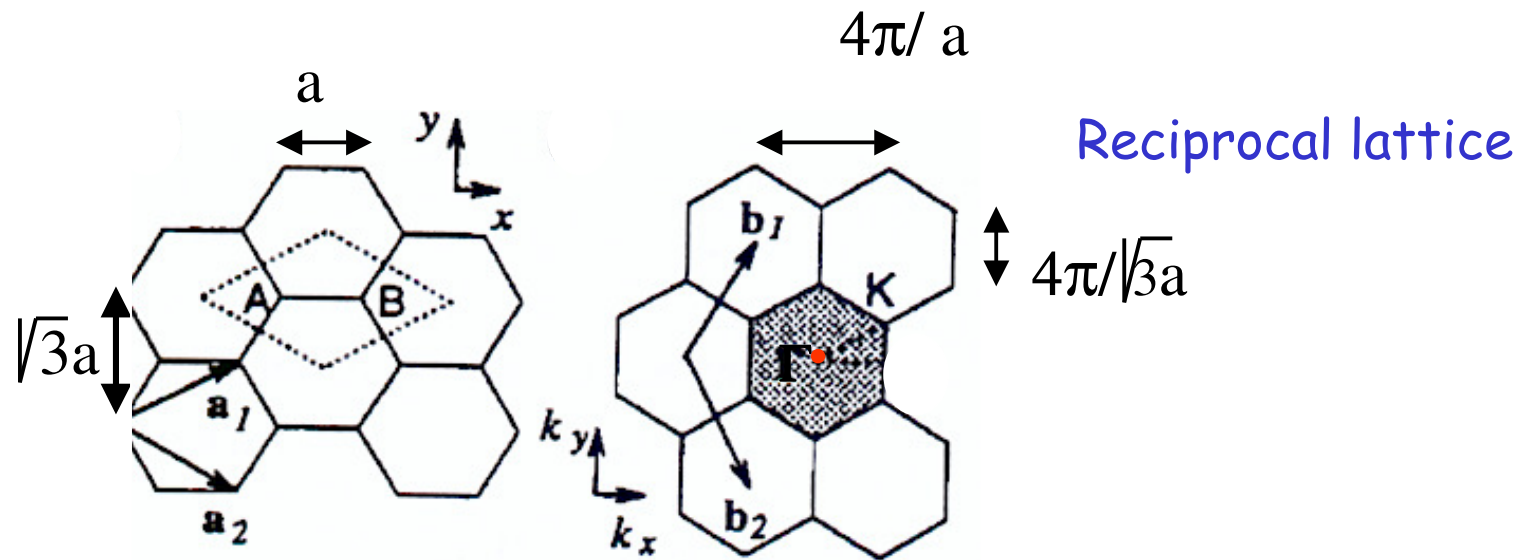
$$\phi_k = (-1)^p h_{pk} / |h_{pk}|$$

$$E_p(k) = (-1)^p t |h_{pk}|$$

Diagonalisation
of Bloch waves
within a cell

Ψ
p
:

Relation between helicity and metallic character



$$\vec{C} = n \vec{a}_1 + m \vec{a}_2$$

$$\vec{C} \times \vec{\Gamma K} = 2\pi (m-n) / 3$$

Ψ

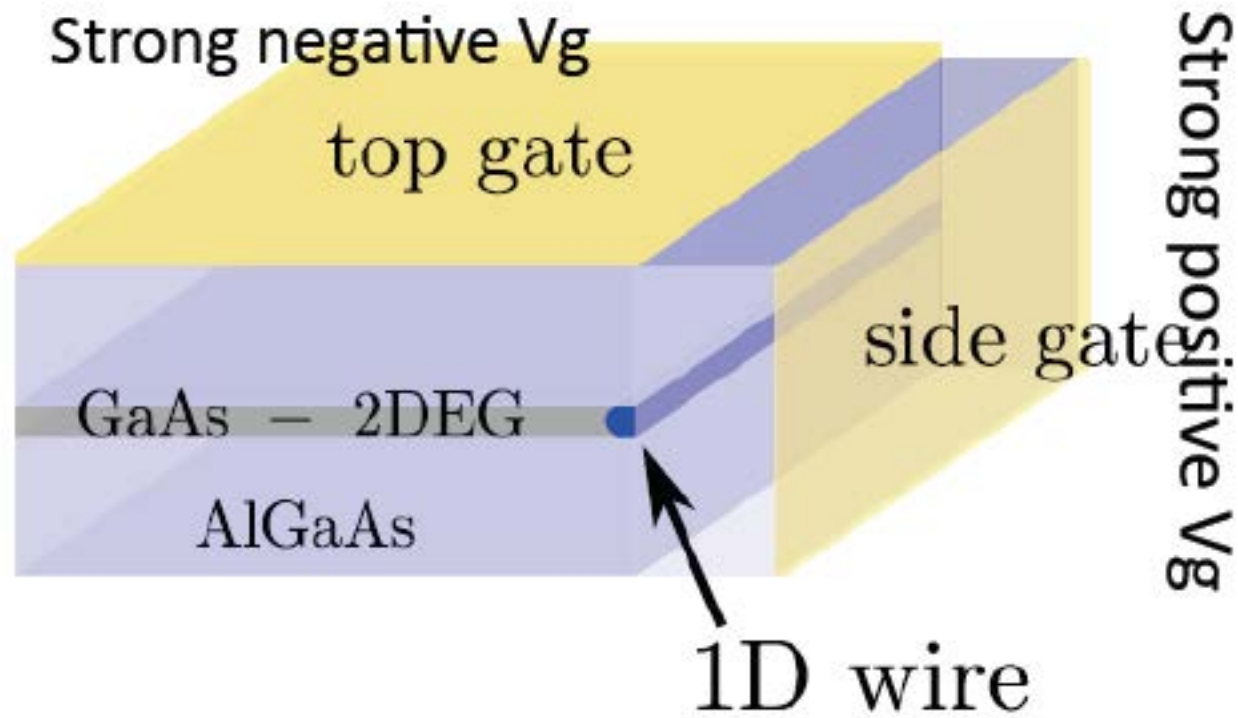
p

:

Metallic tubes are such that $m-n$ is multiple of 3

Cleaved edge GaAs/GaAlAs

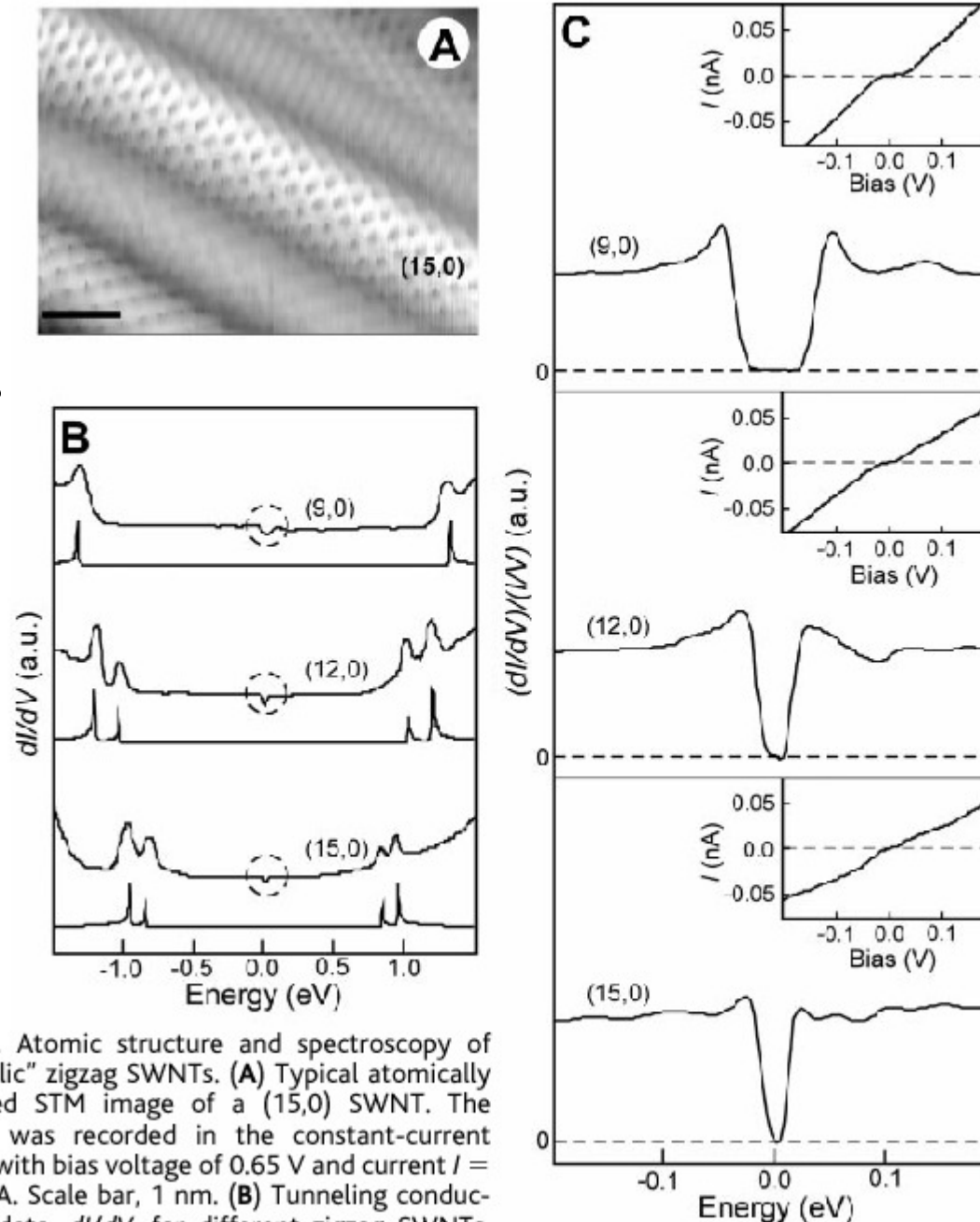
Yacoby, de Piciotto



Tubes Zig/Zag
 Never really metallic!

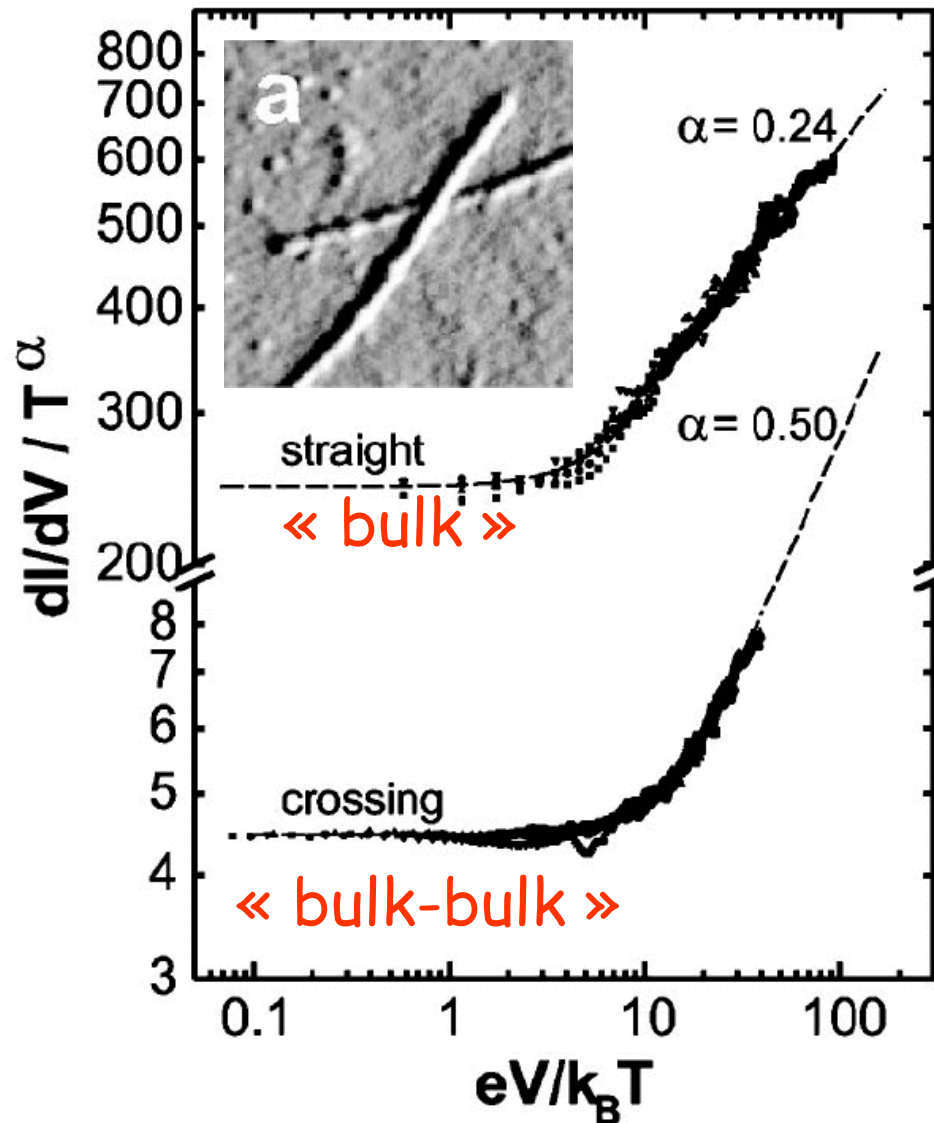
C. Lieber 2001

Small gaps due the
 Distortion of hexagons
 On a cylinder



. Atomic structure and spectroscopy of "zigzag" SWNTs. (A) Typical atomically resolved STM image of a (15,0) SWNT. The image was recorded in the constant-current mode with bias voltage of 0.65 V and current $I = 1$ nA. Scale bar, 1 nm. (B) Tunneling conductance data, dI/dV , for different zigzag SWNTs,

Nanotube crossing



Crossing obtained by AFM manipulation

$$\alpha_{\text{bulk-bulk}} = 2 \alpha_{\text{bulk}}$$

$$= (1/g + g - 2)/4$$

⇒ $g = 0.27$

Postma *et al.*, PRB (2000)