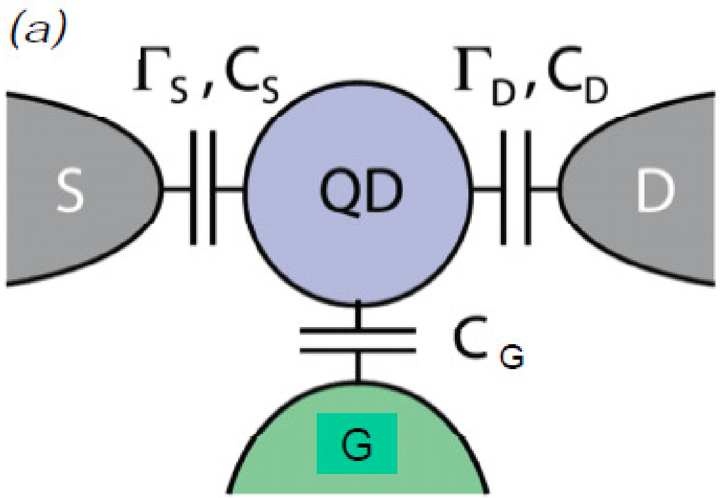
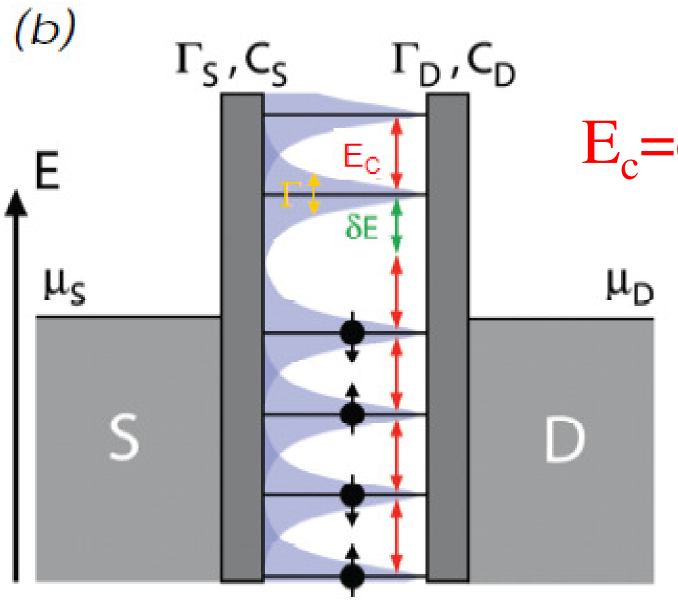
Boites Quantiques :
Du blocage de Coulomb à l'effet Kondo

Limite électrodes de très faible transmission:
Spectre d'addition et énergie de charge
Transmission moyenne: Effet Kondo
Expériences GaAs et nanotubes de carbone
Electrodes supraconductrices:
Compétition effet Kondo et effet Josephson

Boite Quantique: Energies caractéristiques

$$H = H_{leads} + H_{dot} + H_{tunneling}.$$





$E_c = e^2/C$

$$H_{dot} = \sum_{ns} \epsilon_n d^+_{ns} d_{ns} + E_C(\hat{N} - N_0)^2$$

δE= espacement entre niveaux électroniques de la boite

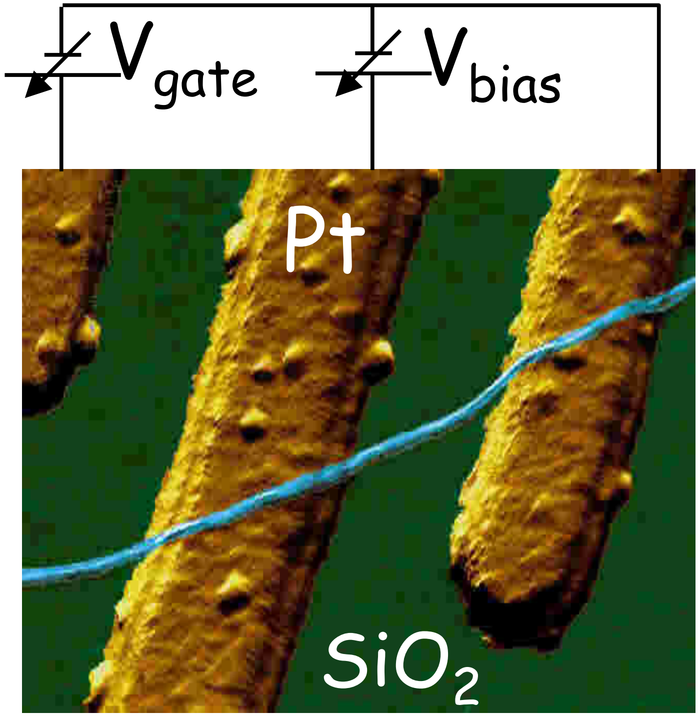
$$E_{add}(N) = \mu(N+1) - \mu(N) = E_C + \delta E$$

$$H_{leads} = \sum_{\alpha ks} \xi_k c^+_{\alpha ks} c_{\alpha ks}$$

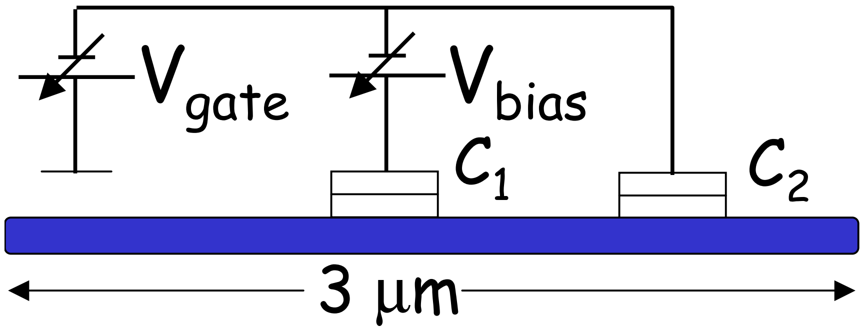
$$H_{tunneling} = \sum_{\alpha kns} t_{\alpha n} c^+_{\alpha ks} d_{ns} + H.c.$$

$$\Gamma_n = \sum_{\alpha} \Gamma_{\alpha n} = \sum_{\alpha} \pi\nu |t^2_{\alpha n}|$$

Nanotubes on Tunnel contacts $(R>>R_Q)$



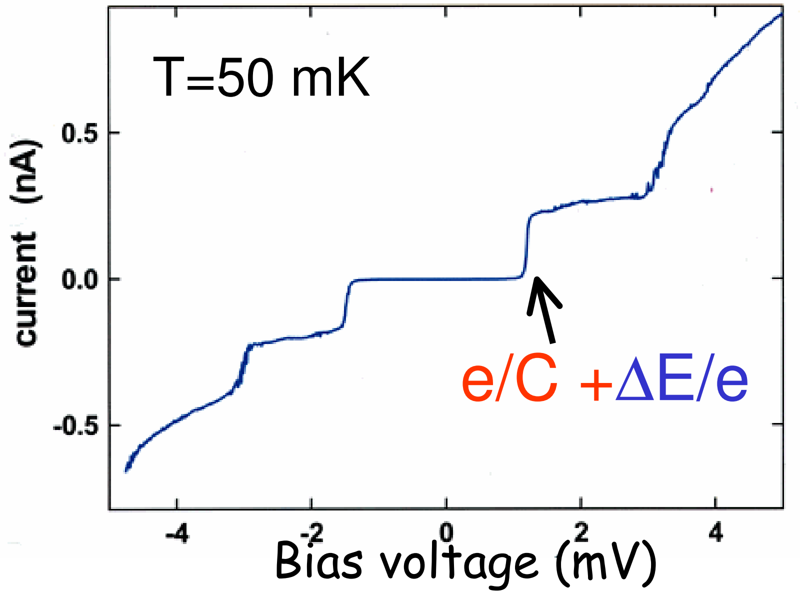
Tans *et al.*, Nature (1997)

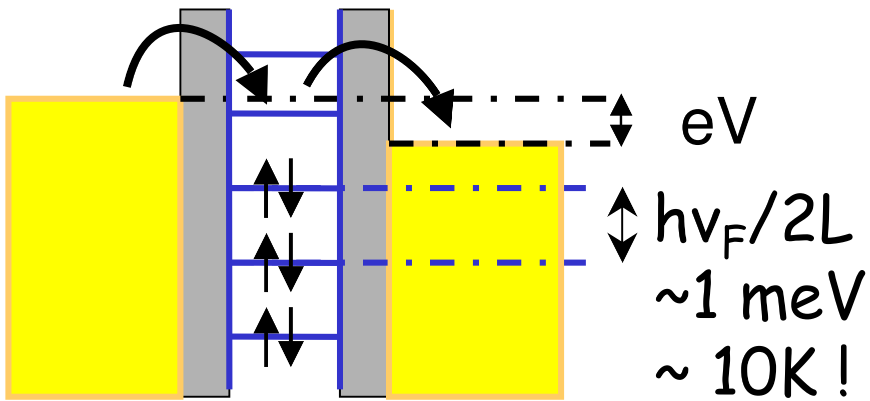


Charging energy $E_C = e^2/C$

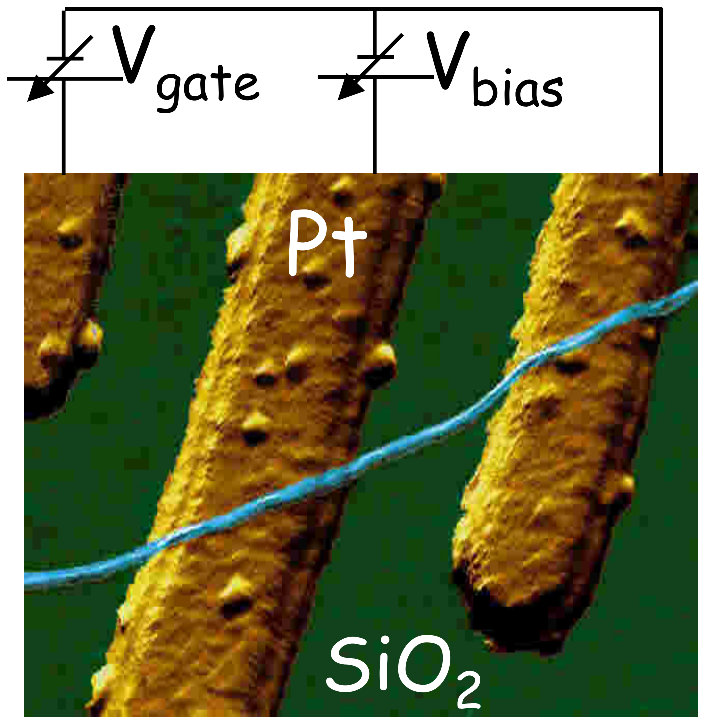
Spectroscopy of electronic levels

At low temperature $T << E_C$
Coulomb Blockade staircase

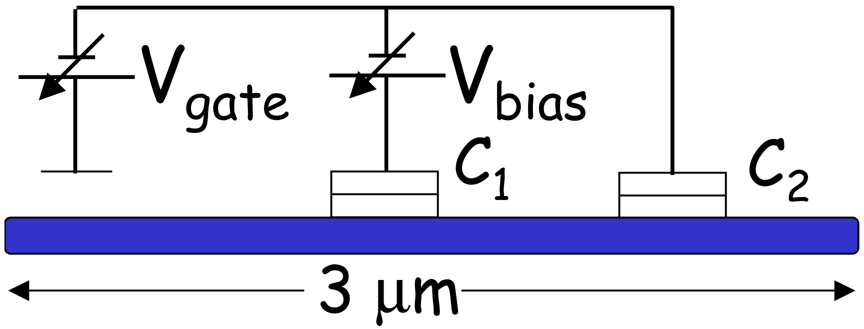




Nanotubes on Tunnel contacts ($R \gg R_Q$)



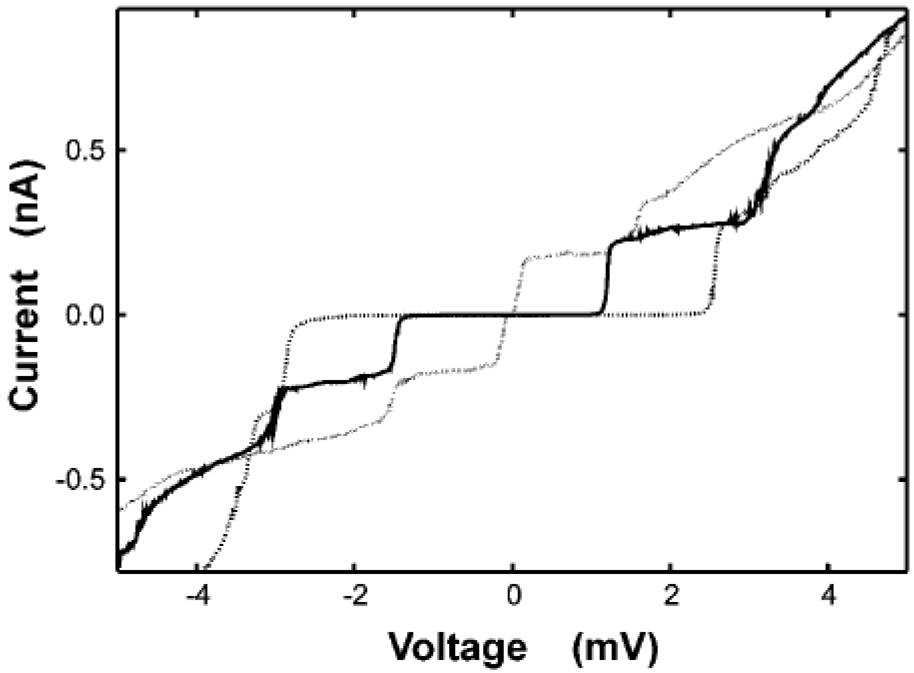
Tans *et al.*, Nature (1997)

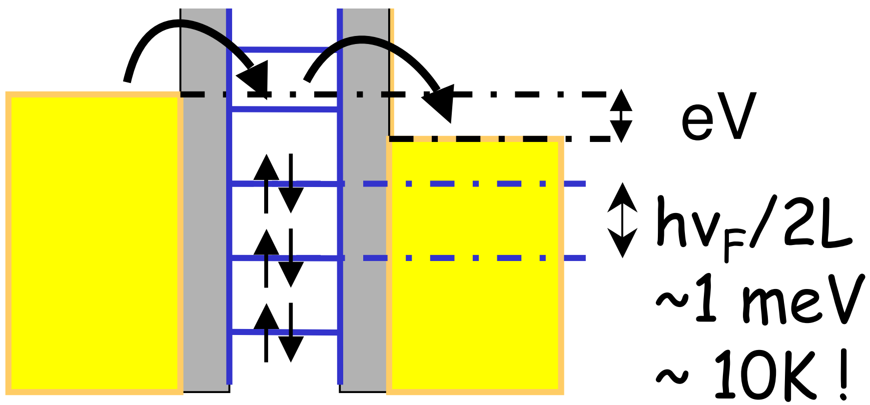


Charging energy $E_C = e^2/C$

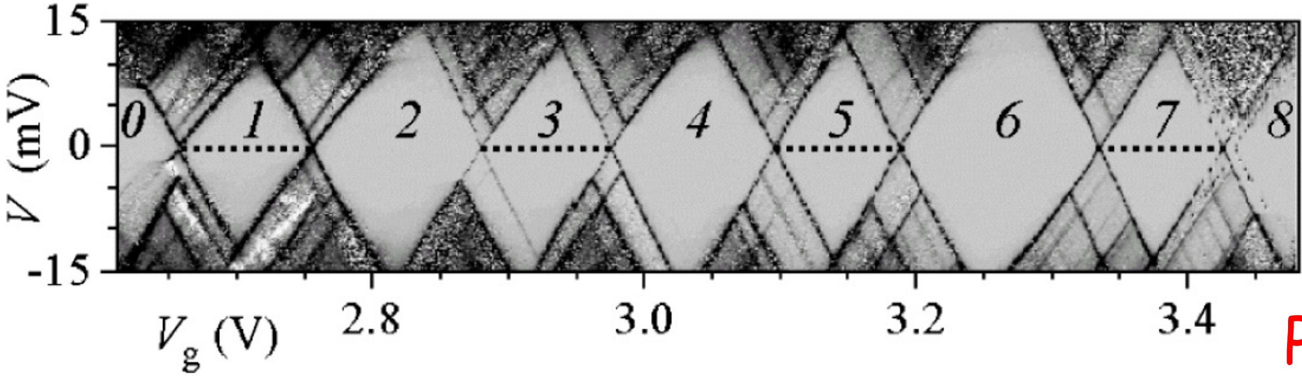
Spectroscopy of electronic levels

At low temperature $T \ll E_C$
Coulomb Blockade staircase

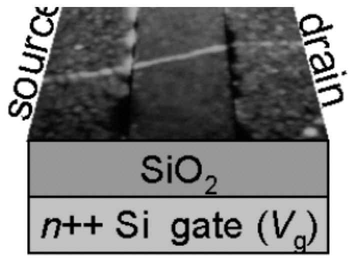




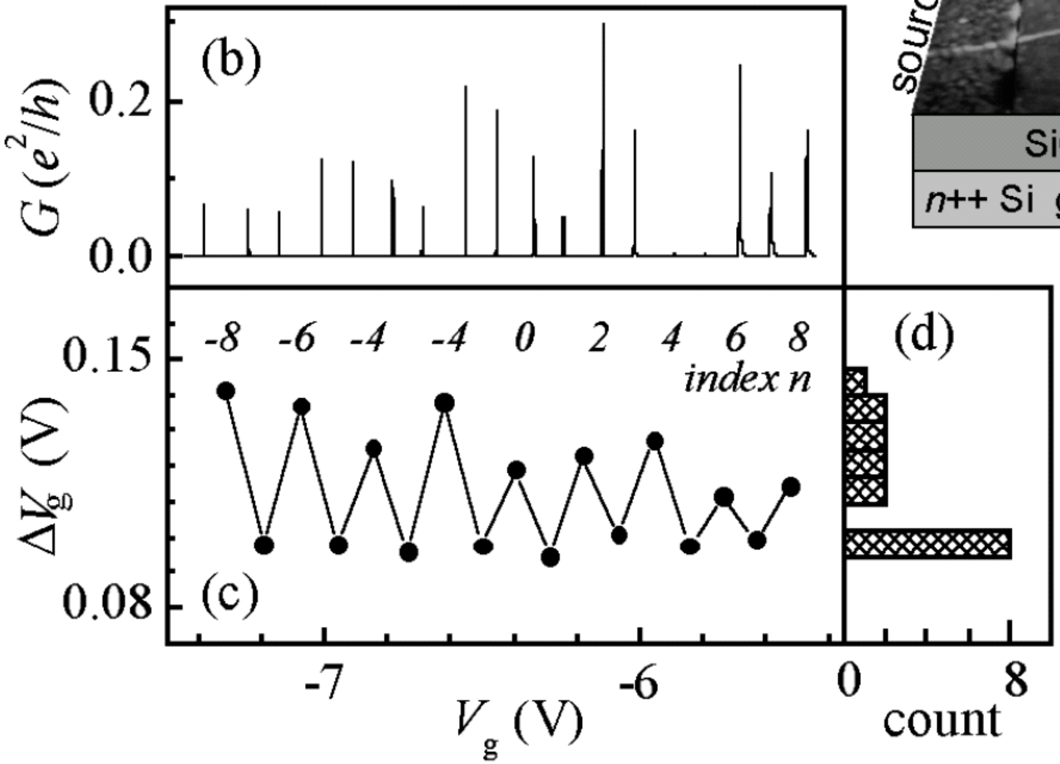
Blocage de Coulomb et transistor à 1 électron



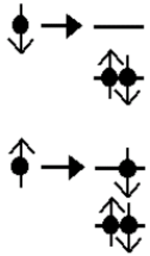
Cobden and Nygard, PRL (2002)



Pics de conductance fonction de Vg
Largeur ~ Transmission des contacts



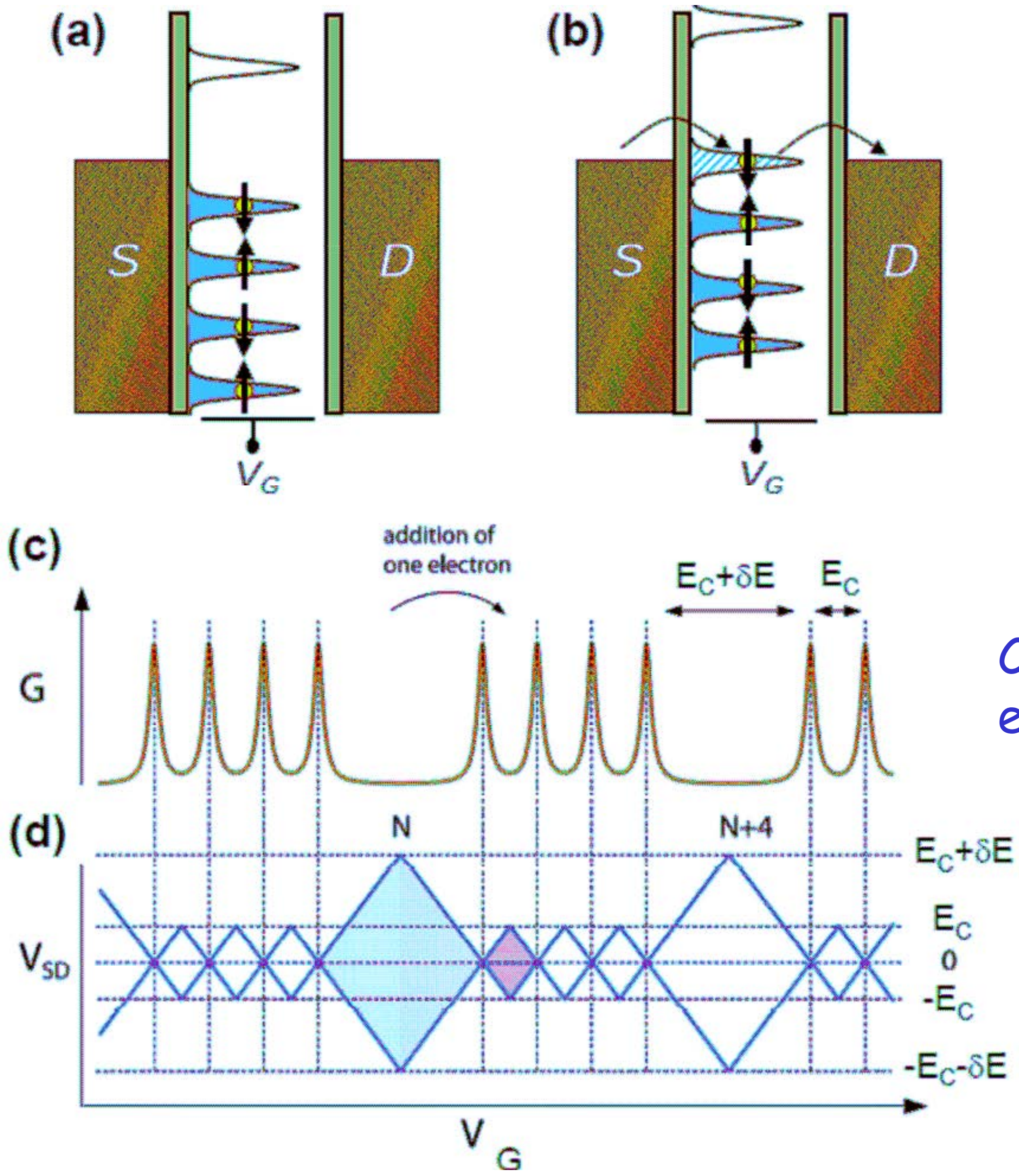
Dégénérescence de spin



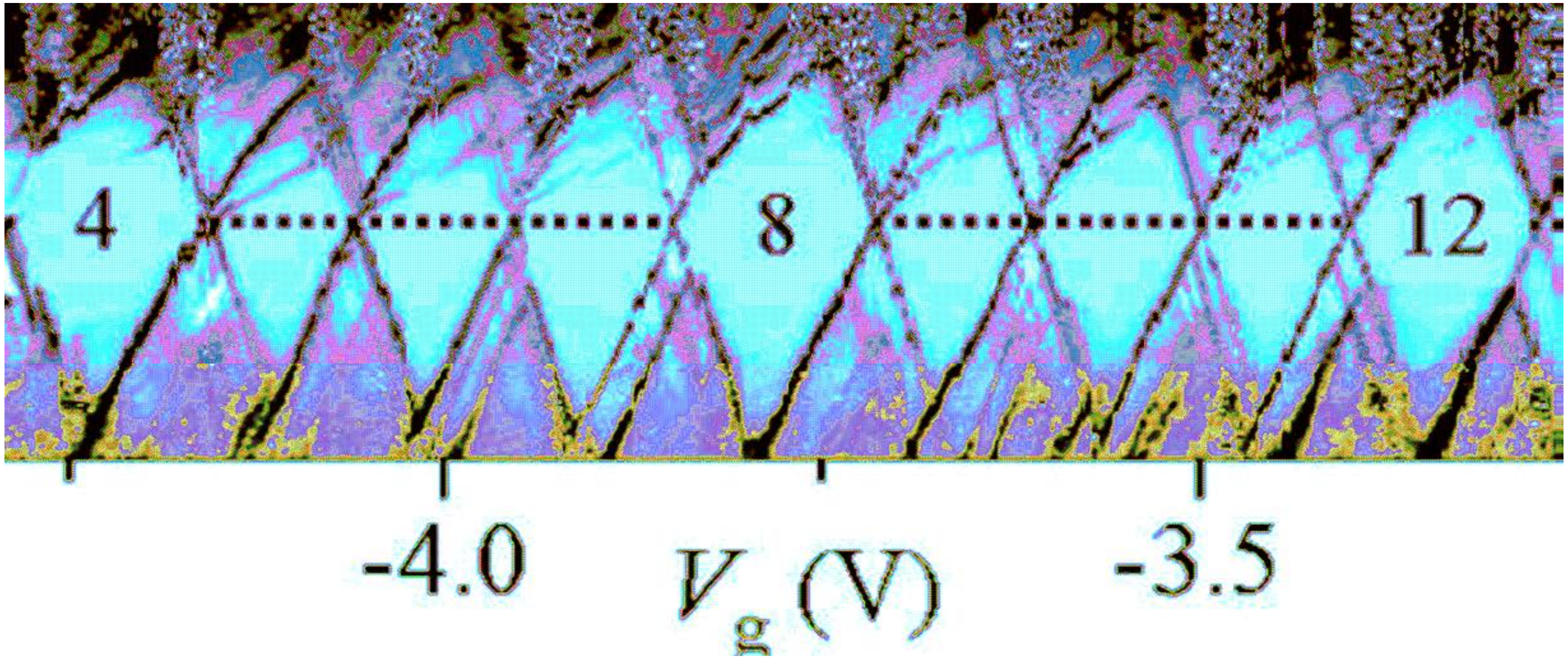
N impair $\Delta v_g \propto e/C$

N pair $\Delta v_g \propto e/C + \Delta E/e$

Dégénérescence orbitale

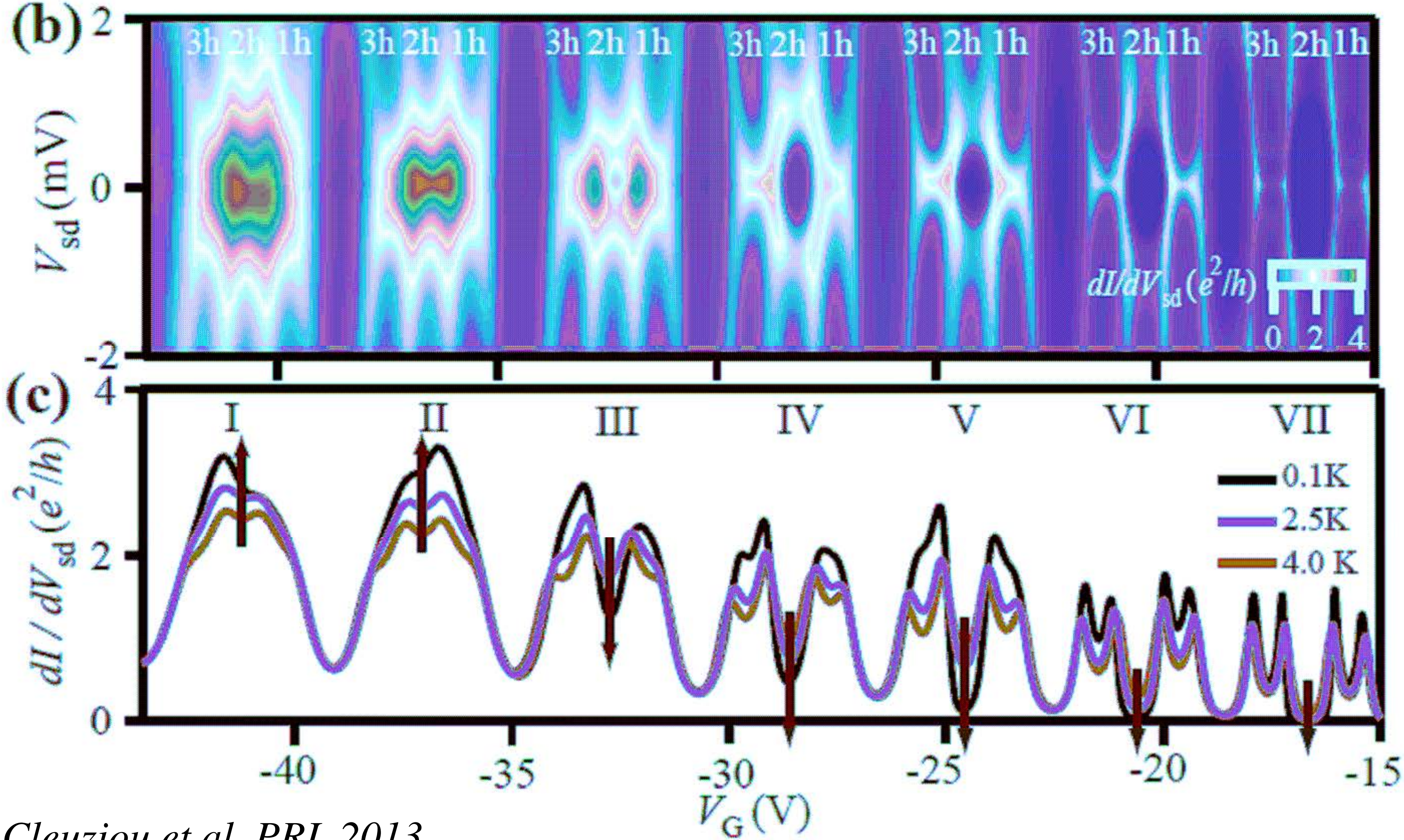


Chaque niveau électronique est 4 fois dégénéré



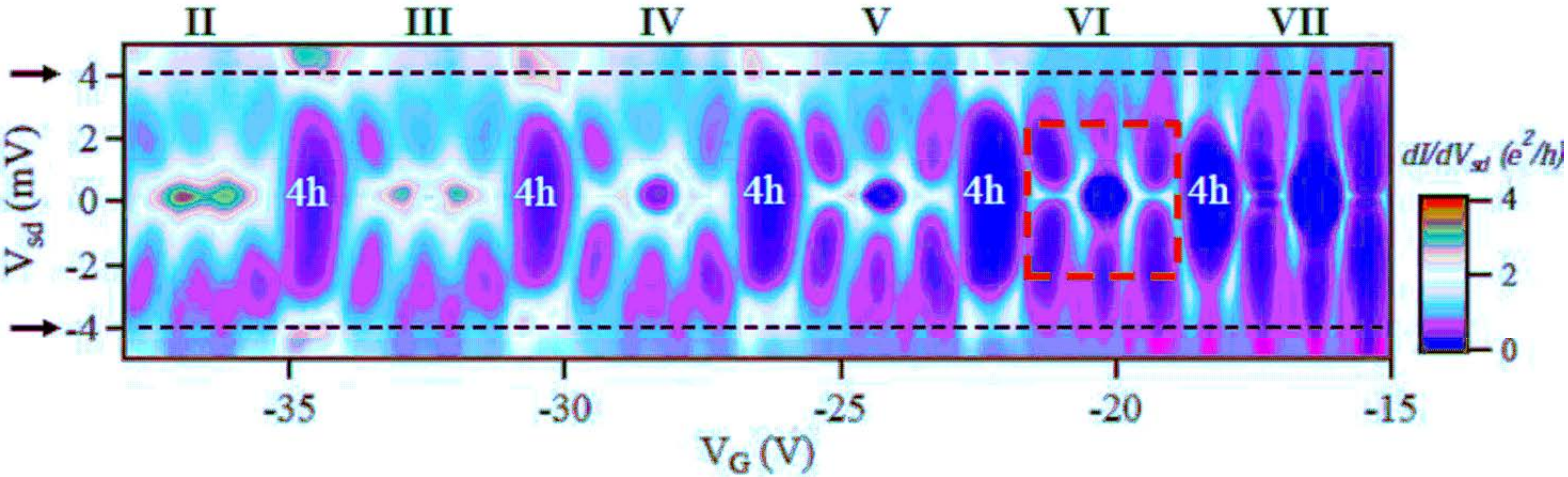
4
8
12
-4.0
V_g (V)
-3.5

Evolution du blocage de Coulomb pour de électrodes plus passantes



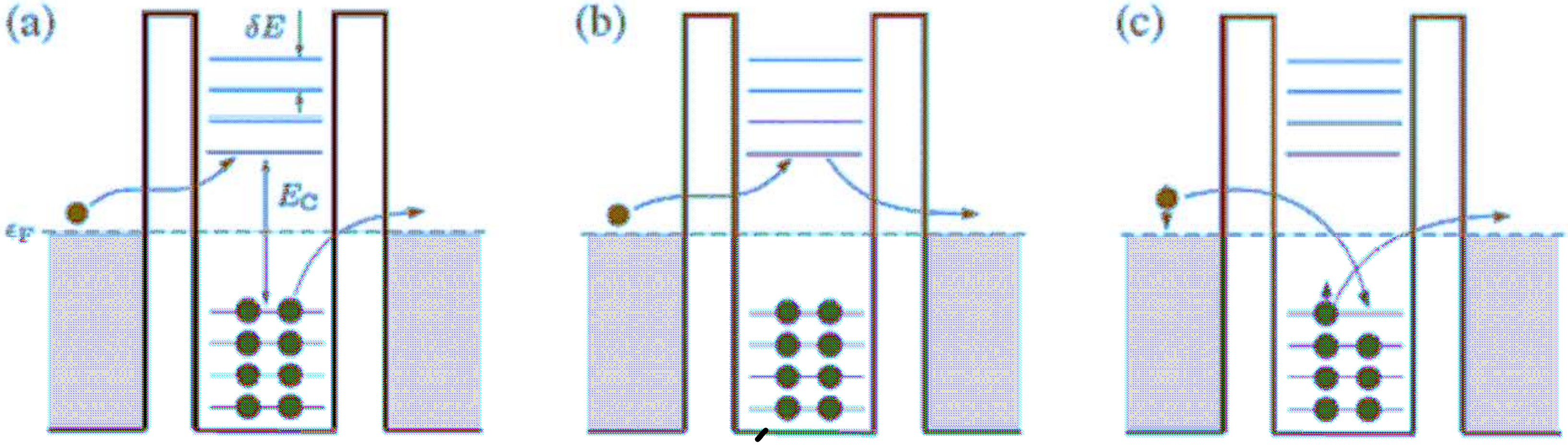
Cleuziou et al. PRL 2013

Augmentation de la conductance à basse température pour un nb impair d'électrons



Cleuziou et al. PRl 2013

Au delà de l'ordre 1: cotunneling



Inélastique

Élastique

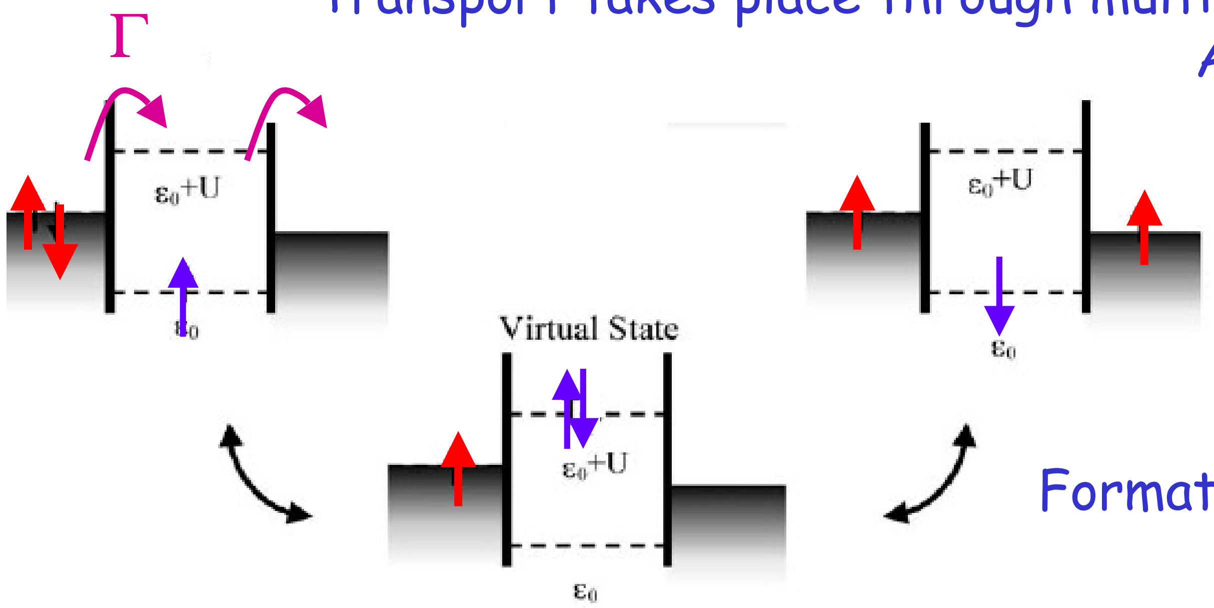
Elastique avec retournement de spins

Pustilnik, Glazman J.Phys C 2004

Contacts with intermediate transmission
Quantum dot with odd number of electrons

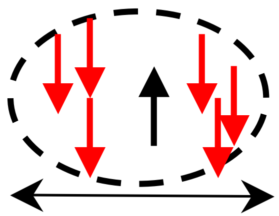
Transport takes place through multiple order spin flip events

Anderson Impurity model



$$H_{eff} = J_{eff}\ \vec{\sigma} \cdot \vec{S}$$

Formation of a many body singlet :



$L_K = h\ v_F / T_K$

$J_{eff} = \Gamma / \nu\ U$

ν: DOS $\quad \Gamma = |t|^2 \nu$ width of energy level ε_0

$$T_K = (U\ \Gamma)^{1/2} \exp\left(-1/\ J_{eff}\ \nu\right)$$

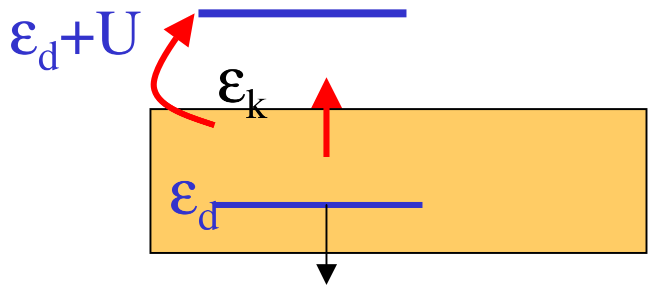
Importance of Coulomb repulsion U

Hamiltoniens Anderson et Kondo

Effet Kondo: Emilie Dupont

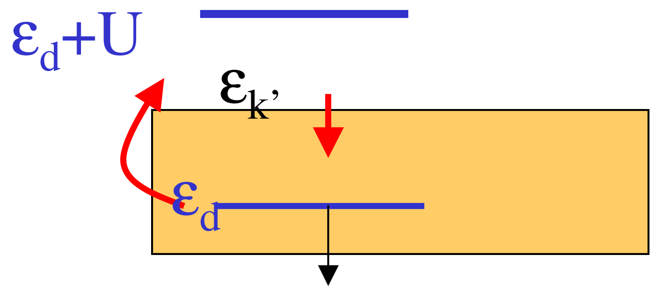
$$H_A = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} a^\dagger_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + \varepsilon_d \sum_{\sigma} a^\dagger_{d\sigma} a_{d\sigma} + \sum_{\mathbf{k},\sigma} \left(V_{\mathbf{k}d} a^\dagger_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + \mathrm{h.c} \right) + U n_{d\uparrow} n_{d\downarrow}$$

$$H = \sum_{\mathbf{k},\mathbf{k}'} J_{\mathbf{k}\mathbf{k}'} \mathbf{S} \sum_{\sigma,\sigma'} a^\dagger_{\mathbf{k}\sigma} \boldsymbol{\sigma}_{\sigma\sigma'} a_{\mathbf{k}'\sigma'}$$



$$T_{(\mathbf{k}\sigma)+(d-\sigma)\rightarrow(\mathbf{k}'-\sigma)+(d\sigma)} = -\frac{1}{2} J_{\mathbf{k}\mathbf{k}'}$$

$$J_{\mathbf{k}\mathbf{k}'} = 2 V_{\mathbf{k}d} V_{d\mathbf{k}'} \left(\frac{1}{\varepsilon_{\mathbf{k}} - \varepsilon_d - U} + \frac{1}{\varepsilon_d - \varepsilon_{\mathbf{k}'}} \right)$$



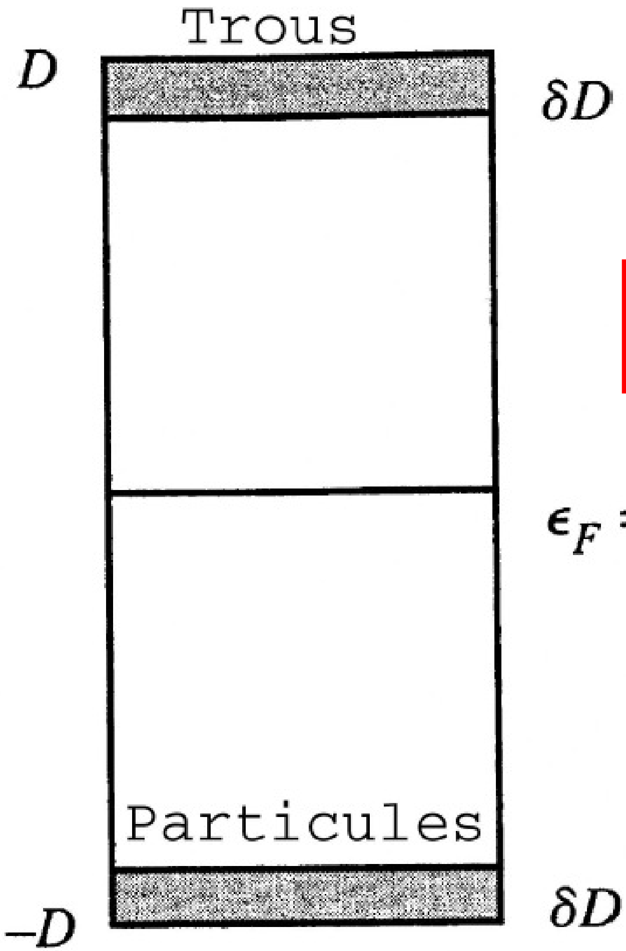
$$J_{eff} = -4 \frac{|V_{\mathbf{k}d}|^2}{U} \propto 1/U$$

Divergence logarithmique du taux de collision à basse température

Formation d'une résonance au niveau de Fermi

$$\Gamma = \Gamma^0 \left(1 - 2 J_0 N(0) \ln \frac{T_F}{T} + \ldots \right)$$

Renormalisation du couplage: Température de Kondo



Calcul à l'ordre 2

$$\frac{dJ}{d\ln D} = \nu \; J^2$$

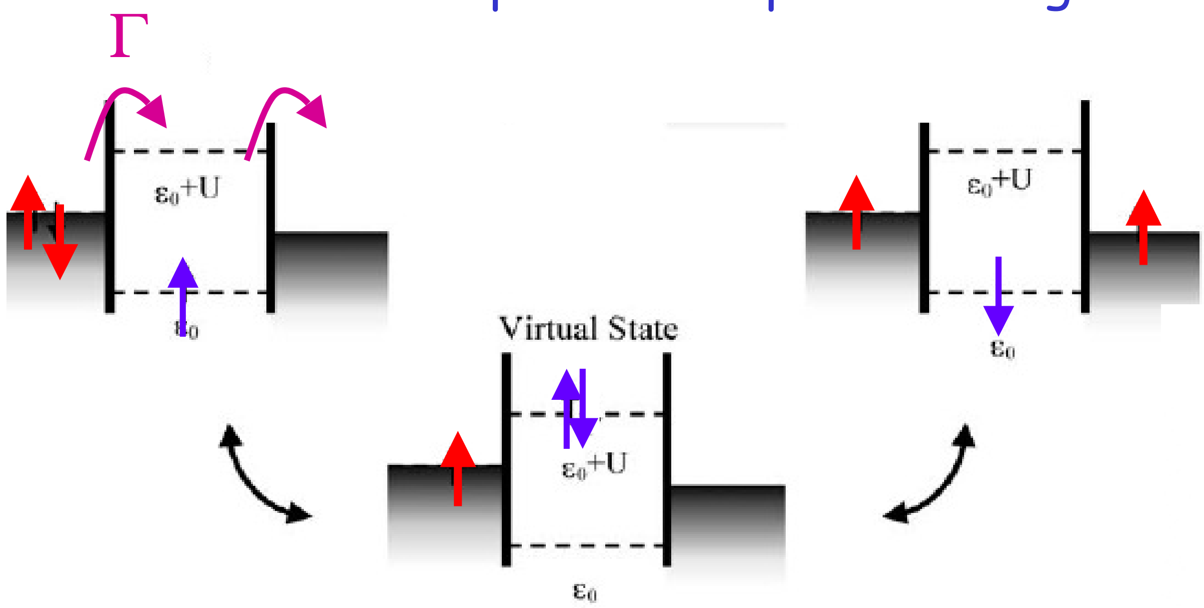
$$D\, e^{-1/(2J\nu)} = \tilde{D}\, e^{-1/(2\tilde{J}\nu)} = \mathrm{Cte} = k_B T_K$$

Scaling en T/T_K
et en eV_{DS}/T_K

Boite quantique D ~ U
Termes d'ordre supérieur
$D \sim U\,(\Gamma/U)^{1/2} = (\Gamma\, U)^{1/2}$
technique adéquate: NRG

Contacts with intermediate transmission
Quantum dot with odd number of electrons

Transport takes place through multiple order spin flip events



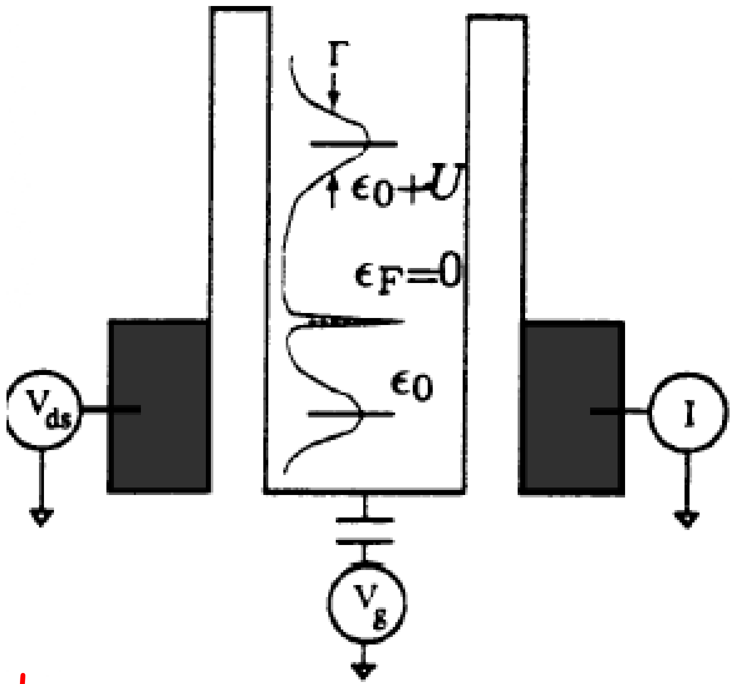
$H_{eff} = J_{eff}\, \vec{\sigma} \bullet \vec{S}$

$J_{eff} = \Gamma / \nu\, U$

ν: DOS $\Gamma = |t|^2 \nu$ width of energy level ε_0

$T_K = (U\,\Gamma)^{1/2} \exp(-1/ J_{eff}\, \nu)$

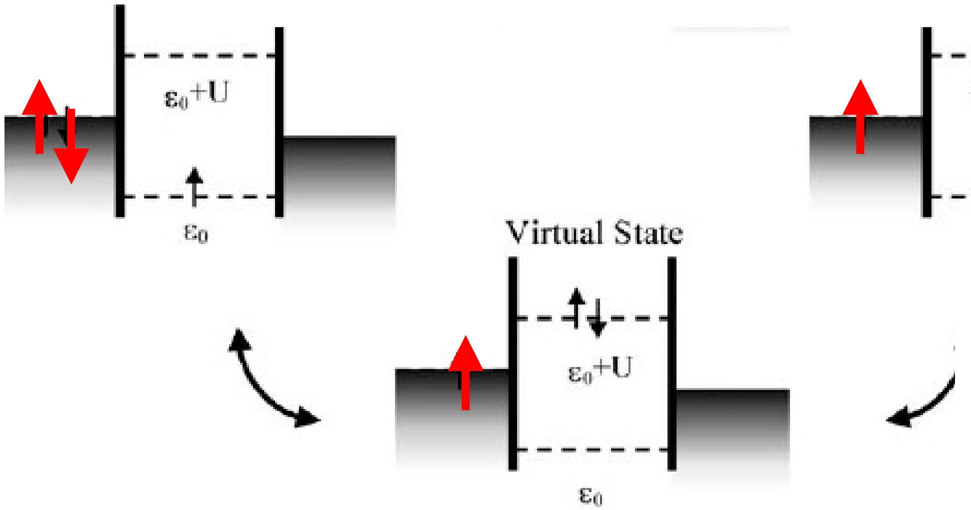
Importance of Coulomb repulsion U

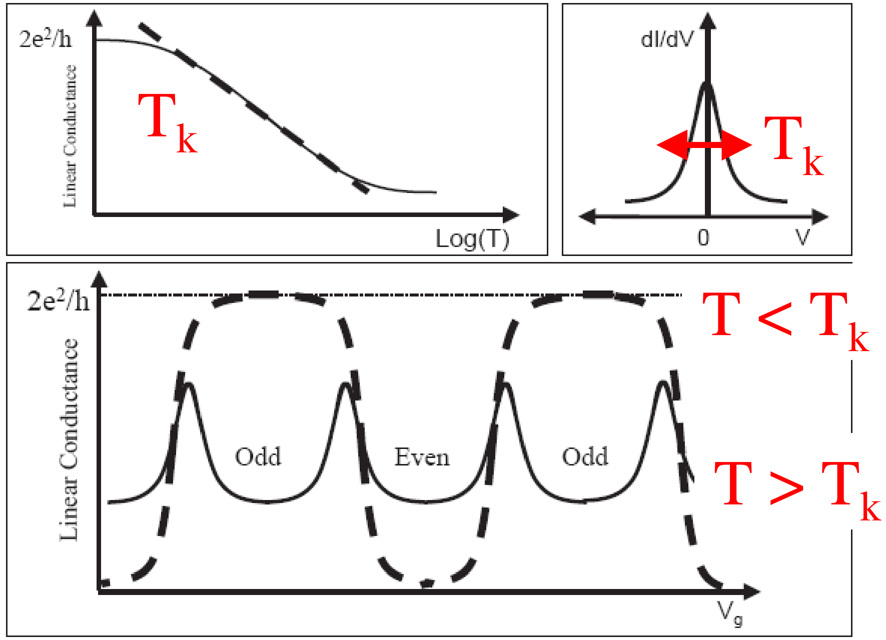


Kondo resonance

Quantum dot with odd number of electrons
Contacts with intermediate transmission t~1

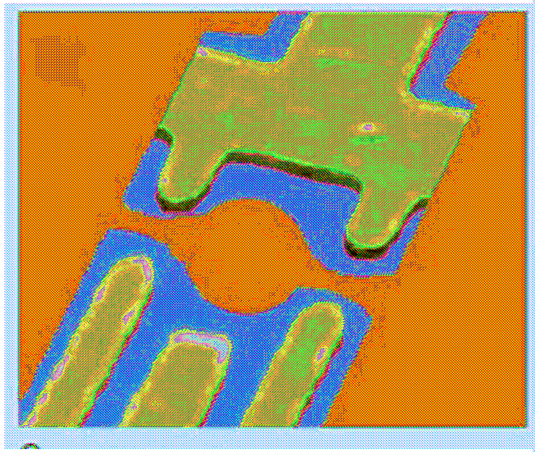
Transport takes place through multiple order spin flip events
Formation of a many body singlet : Kondo resonance

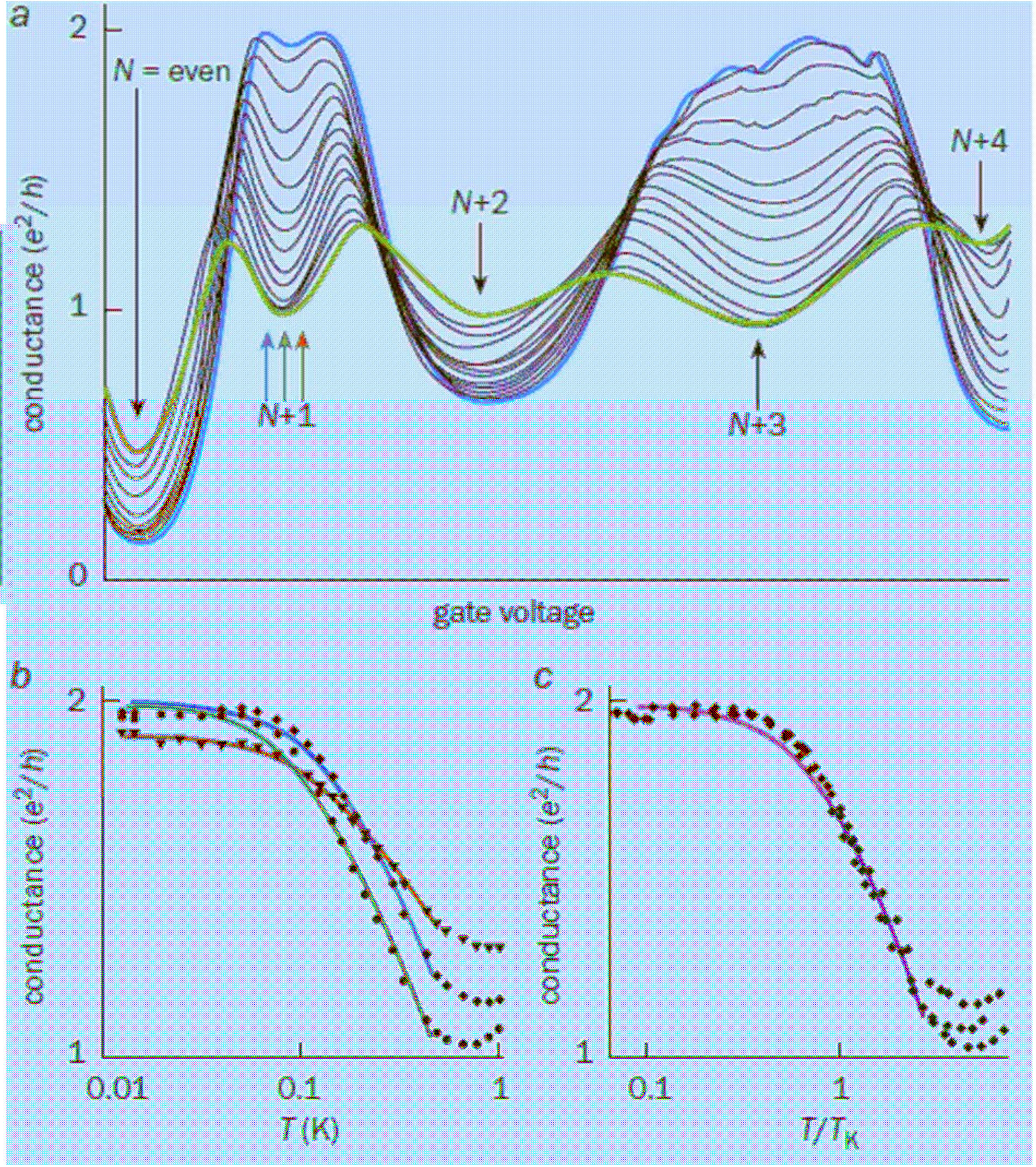




Increase of conductance at low temperature up to $2e^2/h$...

Boite quantique GaAs/GaAlAs

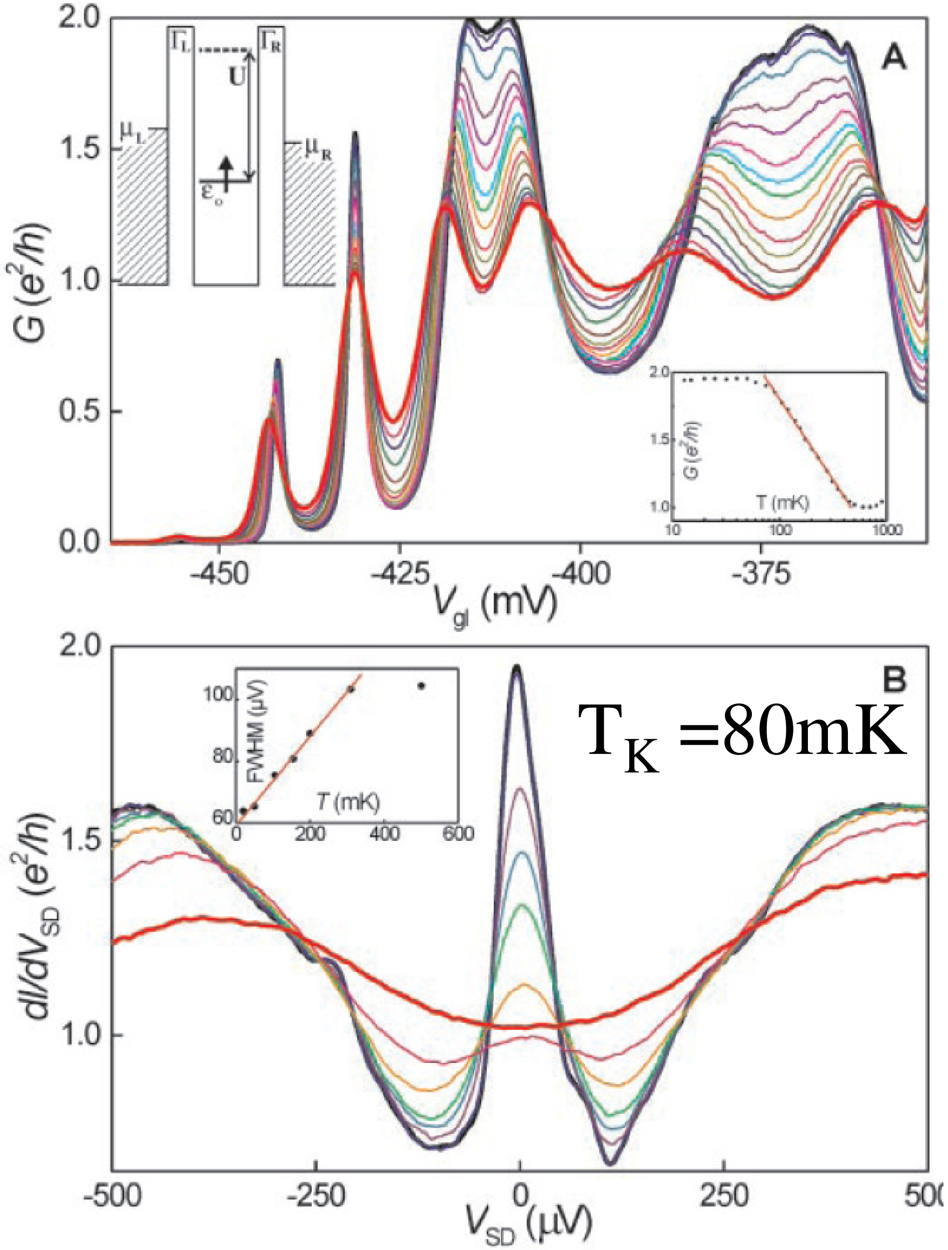




Kouwenhoven 2001

Kondo effect in GaAs/GaAlAs Quantum dots

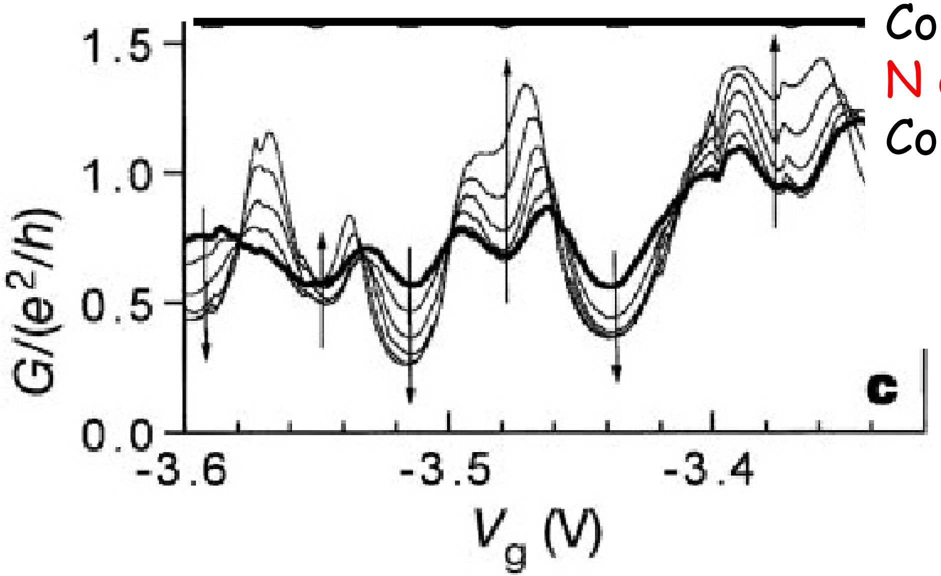
Goldhaber Gordon et al.
Cronenwett et al. 1998



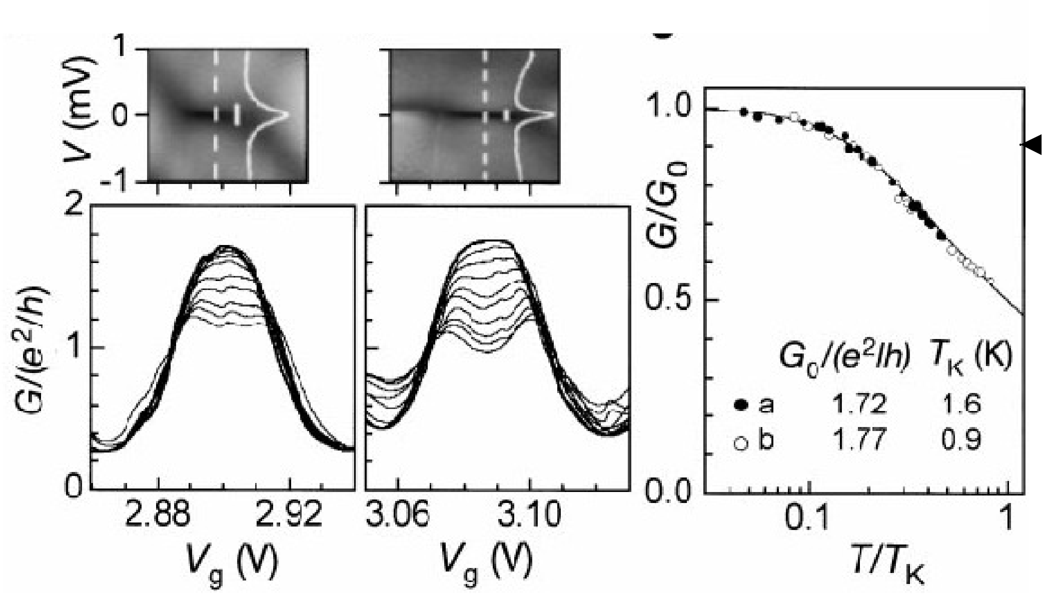
Van der Wiel et al. 2000

Kondo Physics in carbon nanotubes

Cobden *et al.*,
Nature (2001)



N even:
Conductance decreases with temperature
N odd:
Conductance increases with temperature



T_k in the Kelvin range !

Kondo Physics in carbon nanotubes

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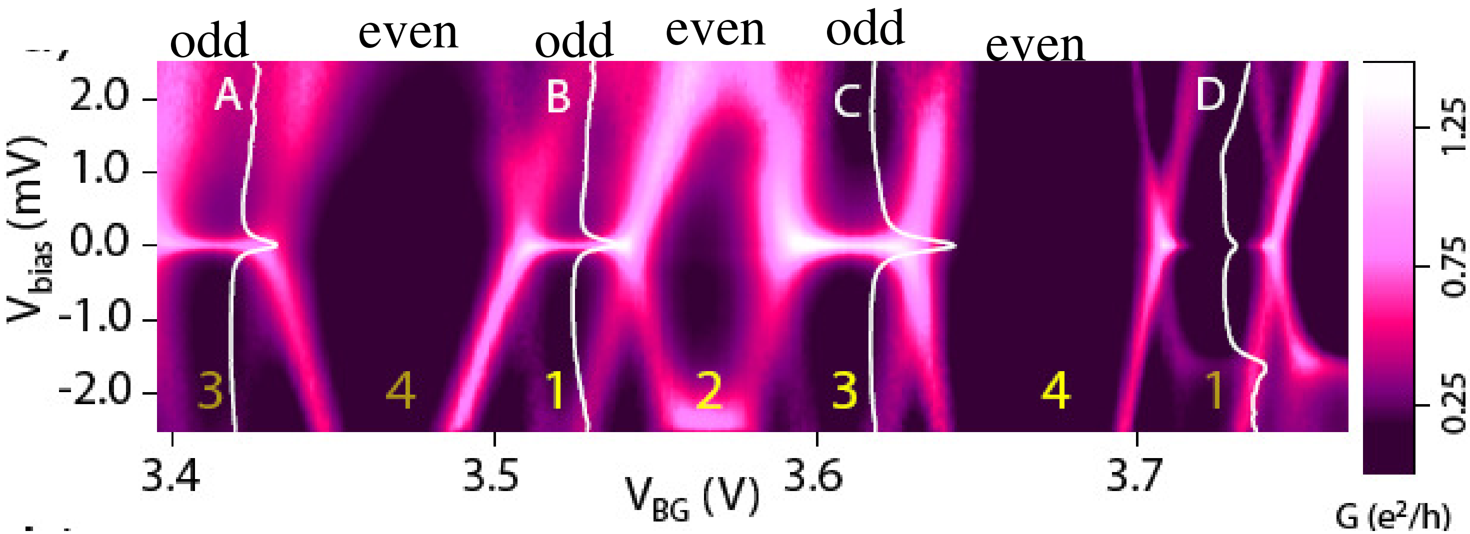
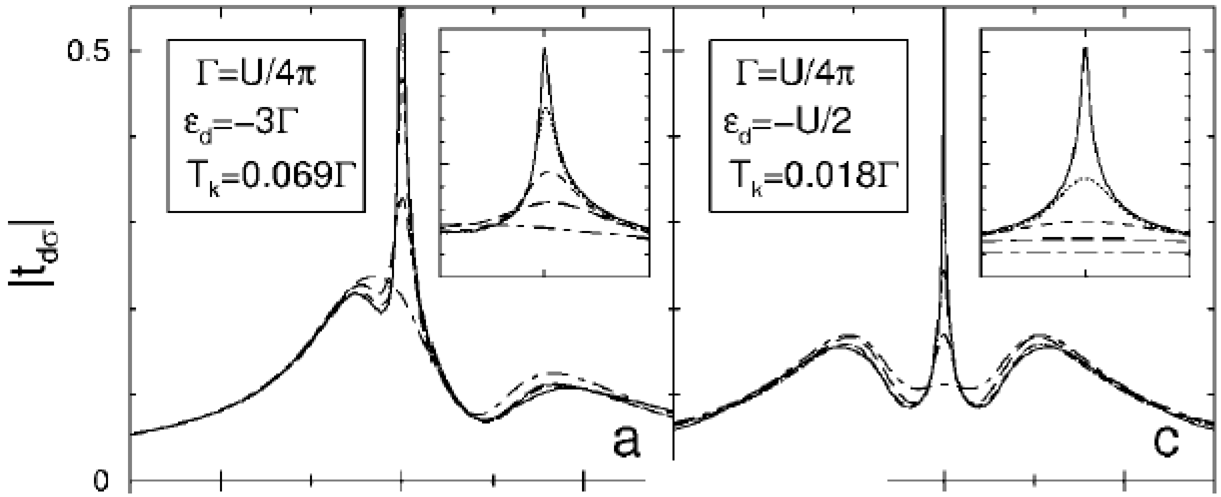


Eichler et al. 2008

T_k in the Kelvin range !

Tuning the Kondo temperature with gate voltage



Gerland et al.

PRL 2000

Minimum in the center of the Kondo ridge

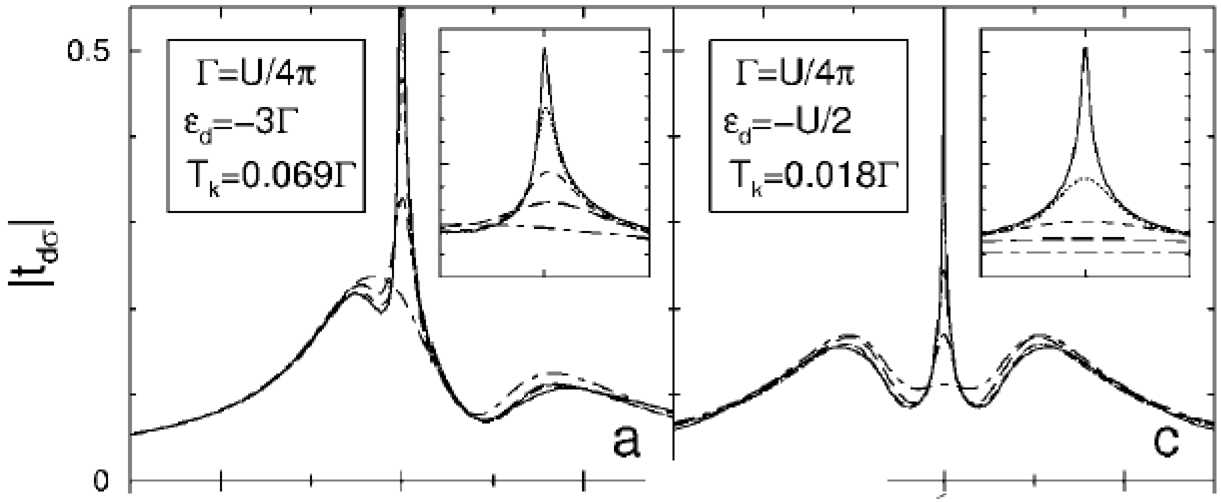
$$H_{\text{QD}} = \varepsilon_0(n_\uparrow + n_\downarrow) + U\, n_\uparrow n_\downarrow$$

$$H_T = \sum_{k,\sigma}(V_k c^\dagger_{k,\sigma} d_\sigma + \text{H.c.}). \quad \Gamma = \pi\rho\,|V|^2.$$

$$T_K = (U\Gamma/2)^{1/2} e^{\pi \varepsilon_d(\varepsilon_d + U)/2\Gamma U}$$

mV

Tuning the Kondo temperature with gate voltage



Gerland et al.

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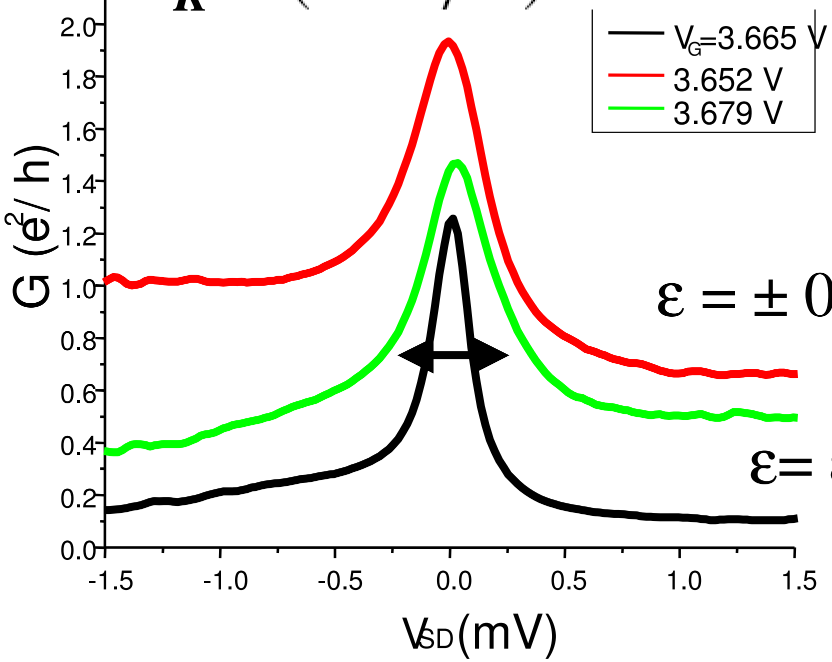
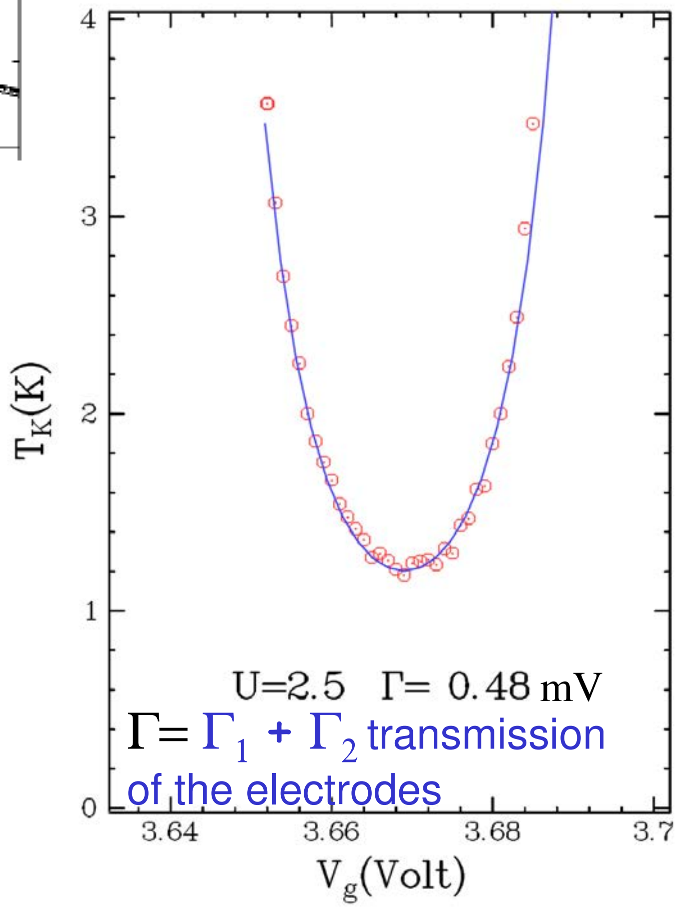
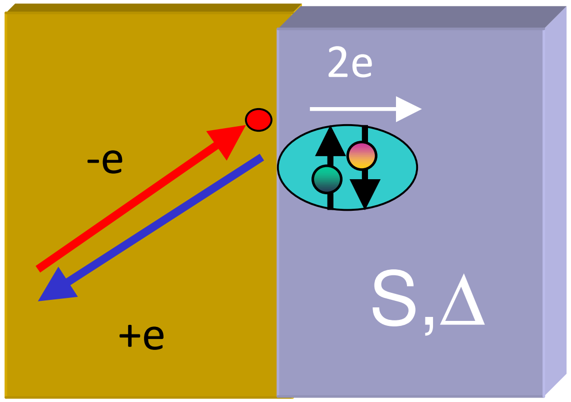


$\varepsilon = \pm 0.25$ U

$\varepsilon = \varepsilon_d$ - U/2 = 0



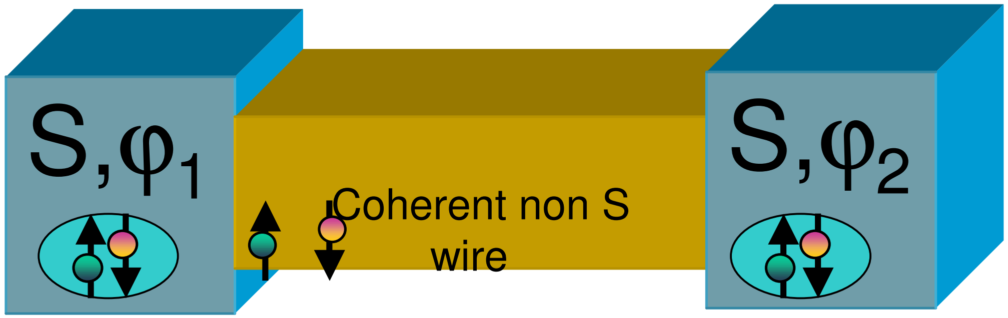
Suppraconducteurs en tant que miroirs à conjugaison de phase



courant NS à V=0:

électron rétro-réfléchi en un trou
(Andreev reflection)

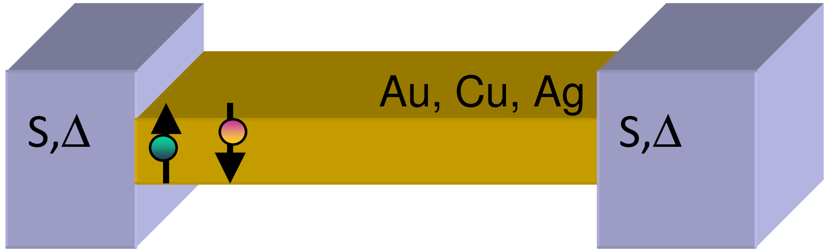
une charge 2e passe de N vers S



Etats confinés: superposition quantique
de fonctions d'ondes électrons et trous
Supercourant à V=0!

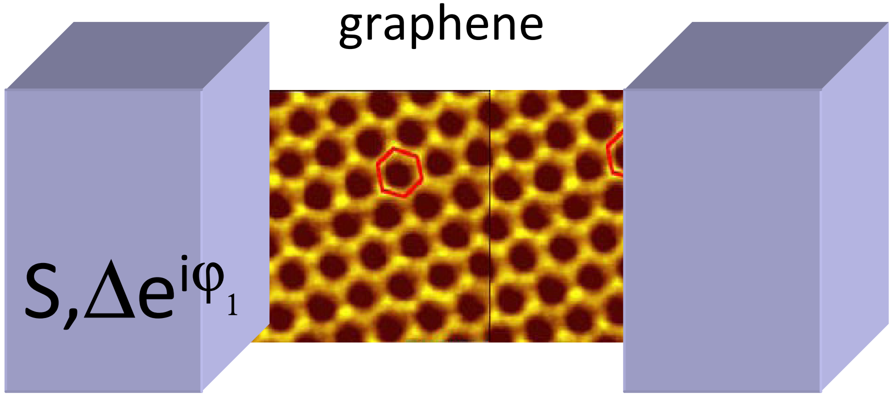
Proximity induced superconductivity in all sorts of conductors

metals

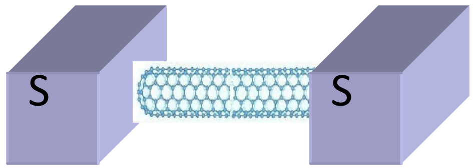


Ferromagnets

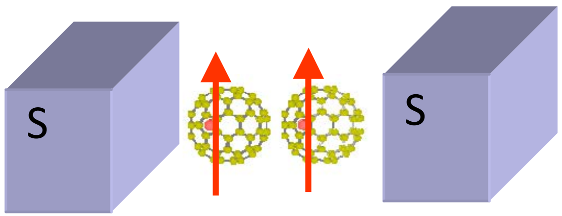
graphene

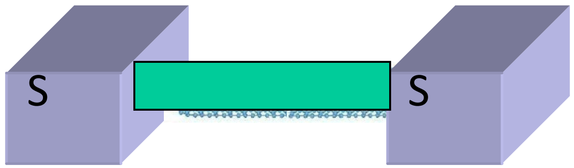


carbon nanotube



Molecules with spins





Bi cristalline nanowire
Surface only is conducting…

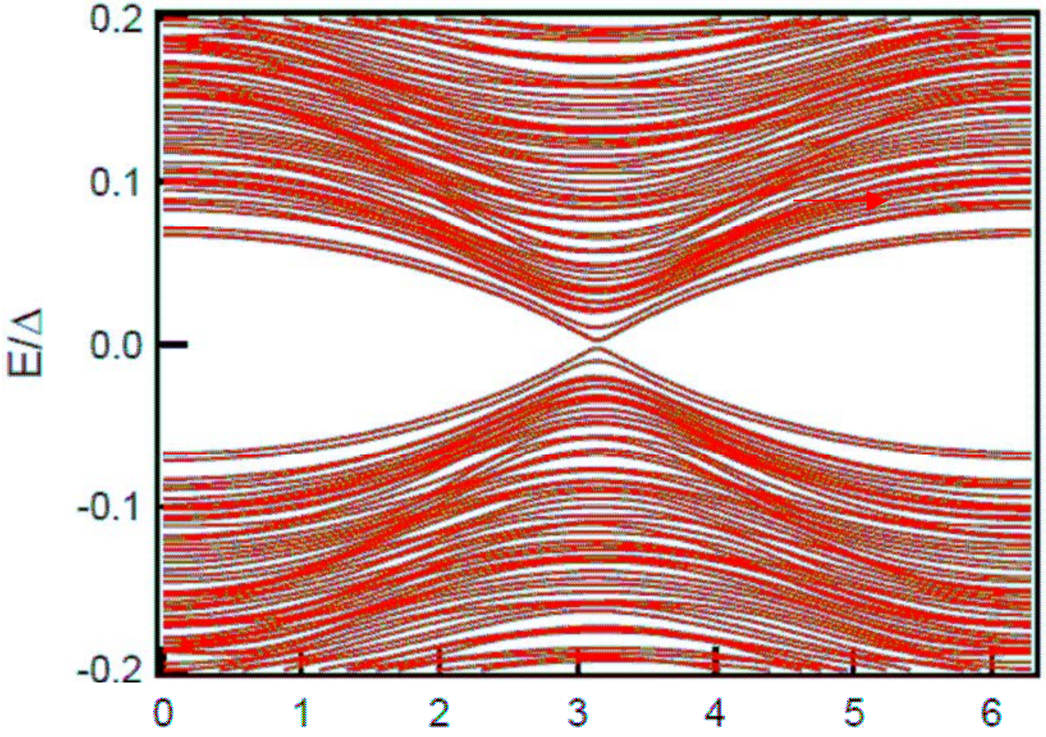
Spectrum of Andreev bound states: long junction

Diagonalisation of the Bogoliubov de Gennes Hamiltonian

$$\mathcal{H} = \begin{pmatrix} H - E_F & \Delta \\ \Delta & E_F - H^* \end{pmatrix} \qquad H = \sum_i \epsilon_i |i><i| + \sum_{i \neq j} t_{ij} |i><j|$$

ξ_s, l_e << L   g= $G_N/(e^2/h)$ =30

$$t_{ij} = t \exp i\phi_{ij}$$



$\varphi = 2\pi\Phi/\Phi_0$

τ_D

Phase dependent minigap

$E_g(\phi) \sim |\cos \phi/2|$

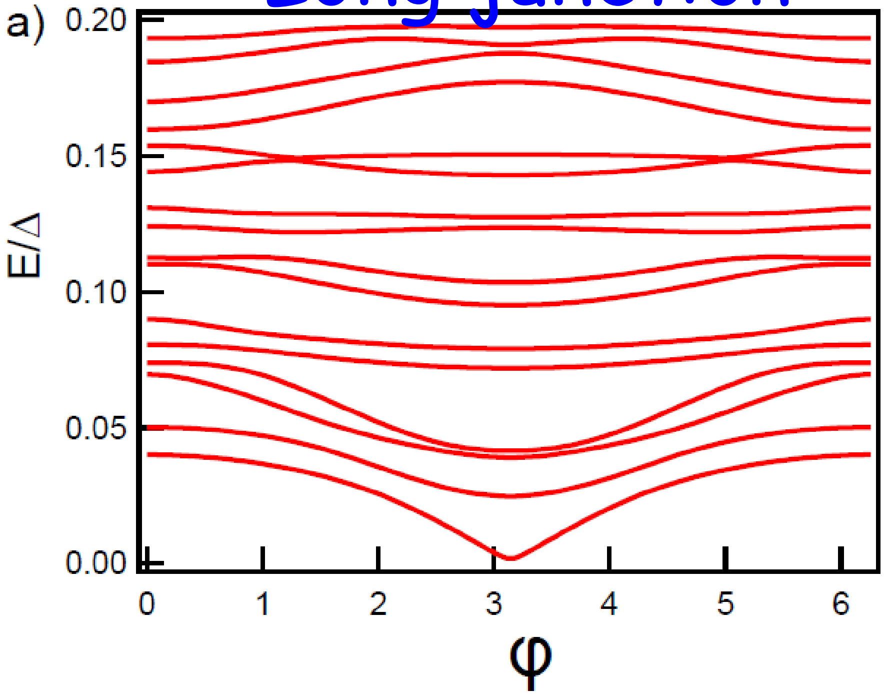
Eg (0)=3.5 E_{th}

$$E_{Th} = \frac{\hbar D}{L^2} = \frac{\hbar}{\tau_D}$$

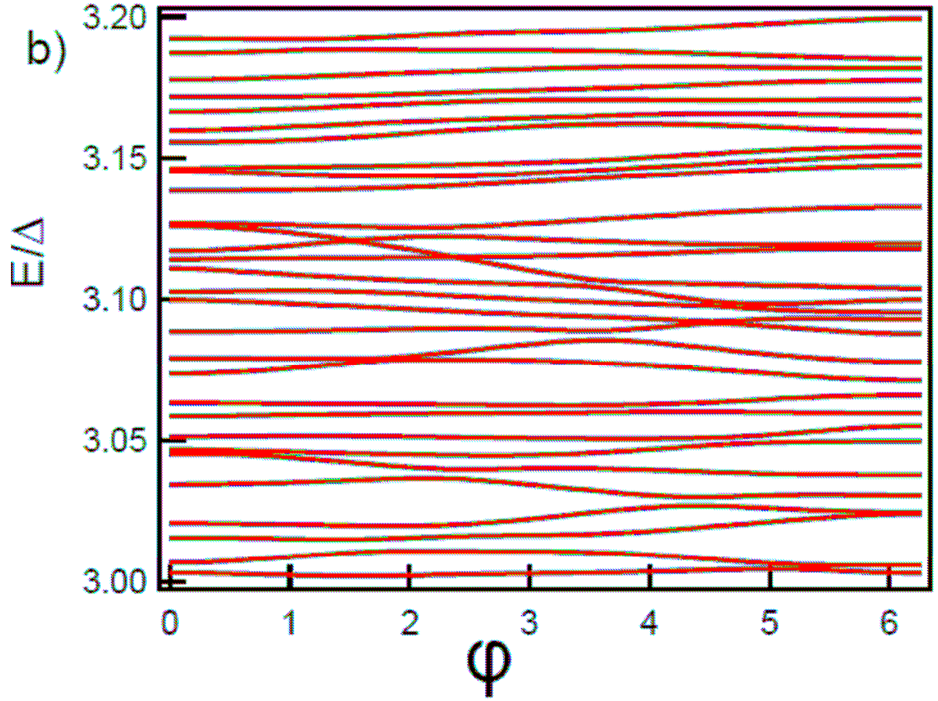
Mean diffusion time through N

Spectrum of Andreev bound states

Long junction



Below Δ: Φ_0 periodic
level spacing N part

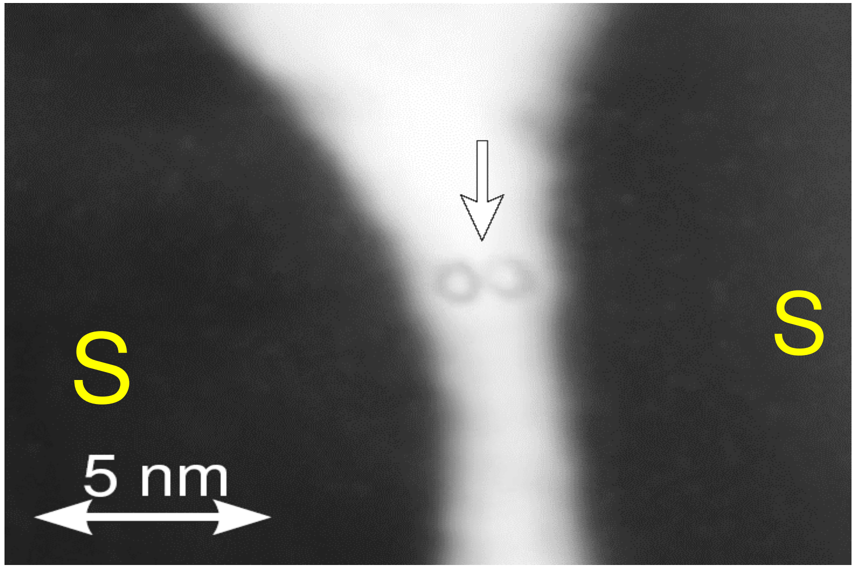


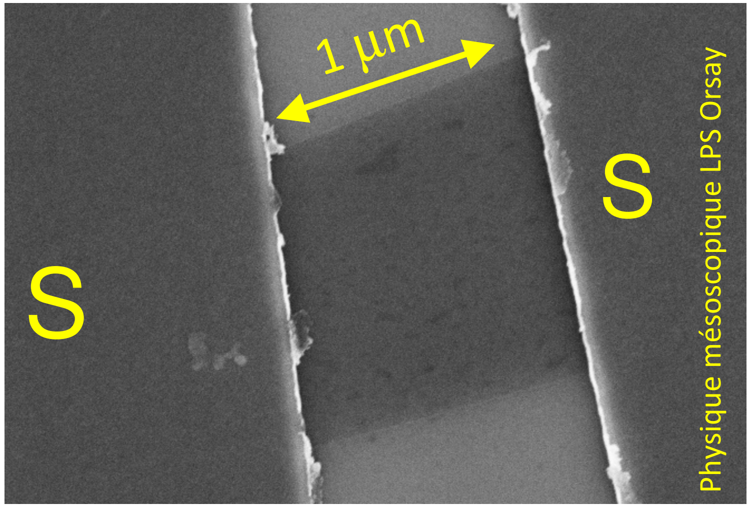
Avove Δ: $2\,\Phi_0$ periodic
level spacing N +S part

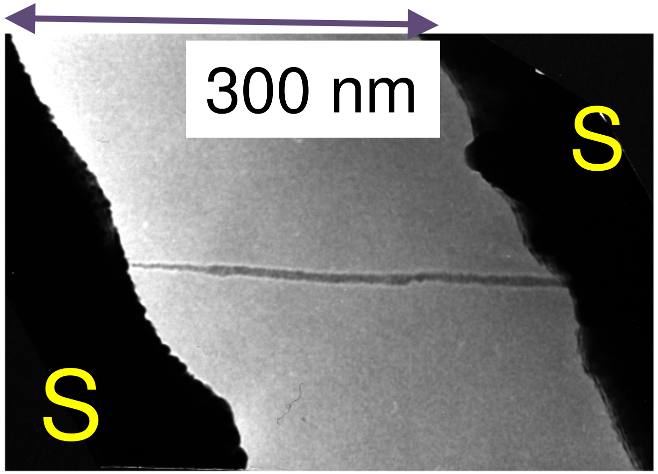
Long junction: no simple relation between Andreev states and eigen transmission channels

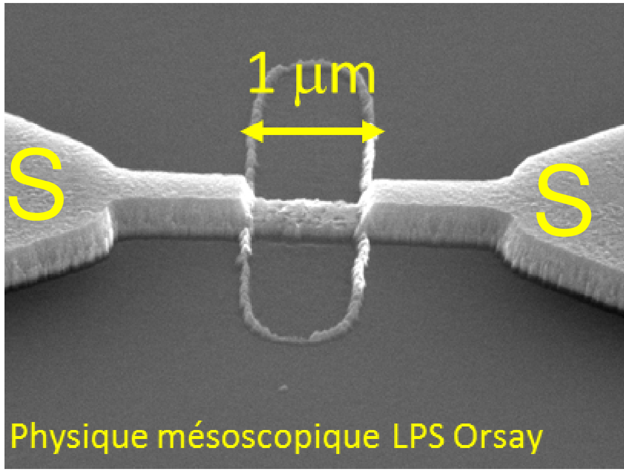
Josephson current $I_s(\varphi) = -\sum_n f_n(\varphi)\, \partial\, \varepsilon_n(\varphi) / \partial\, \varphi$

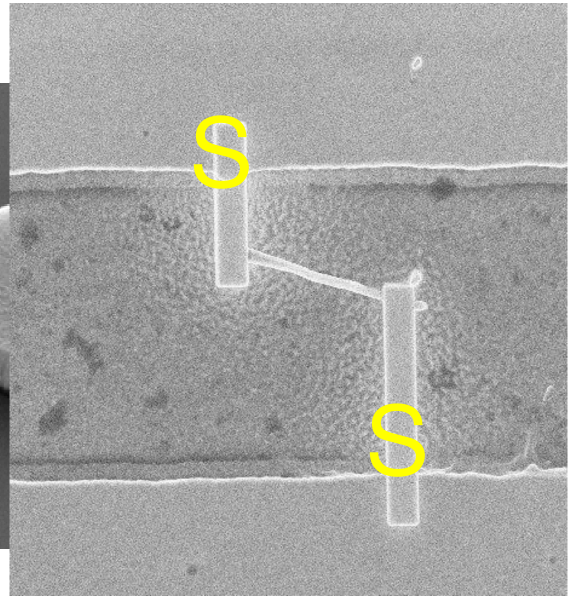
Guessing game… What's what?



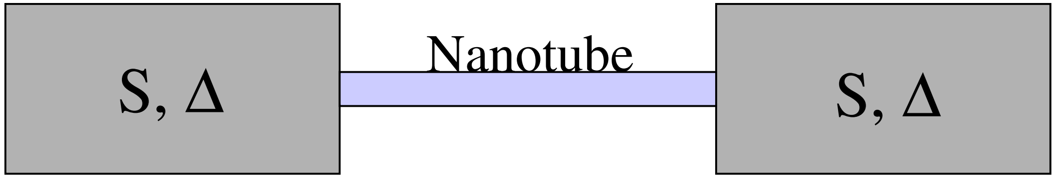


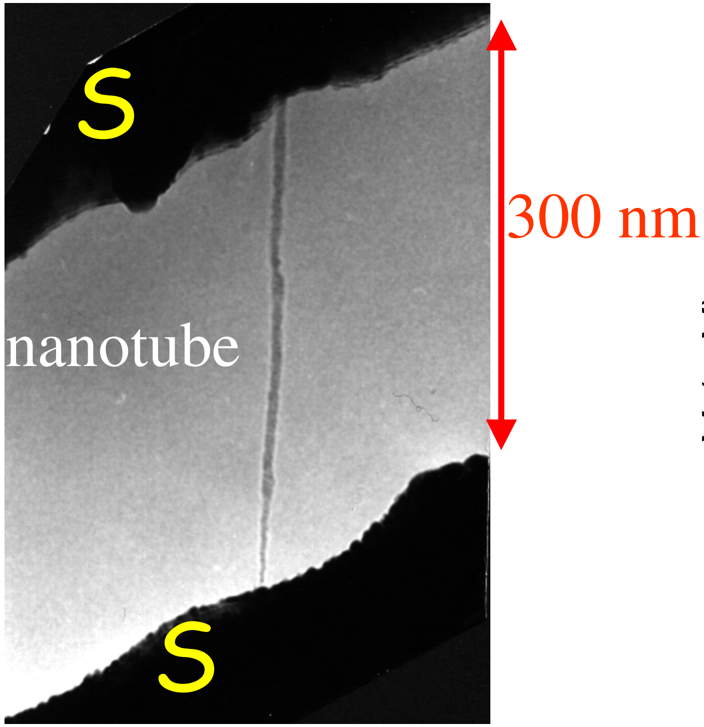


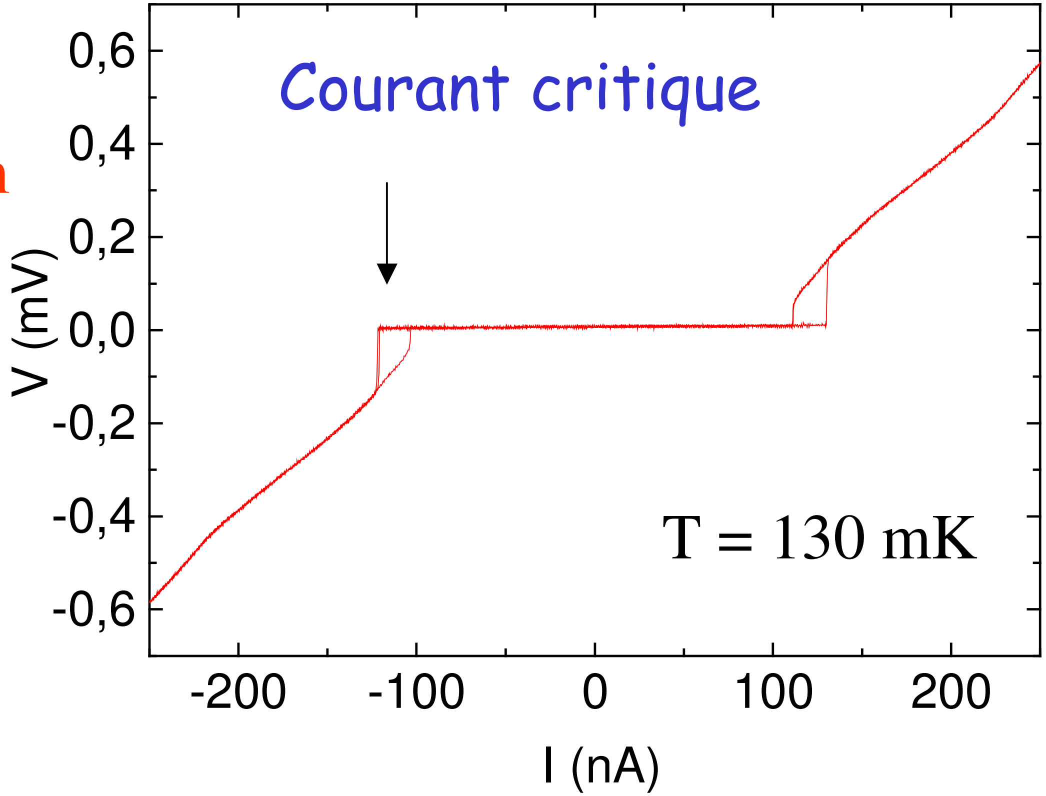




Supercourant dans un nanotube de carbone sur contacts supraconducteurs



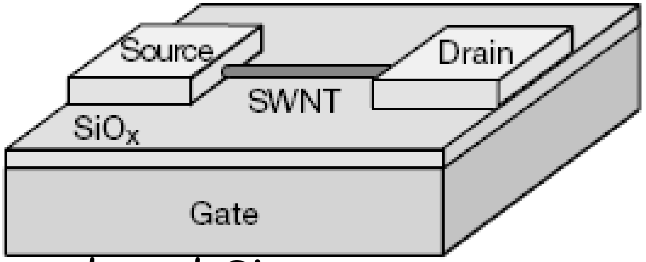


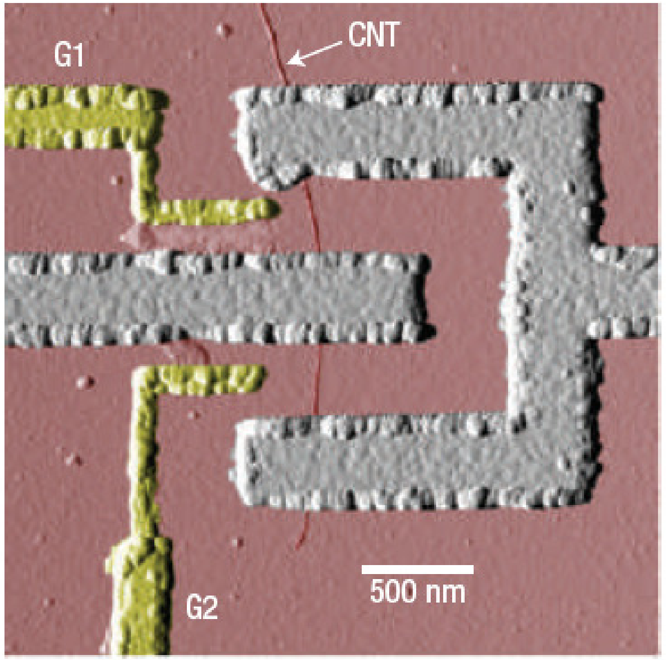


Kasumov et al Science 99

Signature de la transmission cohérente de paires supraconductrices, sonde de la brisure de cohérence de phase

Josephson Transistor with a carbon nanotube



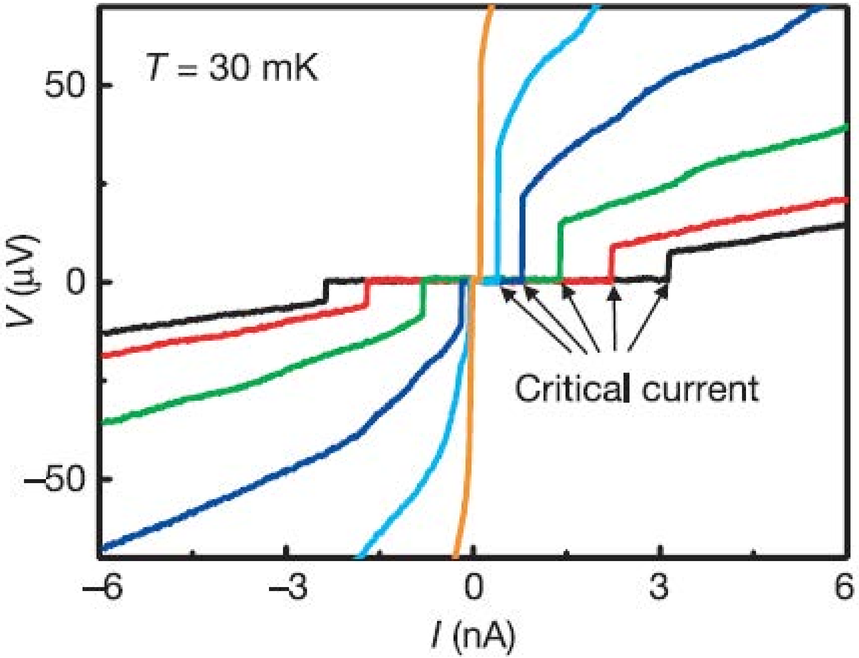


Superconducting Interference Device

Cleuziou et al. Nature Nanotechnology 2007

Superconducting Contacts

Herrero et al. Nature (2006)



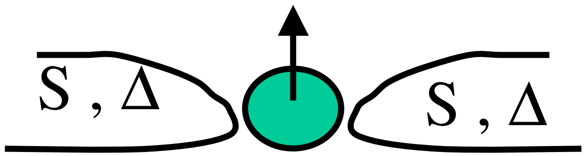
Modulation of Josephson current
With electrostatic gate voltage

Short junctions $L < \xi_S$

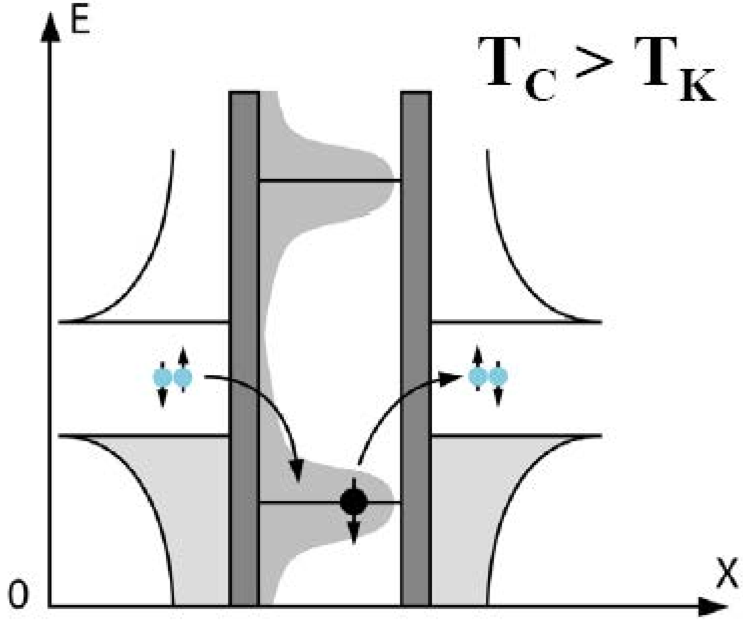
Competition between Kondo and Josephson effects

Glazmann and Matveev 89
Choi et al. 04, Siano et al.05
Karash et al. 08

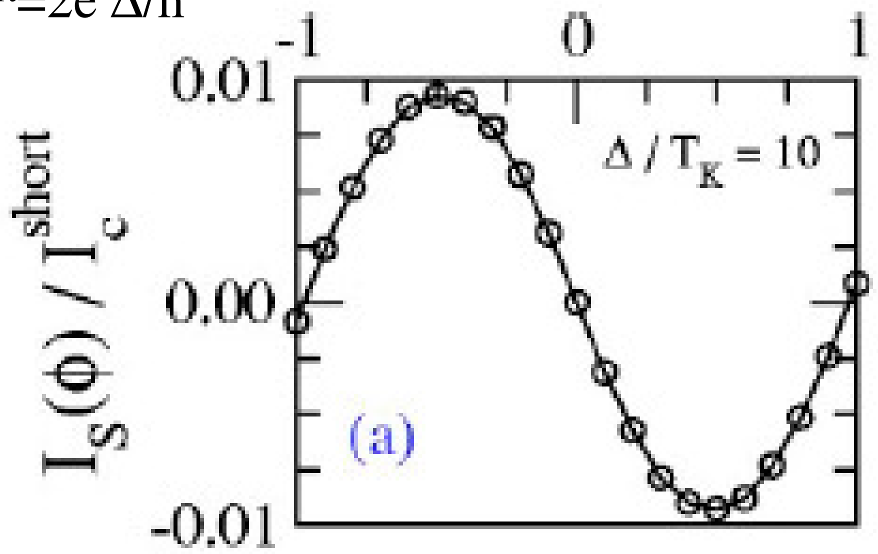
$I_c^{short} = 2e\,\Delta/\hbar$



Anderson impurity



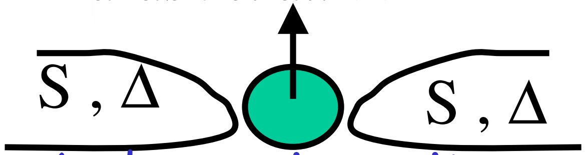
doublet – order of spins is „reversed“



$\Delta > T_k$,
Unscreened magnetic moment
π Junction with small Josephson current

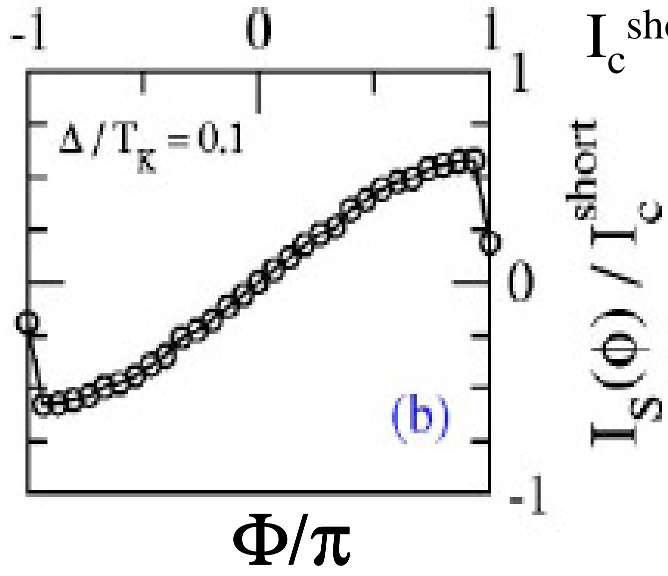
Competition between Kondo and Josephson effects

Glazmann and Matveev 89
Choi et al. 04, Siano et al.05
Karash et al. 08



Anderson impurity
Between S contacts

$\Delta < T_k, \quad \xi > L_K$
Superconductivity and
Kondo effect Cooperate!



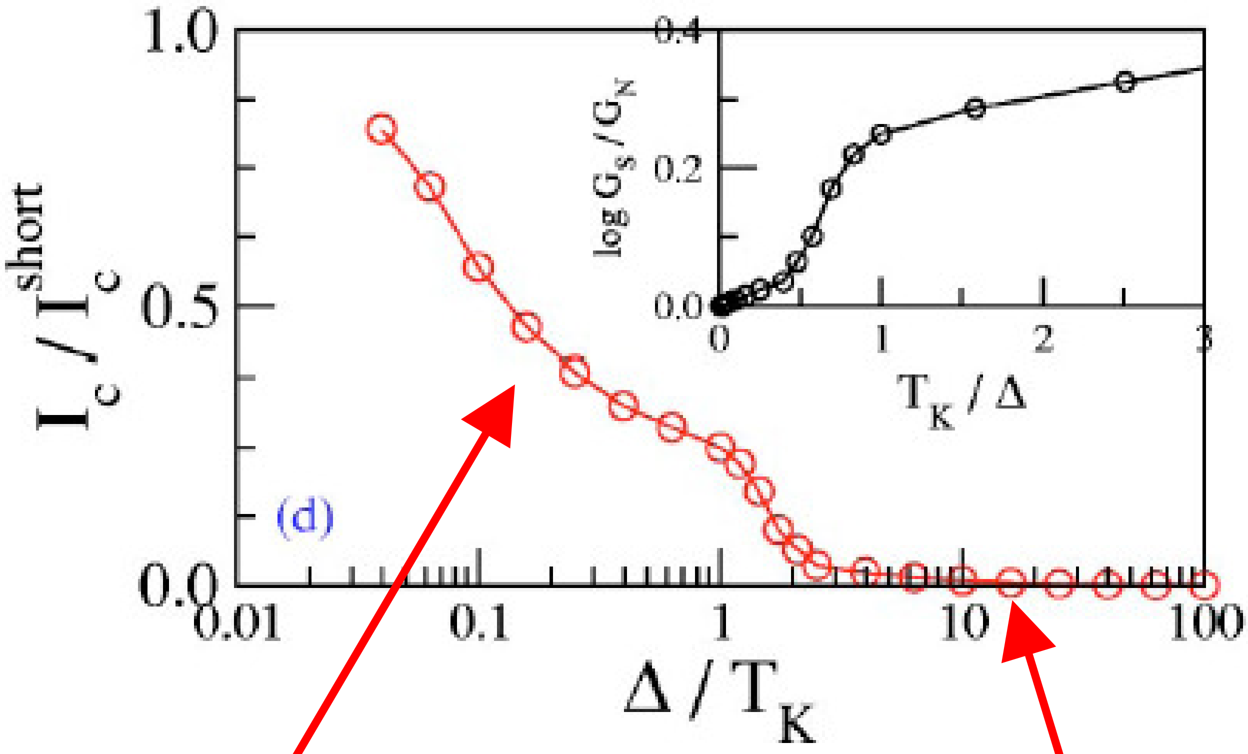
$I_c^{short} = 2e\,\Delta/\hbar$

Large amplitude of Ic independent of the transmission of the electrodes!

Enhanced by Coulomb interactions!

$0\ \pi$ transition when $\Delta > T_K$

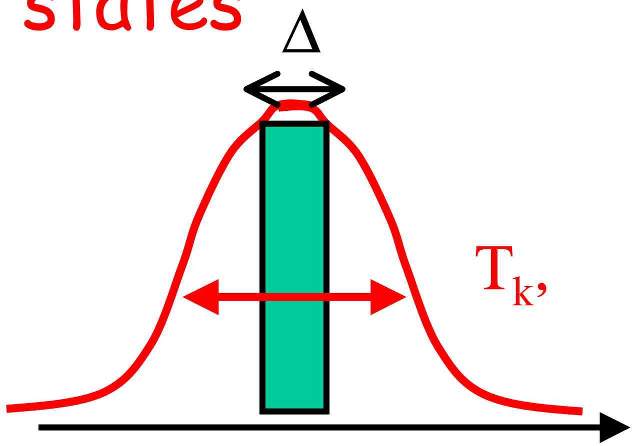
Choi et al. 04

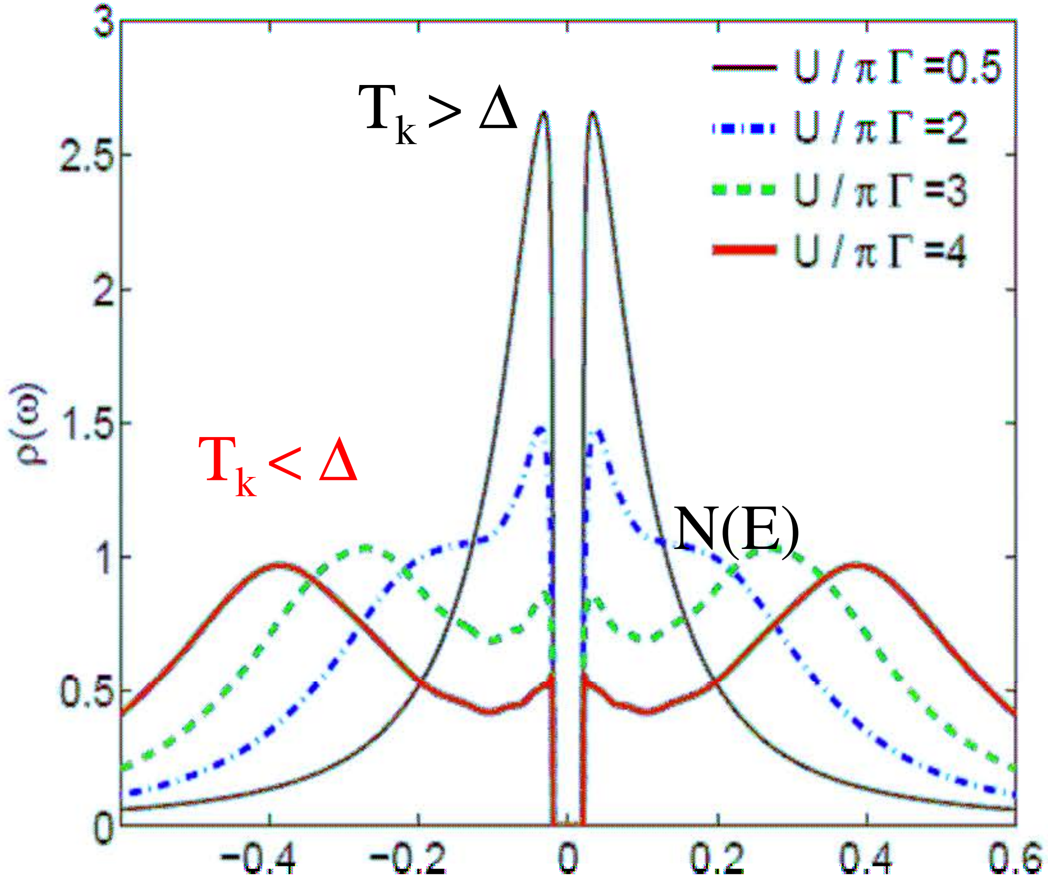


Kondo and proximity effect cooperate

π junction

Evolution of the density of states





$\Delta < T_k$

small perturbation

in the DOS

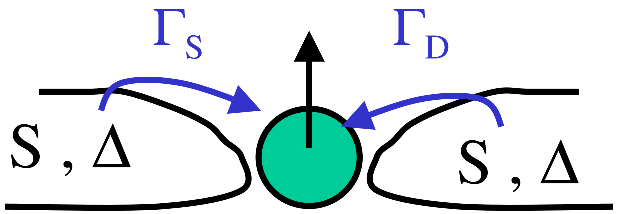
quasiparticles can screen

the magnetic impurity

Bauer et al. J. Phys. (2007)
NRG calculations

Functional RG calculations

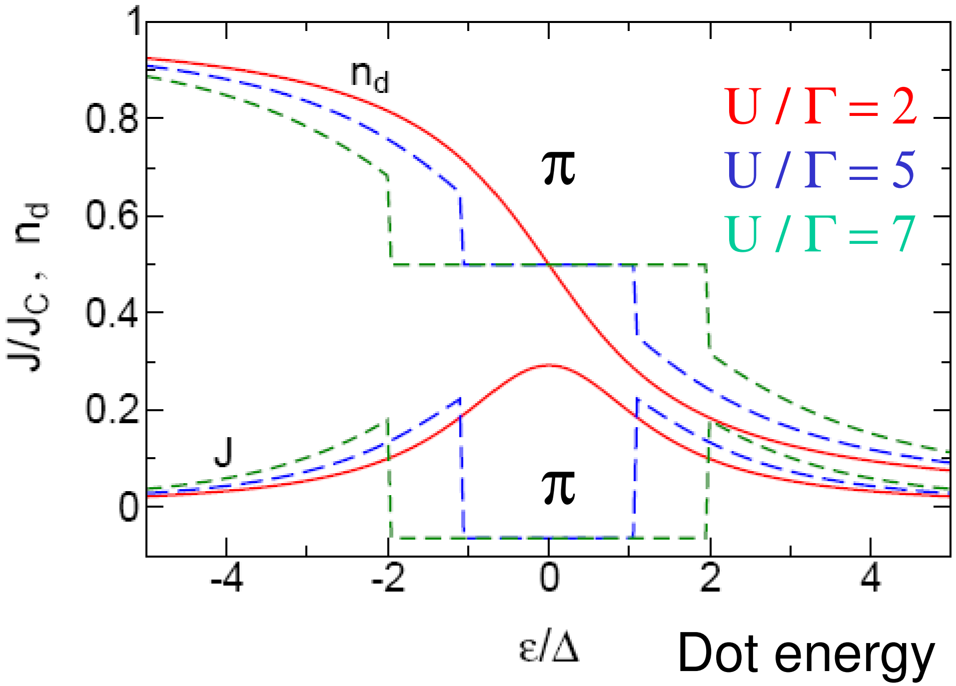
Karrasch et al. 2008 Anderson impurity between S electrodes



$\Gamma = \Gamma_S + \Gamma_D$ transmission of electrodes

U charging energy

$\Gamma_S = \Gamma_D$



Occupation number

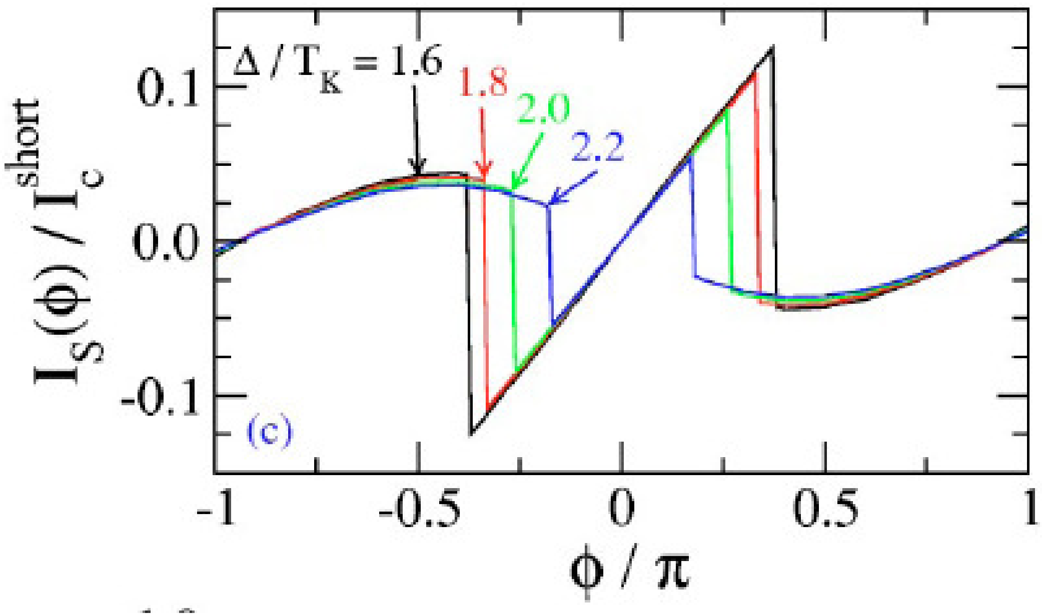
Supercurrent

Dot energy

$0,\pi$ transition tuned with the gate voltage

Current phase relation in the vicinity of the 0/pi transition..

Choi et al. 2004
Karrasch et al 2008



Phase bias induced
Singlet doublet transition

Phase bias induced Singlet doublet transition

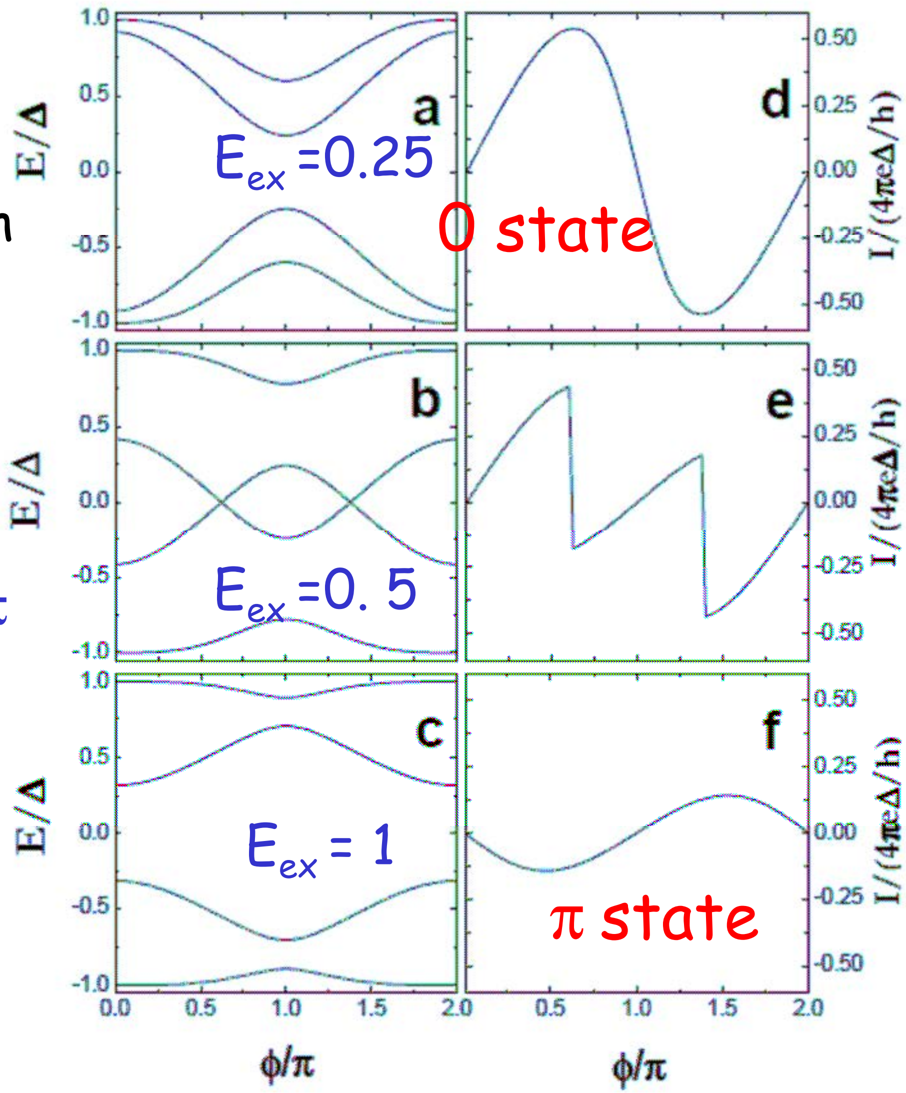
Vecino et al. PRB 03

Evolution of the Andreev spectrum with exchange mean field energy

$E_{ex} = U(n\uparrow - n\downarrow)$

Crossing of Andreev levels close to $\Phi=\pi$

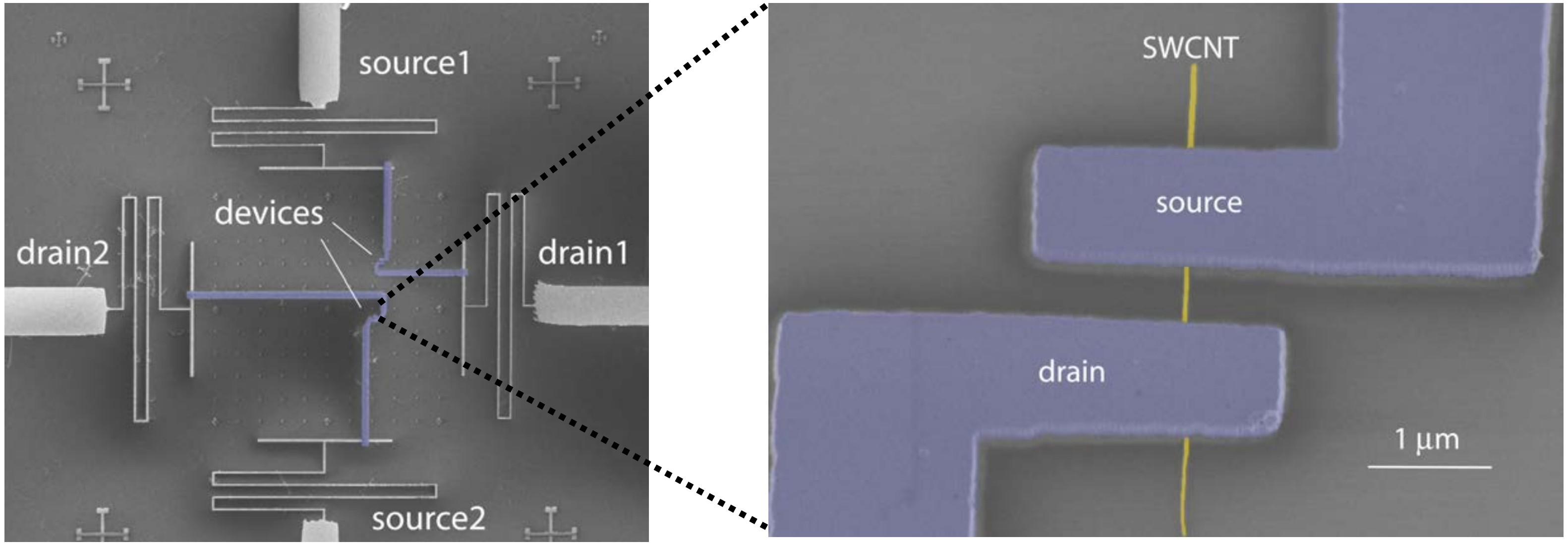
The stability of the π state
Depends on the superconducting phase

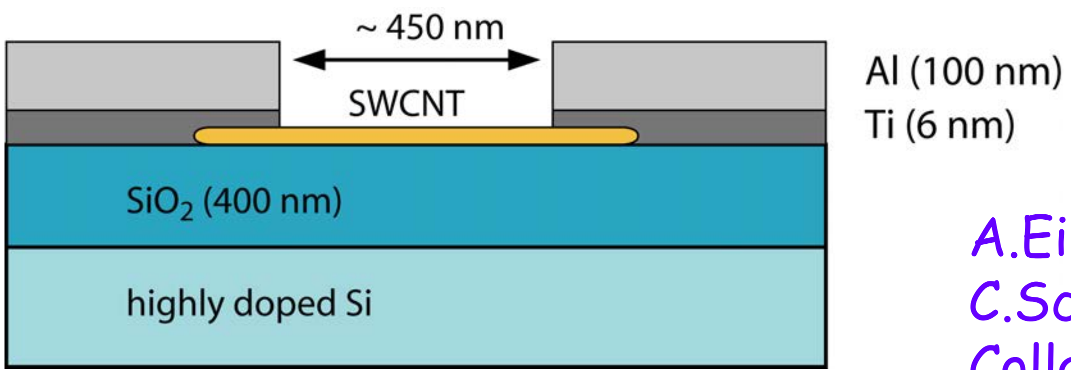


Magnetic state of the junction can be monitored with phase

Tuning Josephson current with Kondo physics

Carbon nanotube on S contacts in a dissipative environment





A.Eichler, R.Deblock, M.Weiss
C.Schönenberger, H.B
Collaboration Orsay Basel

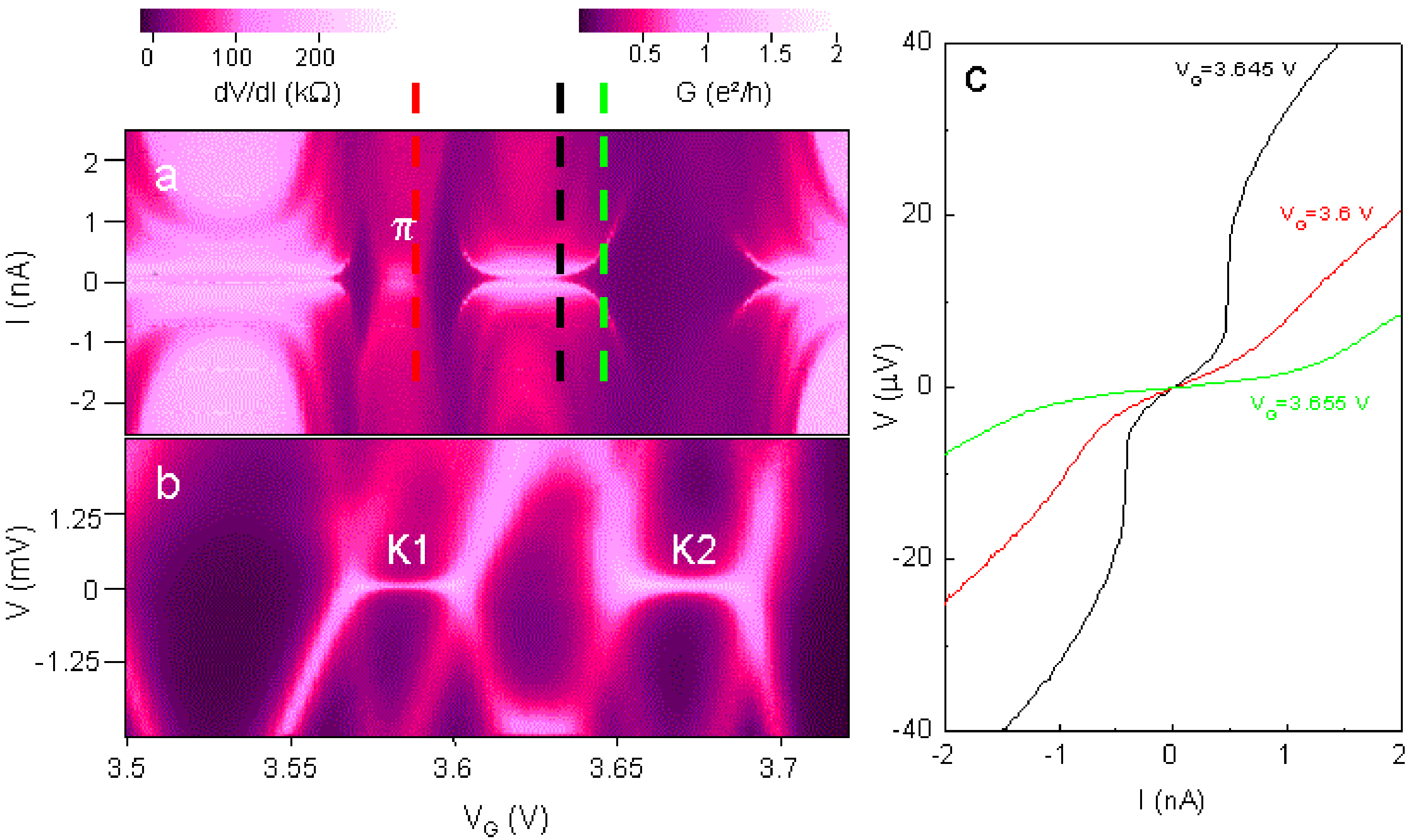
Josephson current and Kondo effect

R(Vg)

S state

G(Vg)

N state



Josephson effect observed in the Kondo regime

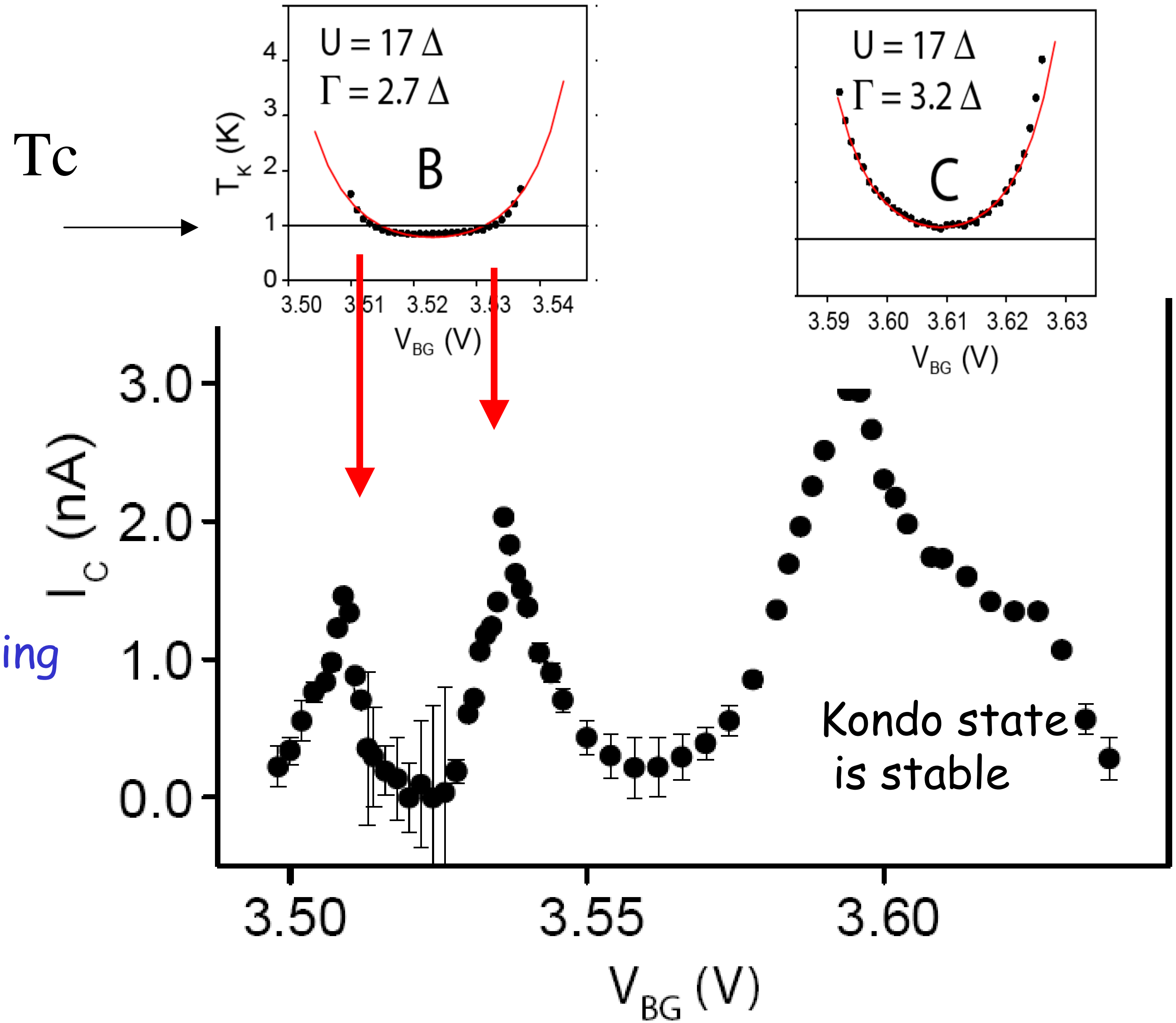
Competition between Kondo and Josephson effects

$T_K \sim Tc$

Supercurrent switching within few mV of gate voltage!

Instability of Kondo screening

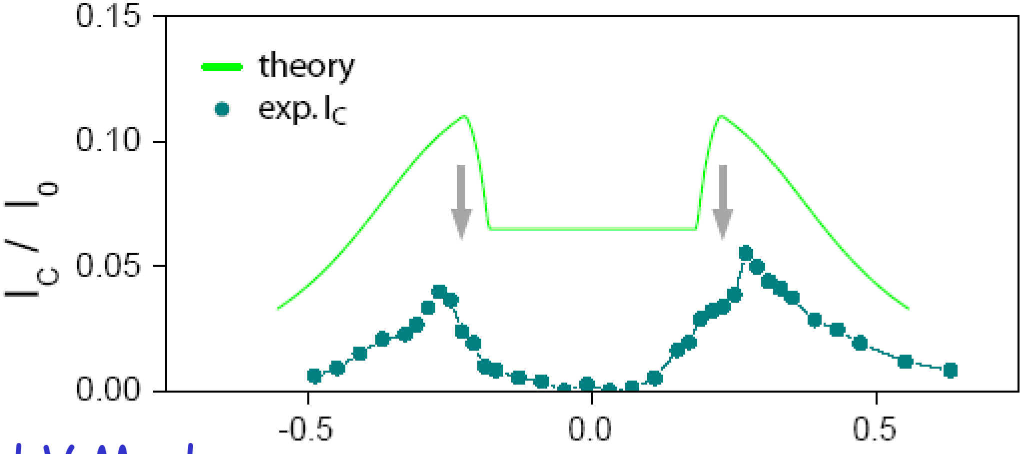
Singlet doublet transition!



Comparison between experiment and theory

$I_0 = e\,\Delta/\hbar$

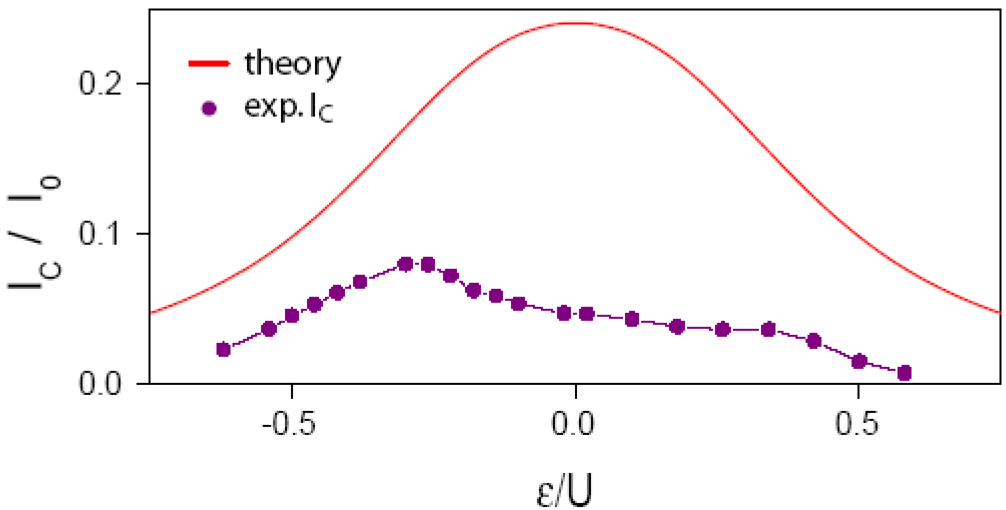
Theory T=0
$\Delta/\Gamma = 0.5$
$\Gamma/U = 0.11$
$\Gamma_R/\Gamma_L = 6$



Experiment
$\Delta/\Gamma = 0.4$
$\Gamma/U = 0.15$
$\Gamma_R/\Gamma_L = 6$

Collaboration
C. Karrasch and V.Meden

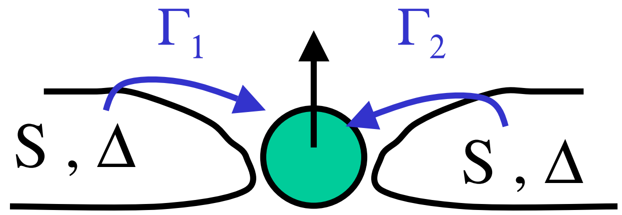
Theory
$\Delta/\Gamma = 0.5$
$\Gamma/U = 0.2$
$\Gamma_R/\Gamma_L = 3$



Experiment
$\Delta/\Gamma = 0.4$
$\Gamma/U = 0.2$
$\Gamma_R/\Gamma_L = 3$

Why are experimental values of Ic too small (facteur 3)?
Finite temperature? Influence of the doped substrate?

Importance of the asymmetry between the electrodes

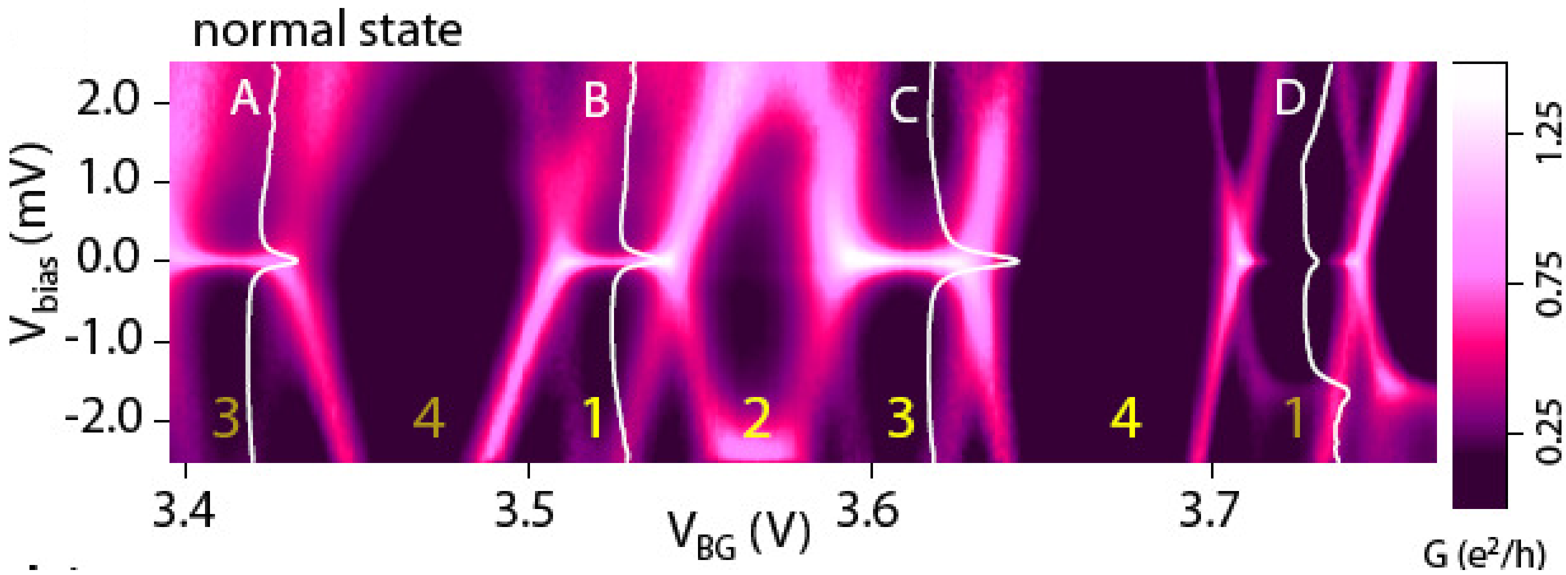


Nearly identical values of T_K and $\Gamma = \Gamma_1 + \Gamma_2$

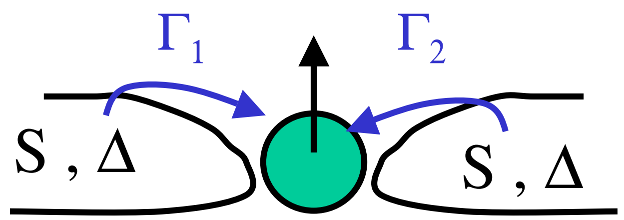
but different values of $\Gamma_1 / \Gamma_2 = 1$

determined from $G_K = 8\frac{\Gamma_1\Gamma_2}{(\Gamma_1+\Gamma_2)^2}$ $T \ll T_K$

$\Gamma_1 / \Gamma_2 = 7$ 6 2.5 70



Importance of the asymmetry between the electrodes



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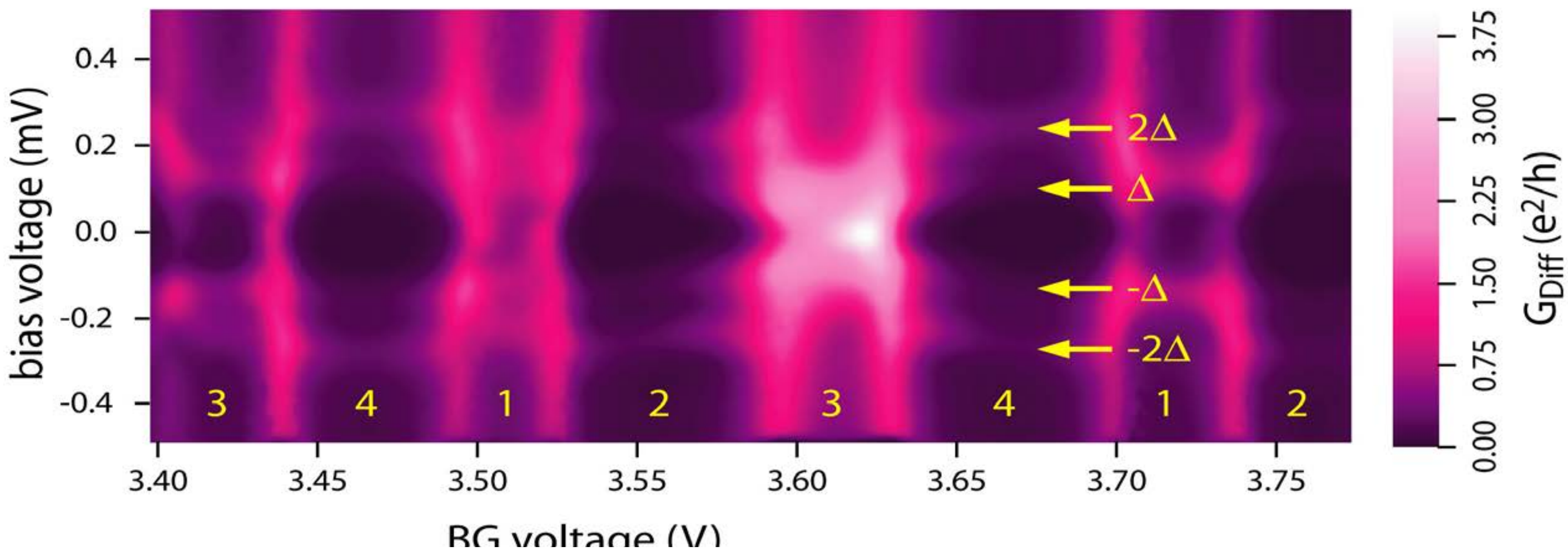
determined from $G_K = 8\frac{\Gamma_1\Gamma_2}{(\Gamma_1+\Gamma_2)^2}$ $T \ll T_K$

$\Gamma_1 / \Gamma_2 = 7$ 6 2.5 70

Supercurrent with 0π transition

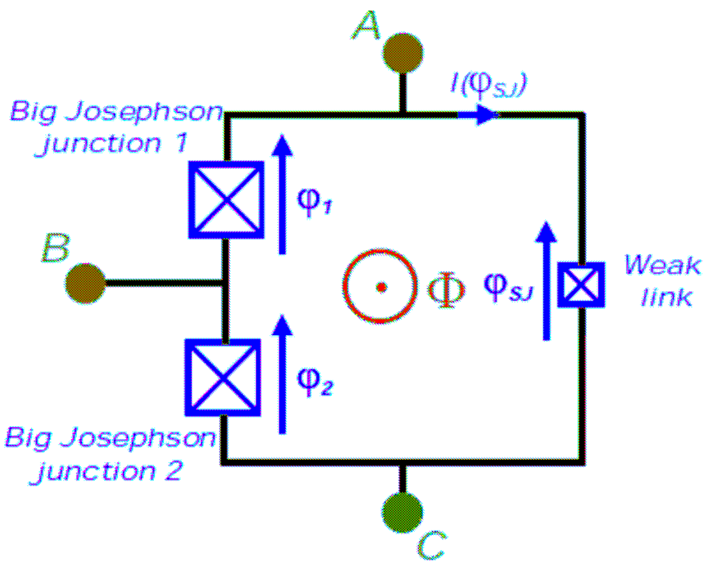
Supercurrent

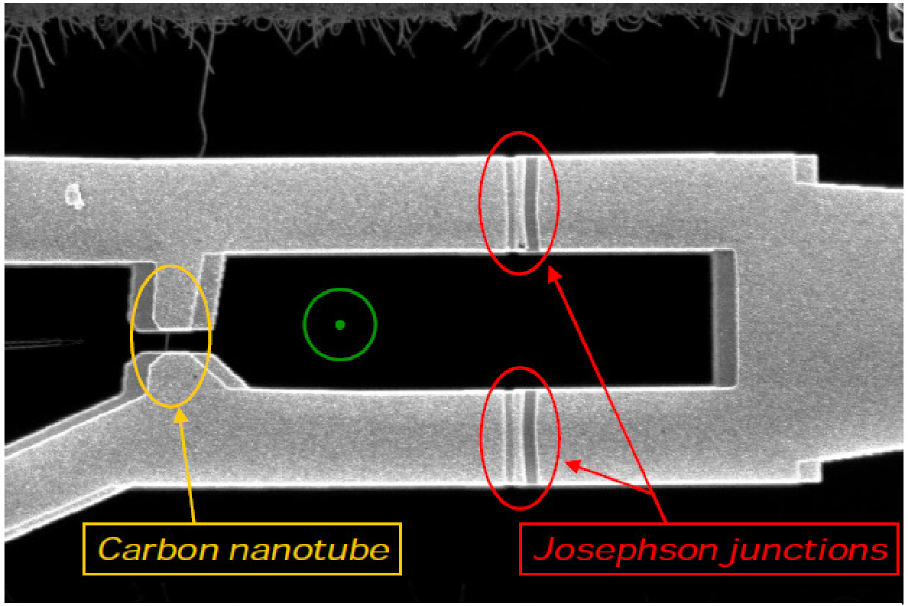
No supercurrent!



Insertion of a carbon nanotube into a SQUID

Inspired from Della Roca et al. 2007





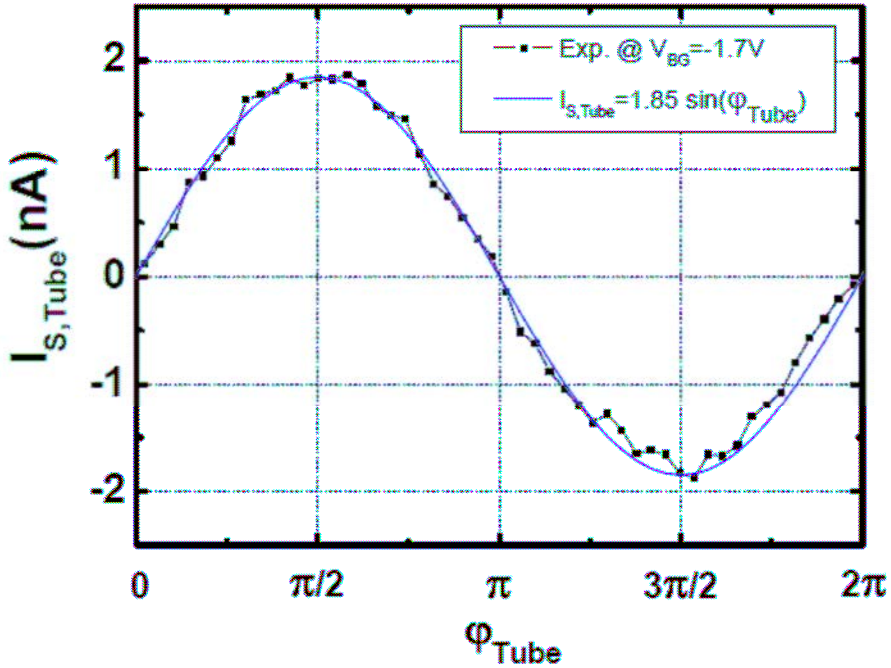
J. Basset et al. 2011
R.Delagrange 2013

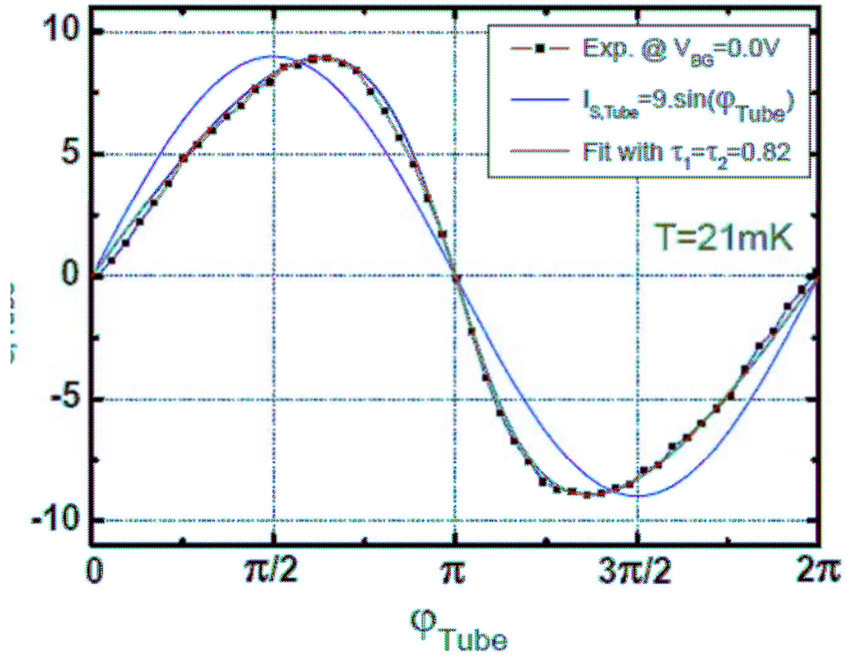
Ic (Junction) >> Ic (tube)

The flux dependence of Ic(SQUID) is dominated by $I_c(f)$ (tube)

Insertion of a carbon nanotube into a SQUID

Highly transmitting electrodes



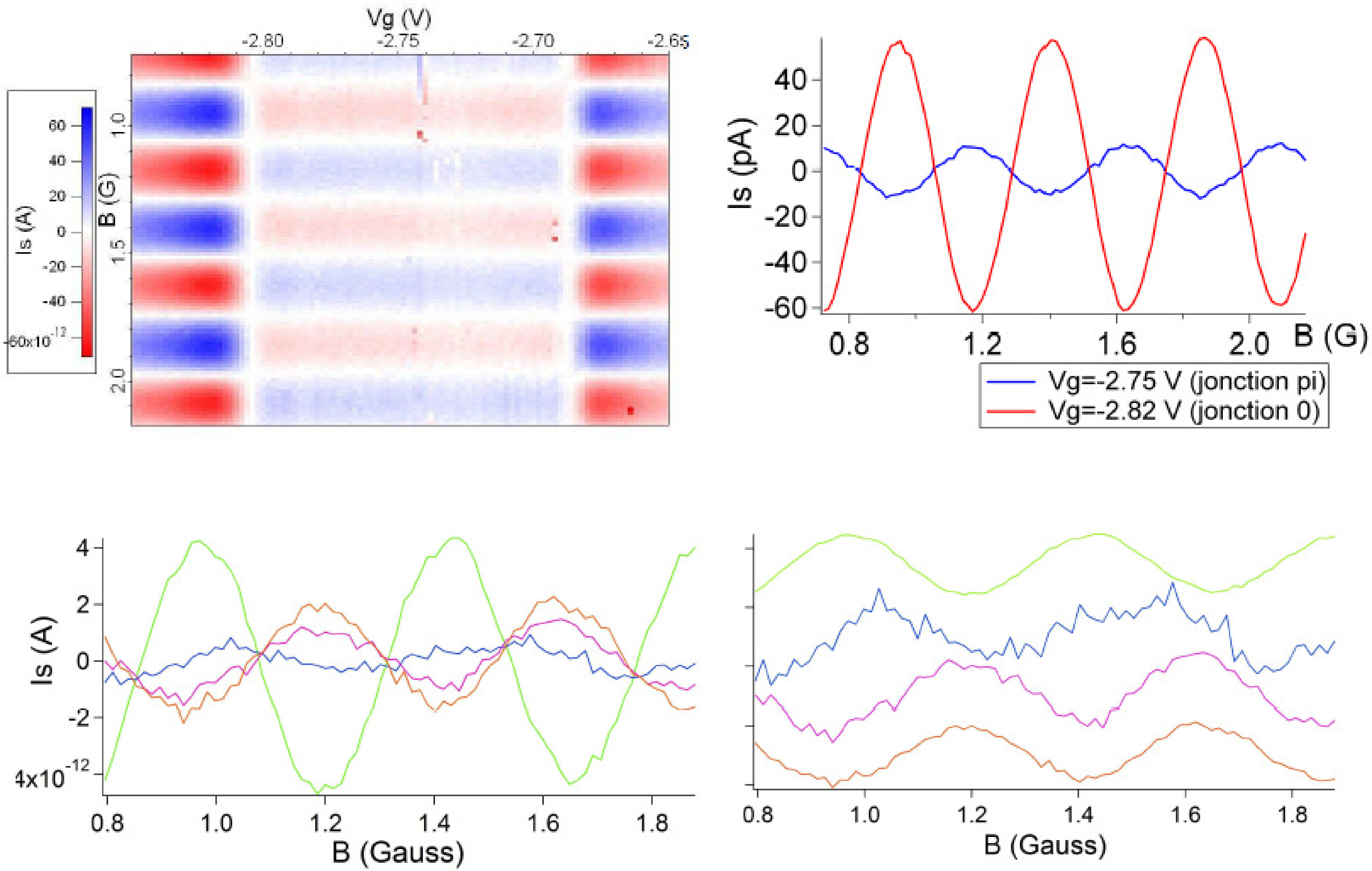


$$I(\varphi) = \sum_{i=1}^{N} \frac{e\tau_i \Delta}{2\hbar} \frac{sin\varphi}{\sqrt{1-\tau_i sin^2(\varphi/2)}}.$$

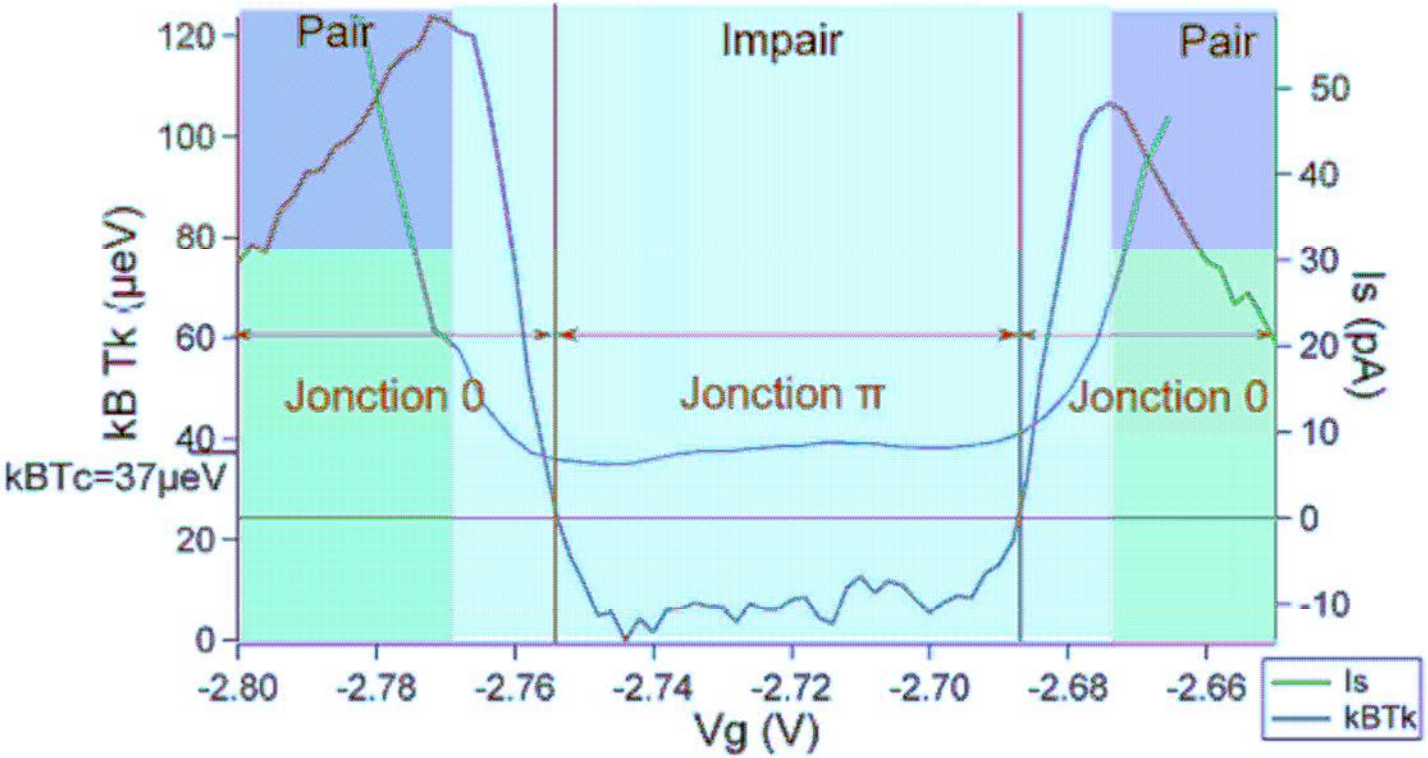
τ_i transmission of the mode i

Anharmonic current phase relation
For large transmissions!
Kondo regime still not observed….

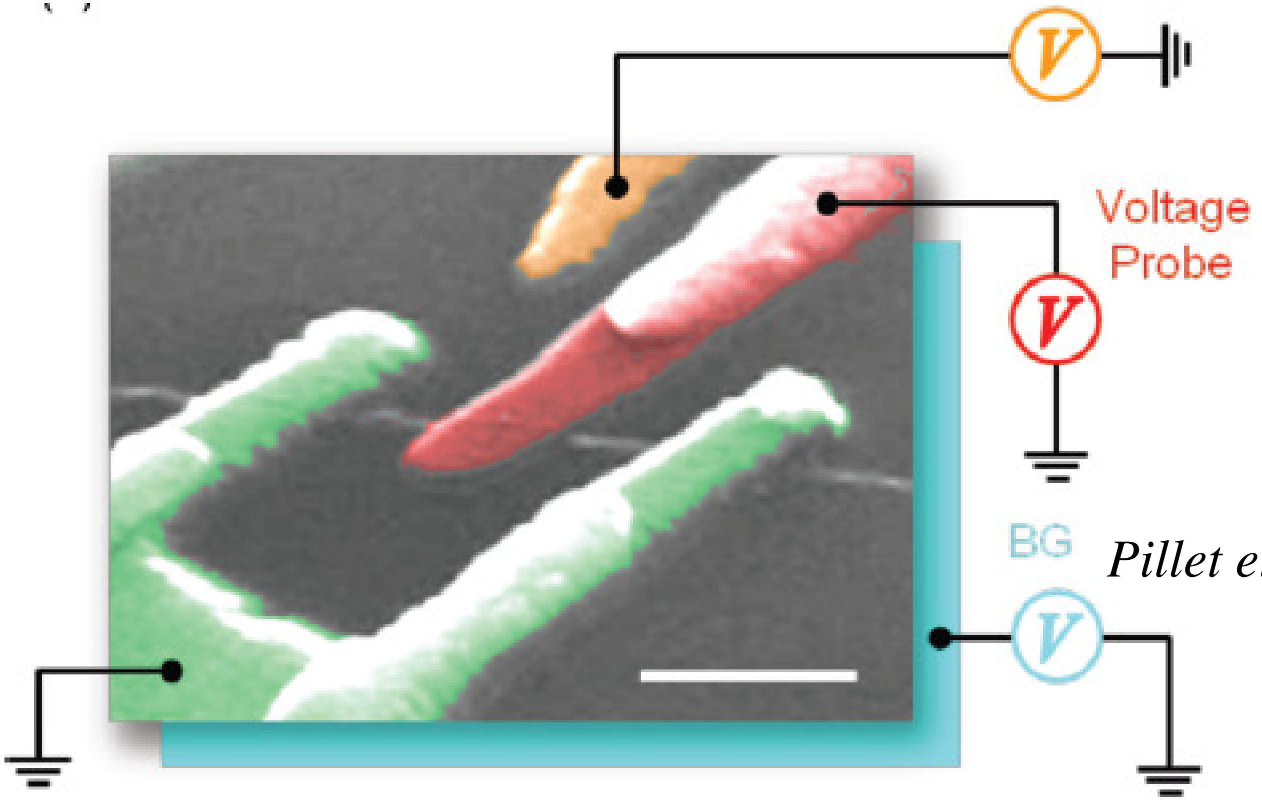
J. Basset et al. 2011



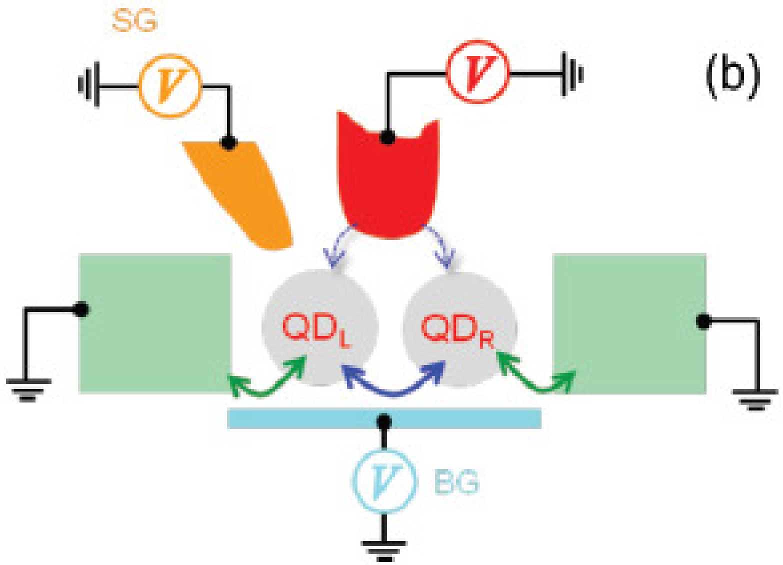
Vg (V)
-2.80
-2.75
-2.70
-2.65
B (G)
1.0
1.5
2.0
Is (A)
60
40
20
0
-20
-40
-60x10^-12
Is (pA)
B (G)
0.8
1.2
1.6
2.0
Vg=-2.75 V (jonction pi)
Vg=-2.82 V (jonction 0)
Is (A)
4x10^-12
B (Gauss)
B (Gauss)



Pair
Impair
Pair
Jonction 0
Jonction π
Jonction 0
kBTc=37µeV
kB Tk (µeV)
Is (pA)
Vg (V)
120
100
80
60
40
20
0
50
40
30
20
10
0
-10
-2.80
-2.78
-2.76
-2.74
-2.72
-2.70
-2.68
-2.66
Is
kBTk

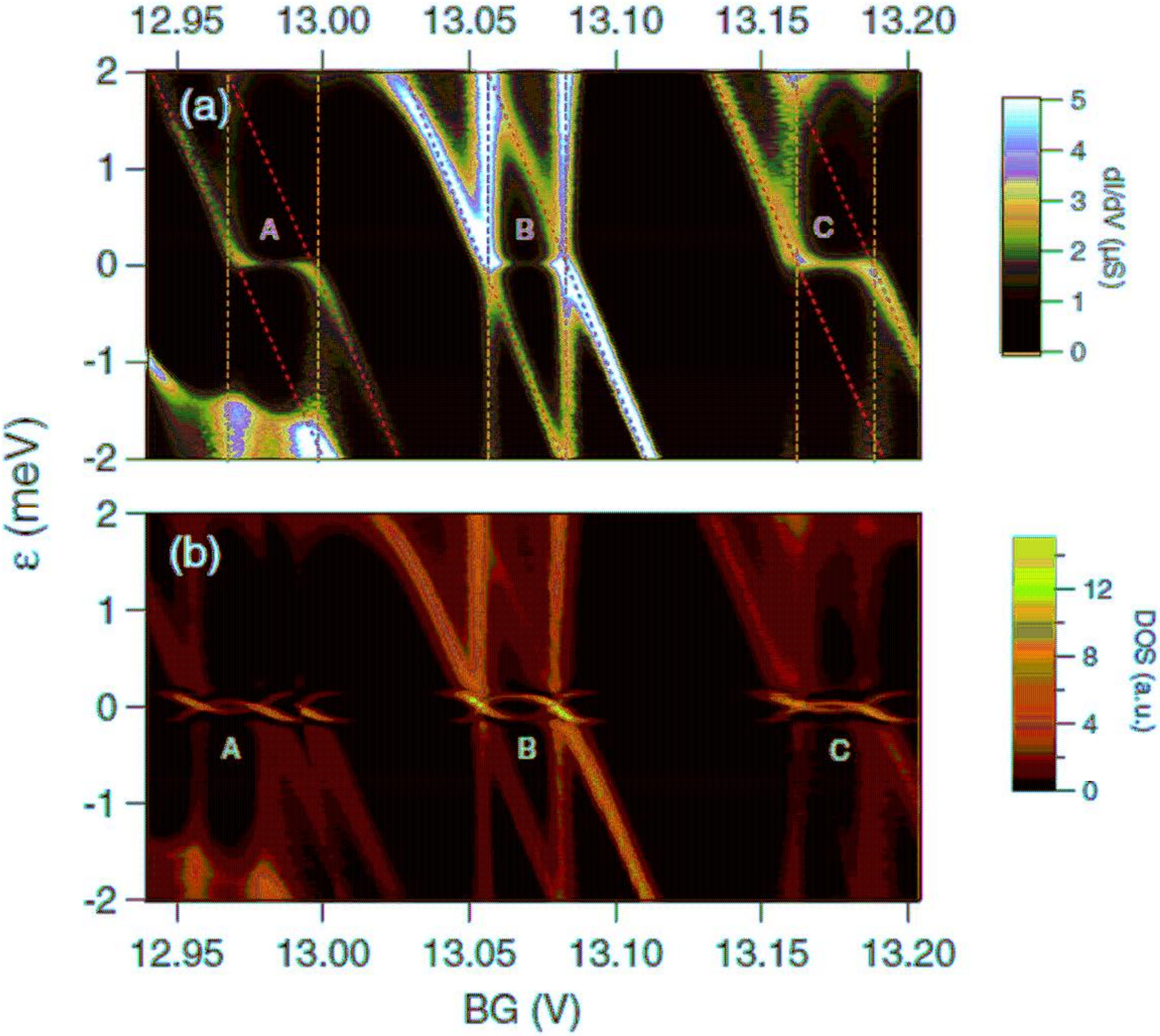


Pillet et al. PRB 2013



Spectroscopie des états D'Andreev d'un nanotube

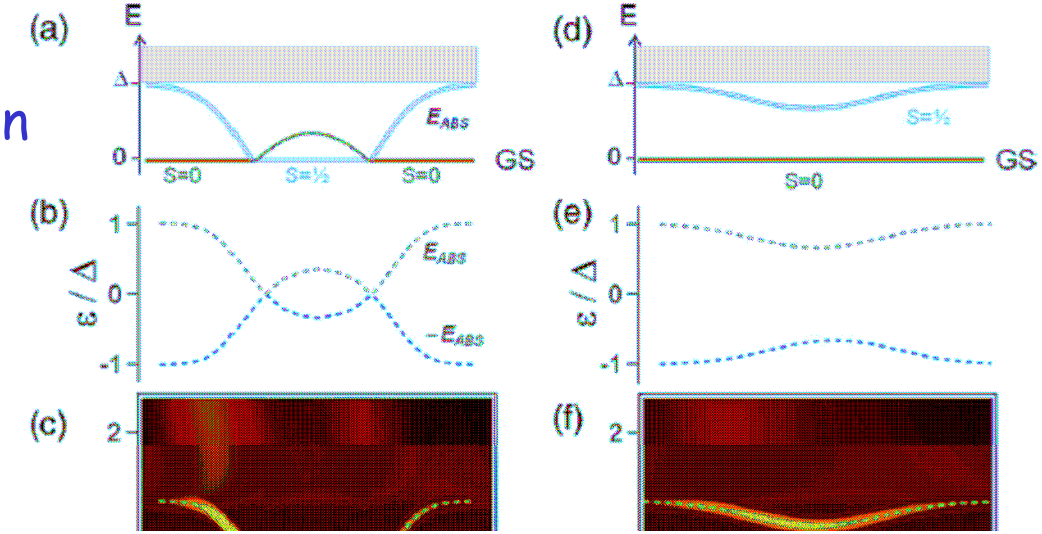
Pillet et al.



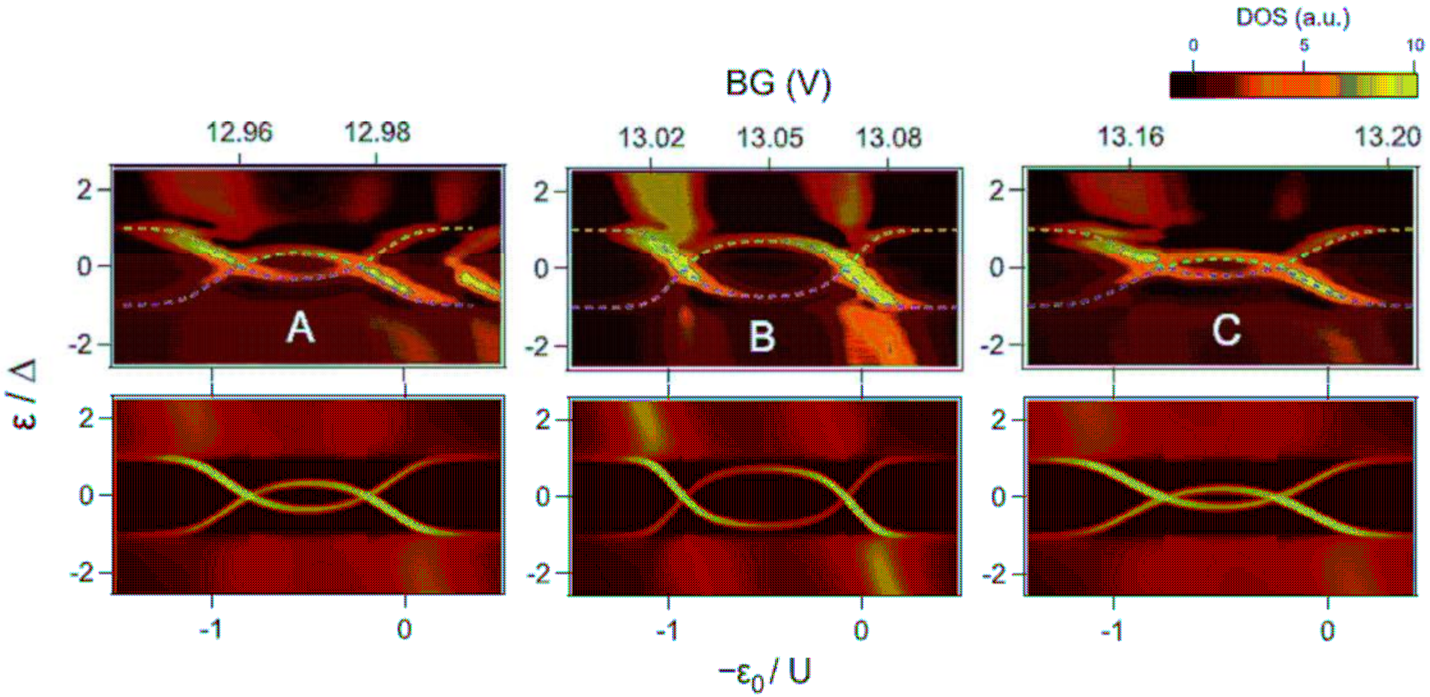
12.95
13.00
13.05
13.10
13.15
13.20
(a)
A
B
C
dI/dV (μS)
(b)
DOS (a.u.)
ε (meV)
BG (V)

Comparaison entre expériences et résultats obtenus par NRG

Transition
Singulet
Doublet



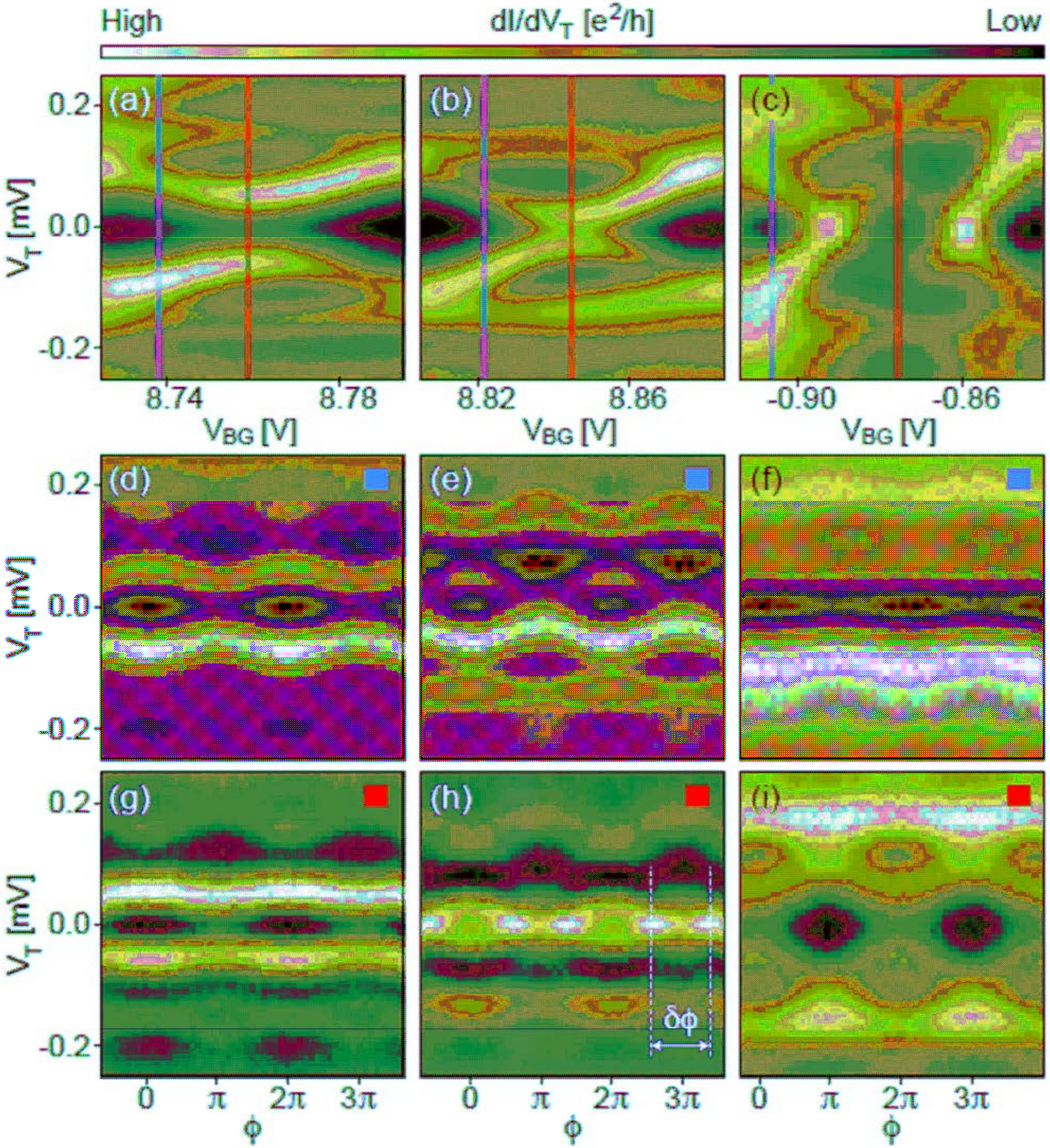
Etat
Singulet
stable



Pillet et al. PRB 2013

InAs Q dot

Croisement
des ABS
Avec la phase



Chang et al.
PRL 2013