

Radiative Heat Transfer at the Nanoscale

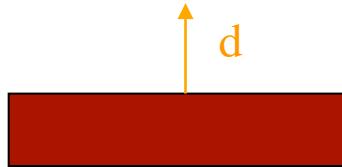
Jean-Jacques Greffet

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Institut Universitaire de France
CNRS**

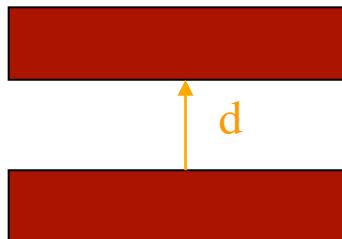
Parc National du Mont Orford, 16-27 septembre 2013

Outline of the lectures

1. Introduction



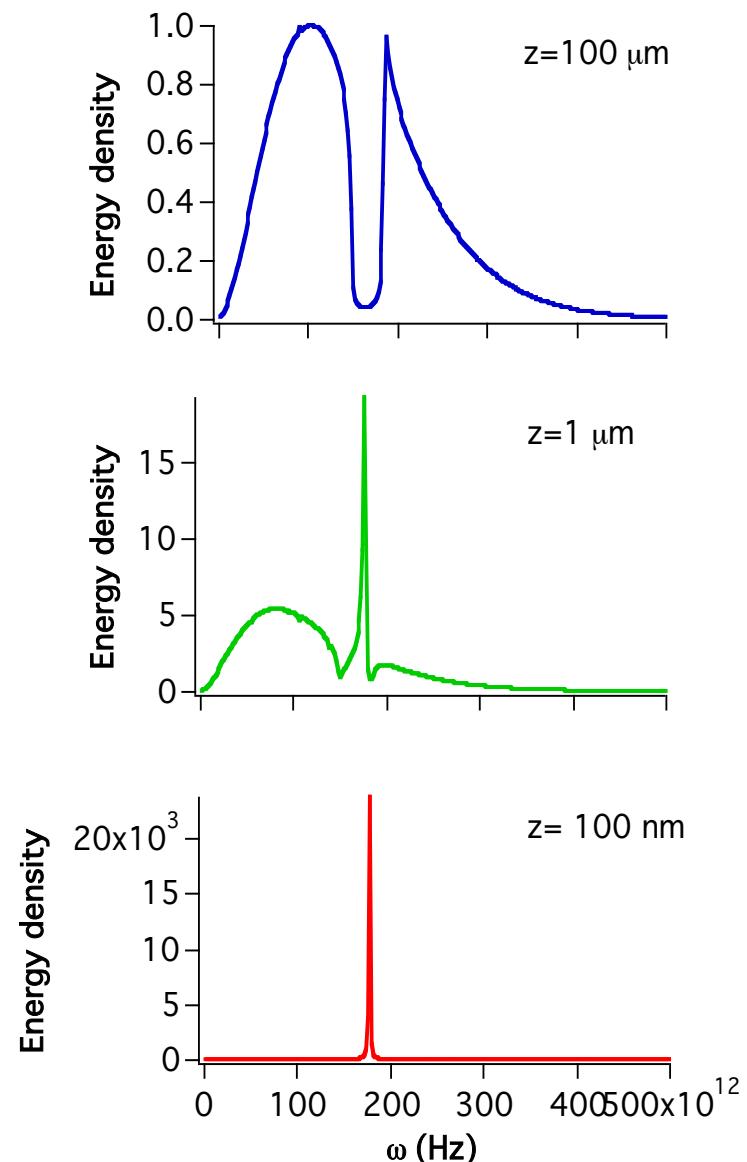
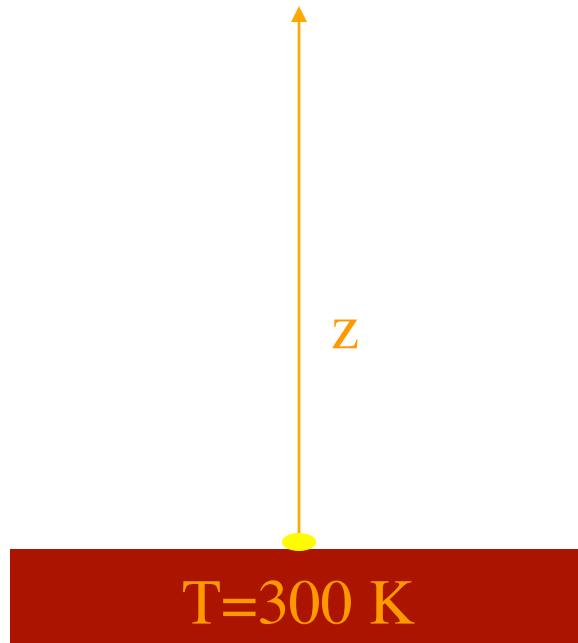
2. Heat radiation close to a surface



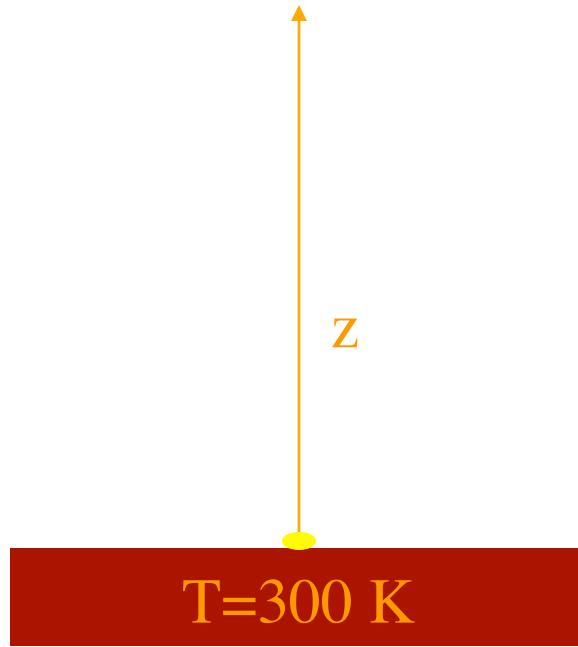
3. Heat transfer between two planes. Mesoscopic approach, fundamental limits and Applications

4. Tailoring spontaneous emission with surface waves

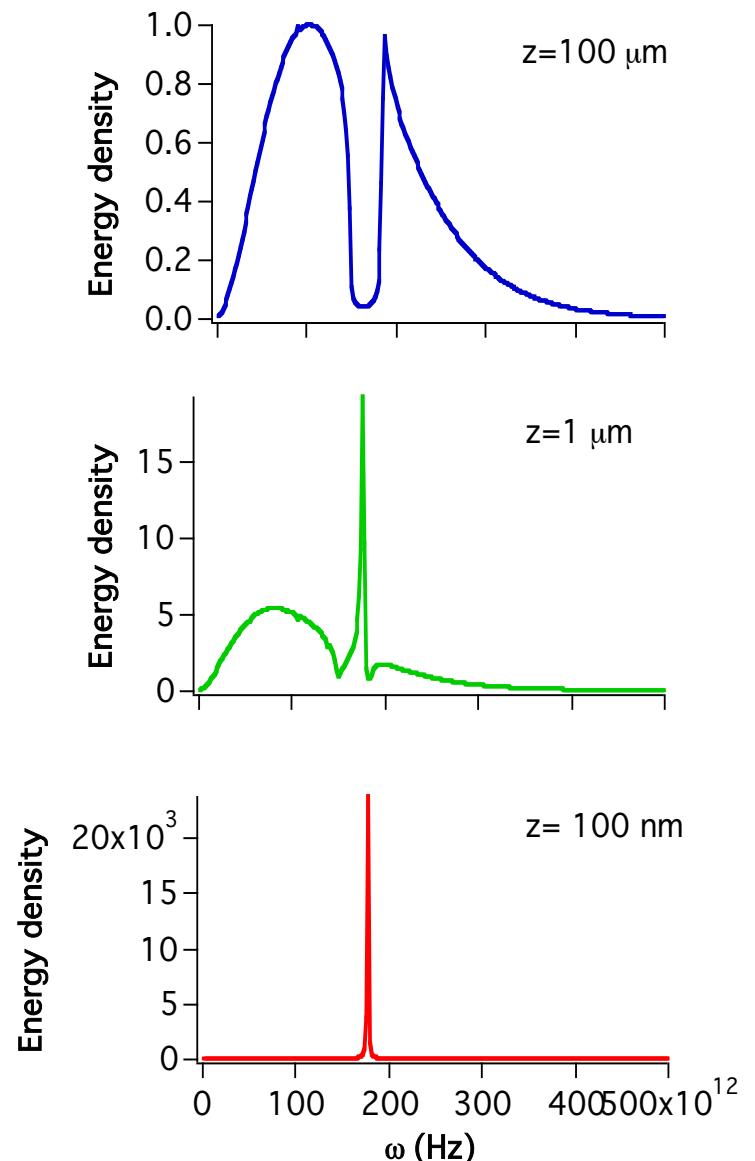
Density of energy near a SiC-vacuum interface



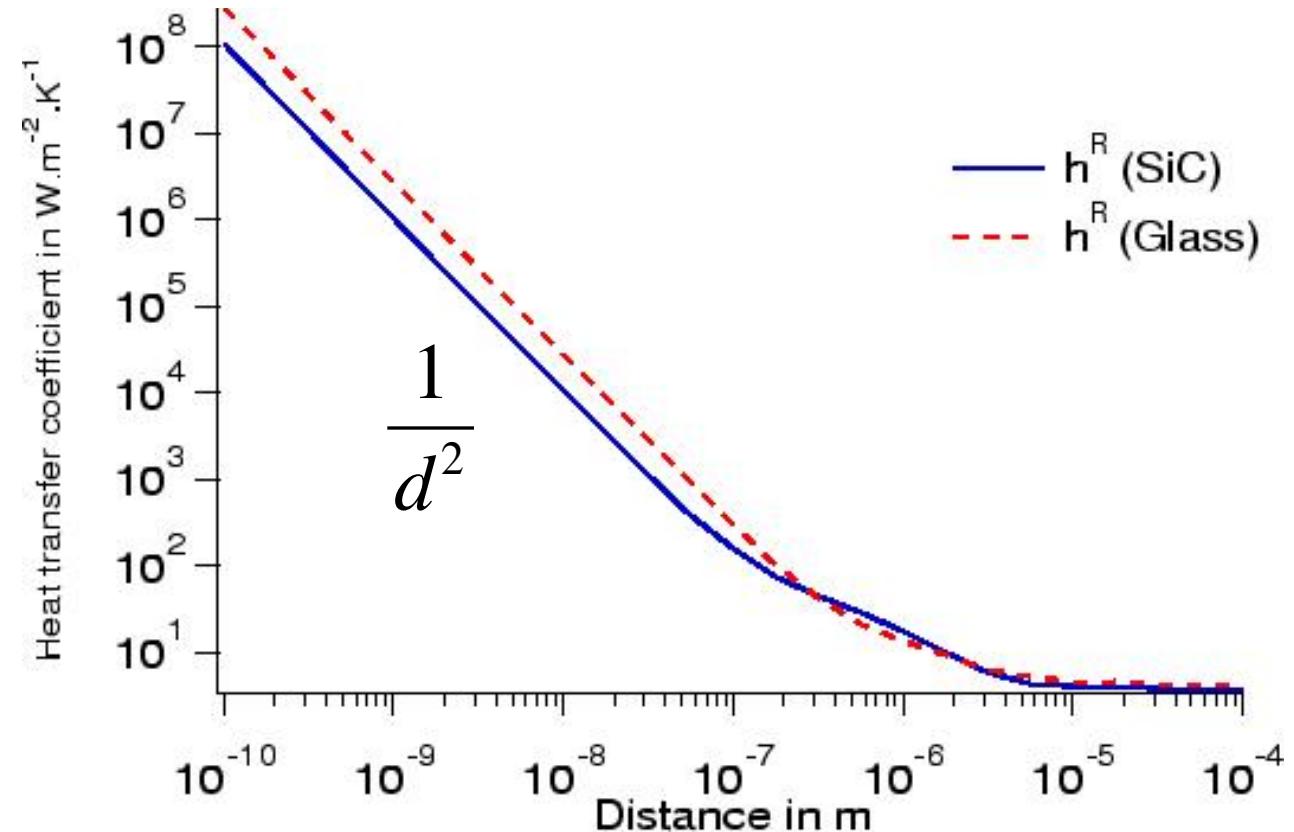
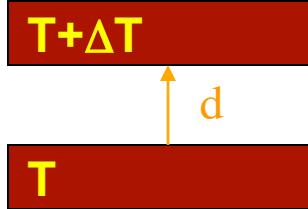
Density of energy near a SiC-vacuum interface



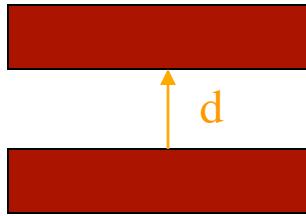
Energy density close to surfaces is orders of magnitude larger than blackbody energy density and monochromatic (temporally coherent)



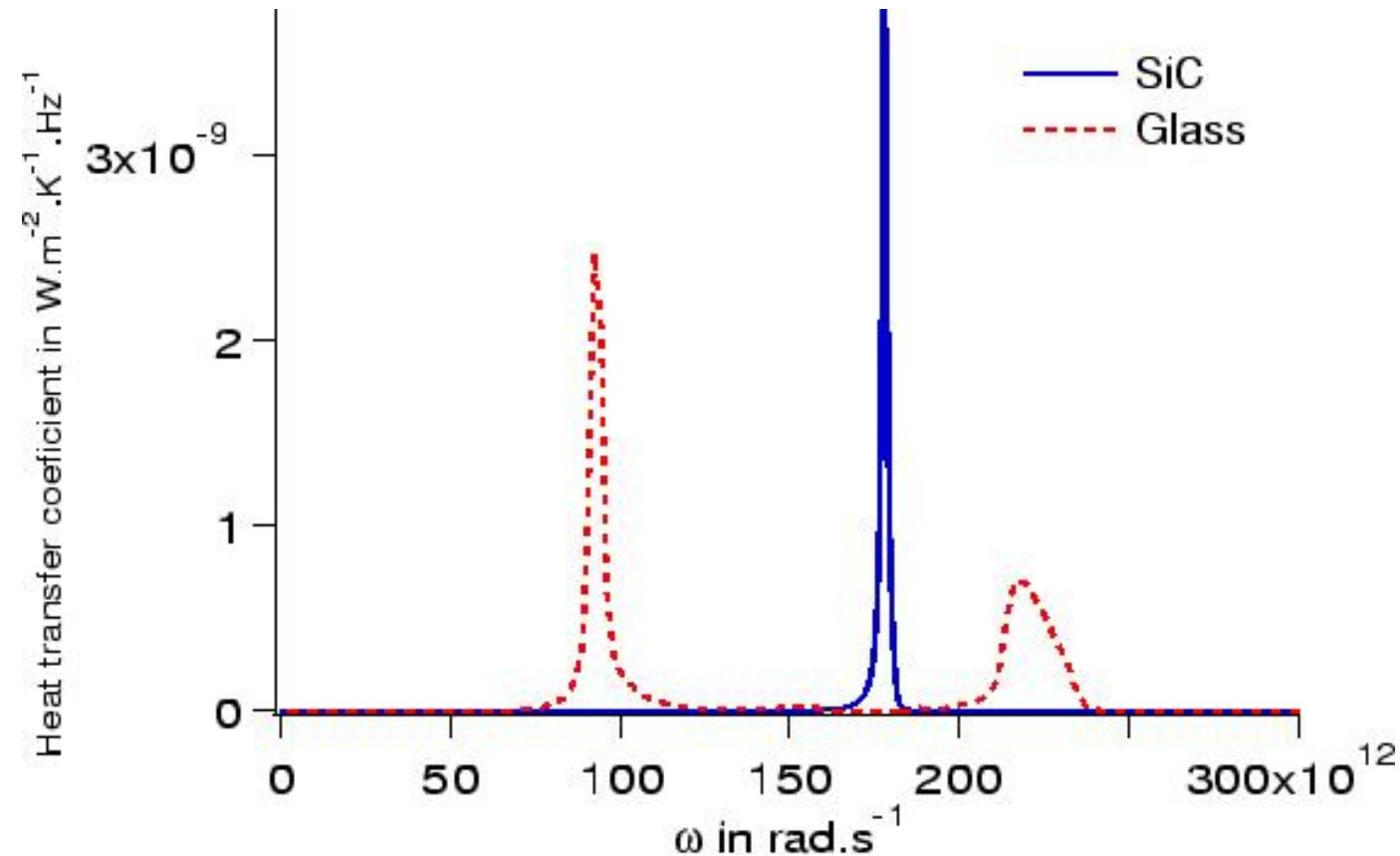
$$\Phi = h^R \Delta T$$



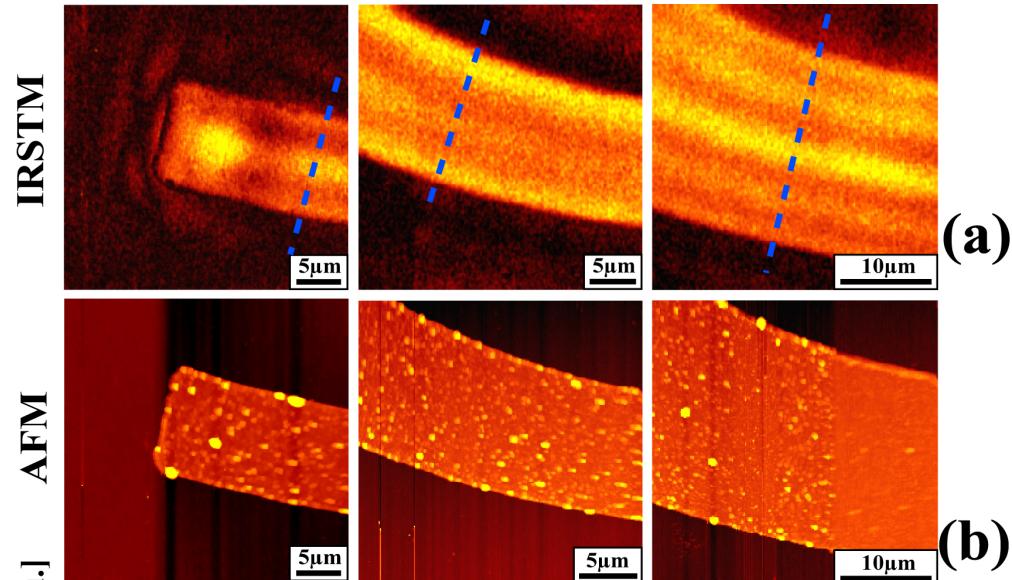
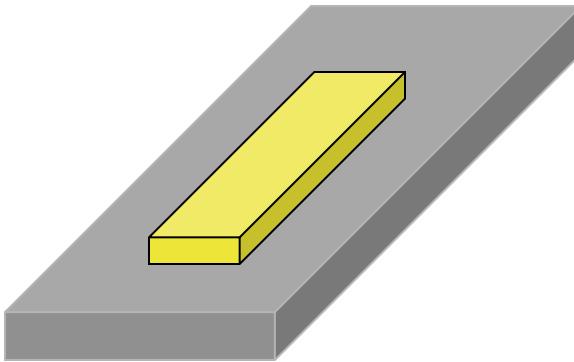
The flux is several orders of magnitude larger than Stefan-Boltzmann law.



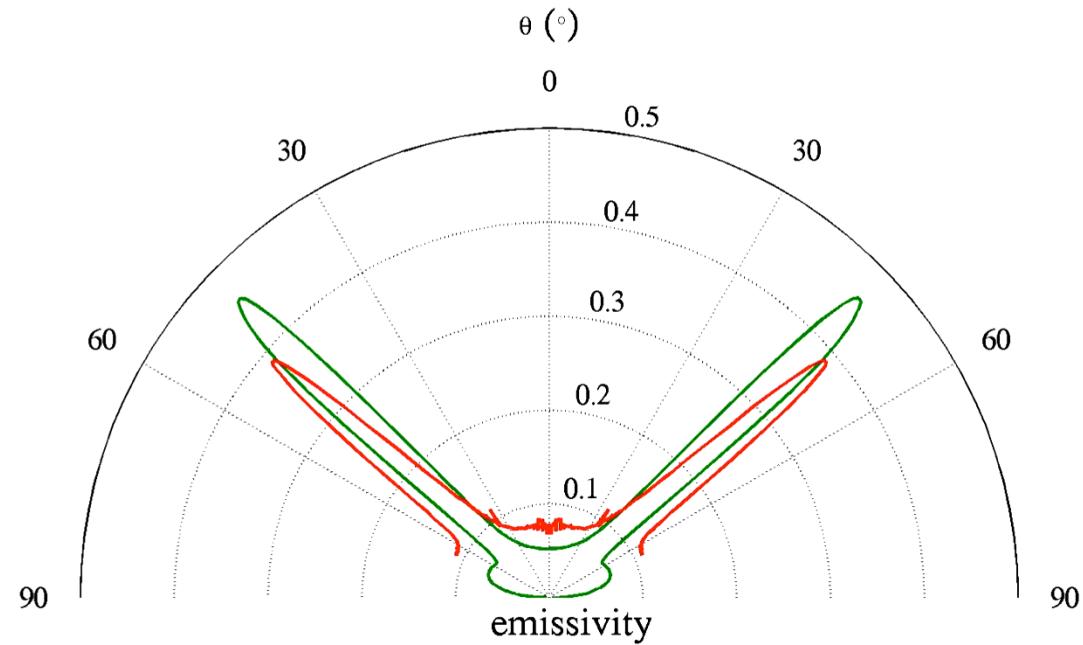
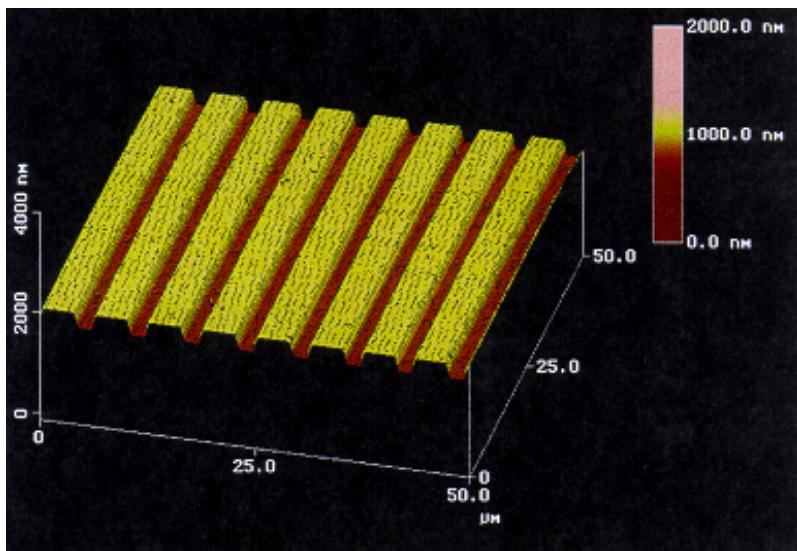
$d=10 \text{ nm}$, $T=300\text{K}$

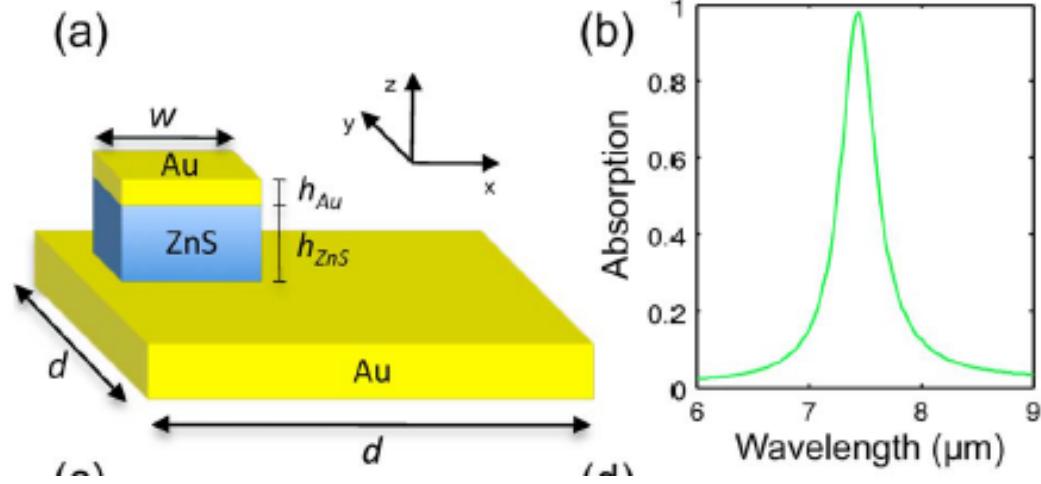
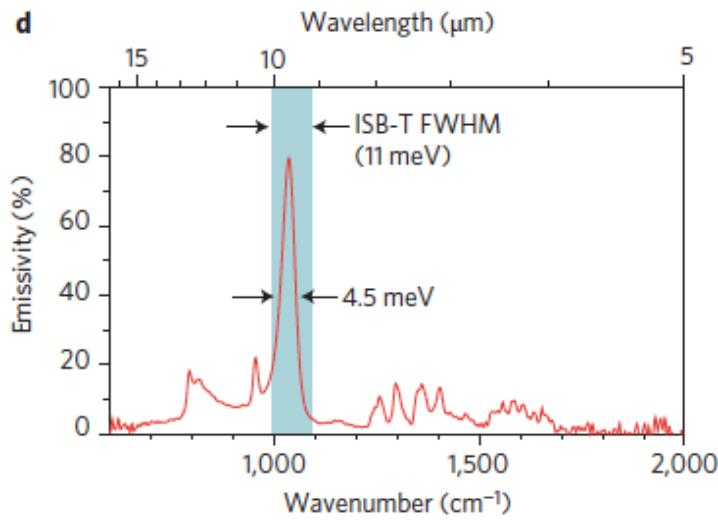


Heat transfer can be quasimonochromatic



**Thermally excited fields can be
spatially coherent in the near field.**





De Zoysa et al. , Nature Photonics (2012)

Bouchon et al. Opt. Lett. 37, p 1038 (2012)

Outline of the lectures

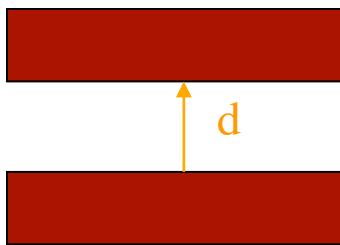
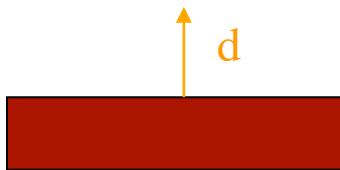
1. Introduction

2. Heat radiation close to a surface

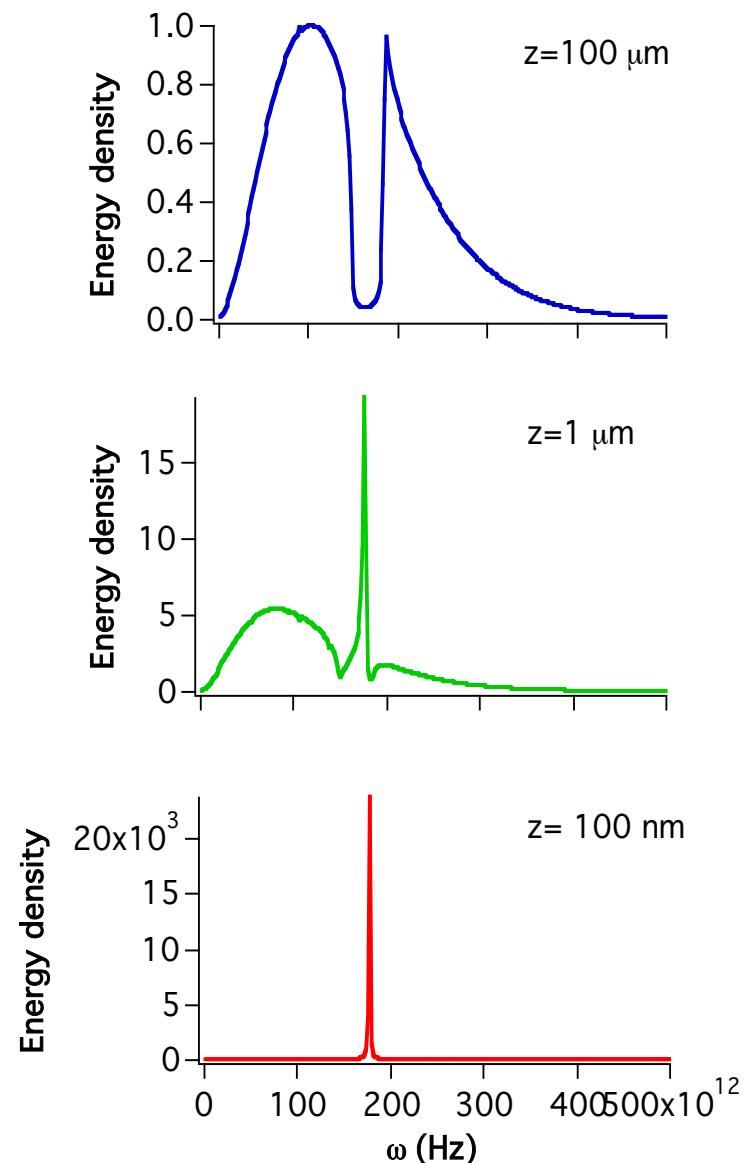
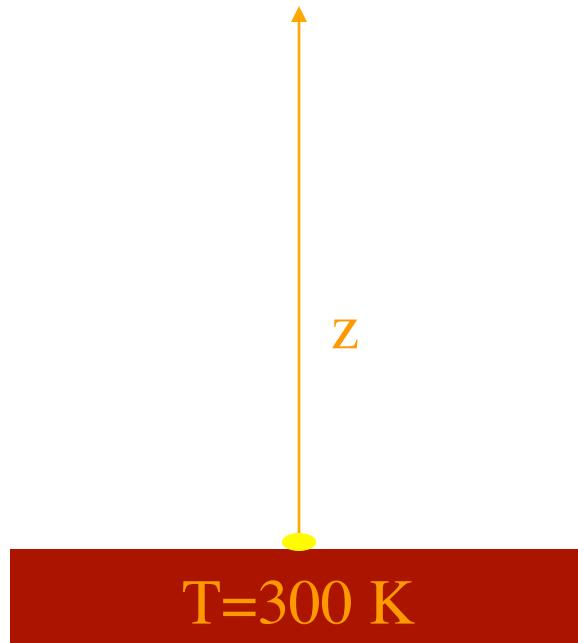
3. Heat transfer between two planes

4. Mesoscopic approach, fundamental limits and Applications

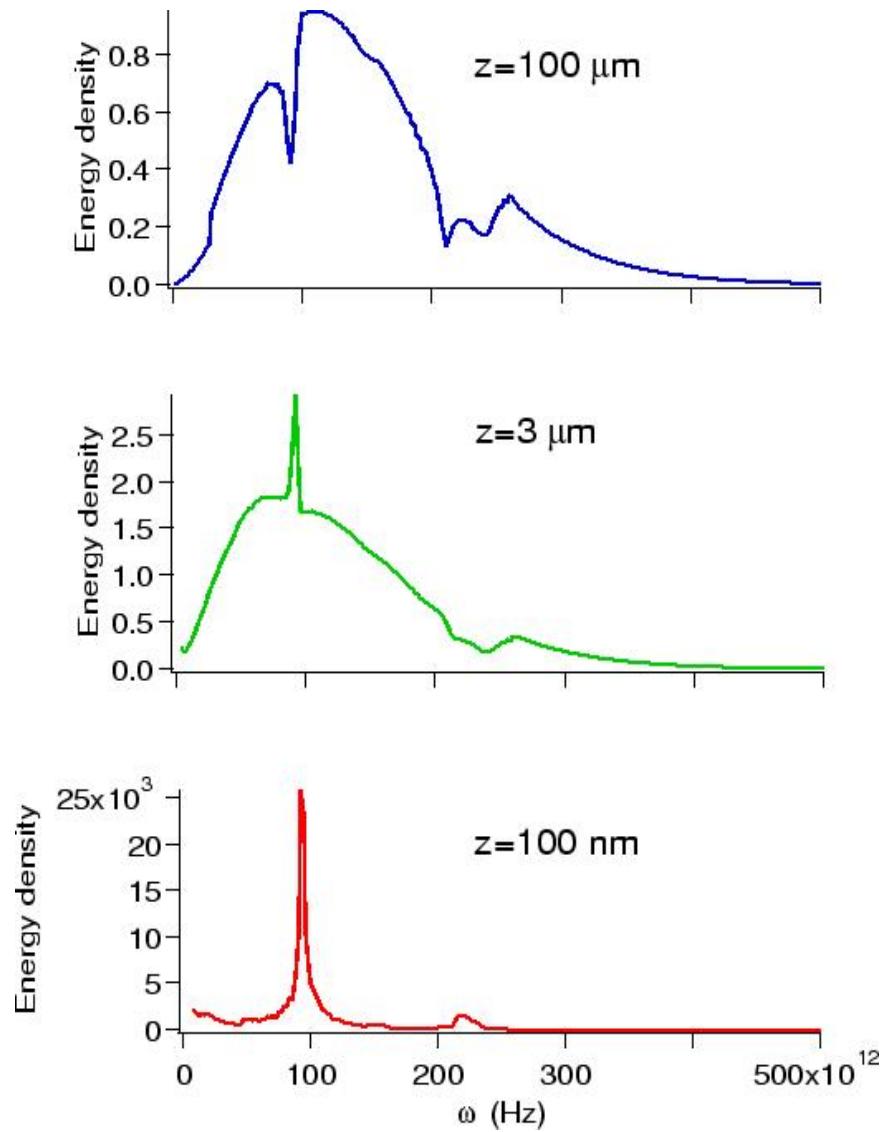
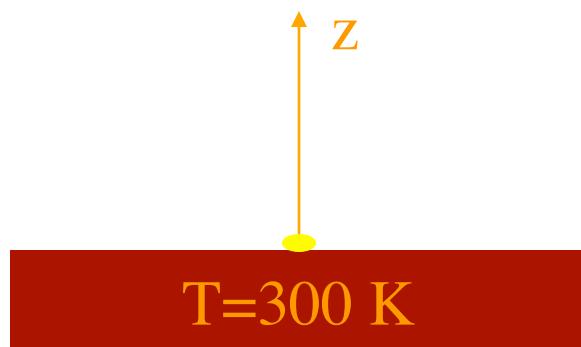
5. Tailoring spontaneous emission with surface waves



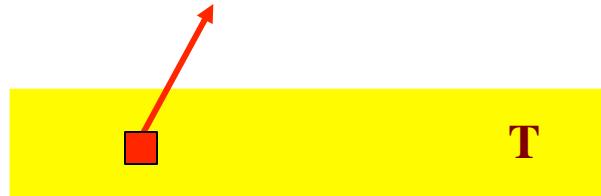
Density of energy near a SiC-vacuum interface



Density of energy near a Glass-vacuum interface



The fluctuational electrodynamics approach



- 1) The medium is assumed to be at local thermodynamic equilibrium volume element = random electric dipole

$$\mathbf{j}(\mathbf{r}, t) \text{ with } \langle \mathbf{j}(\mathbf{r}, t) \rangle = 0$$

- 2) Radiation of random currents = thermal radiation

$$\mathbf{E}(\mathbf{r}, \omega) = i \mu_o \omega \int \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) \cdot \mathbf{j}(\mathbf{r}', \omega) d\mathbf{r}'$$

3) Spatial correlation (cross-spectral density)

$$\langle E_j E^*_k \rangle(\mathbf{r}, \mathbf{r}', \omega) = \mu_o^2 \omega^2 \int G_{jm}(\mathbf{r}, \mathbf{r}_1) G_{kn}^*(\mathbf{r}', \mathbf{r}_2) \langle j_m(\mathbf{r}_1) j_n^*(\mathbf{r}_2) \rangle d\mathbf{r}_1 d\mathbf{r}_2$$

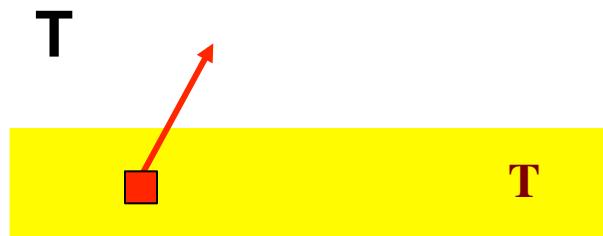
$$U(\mathbf{r}, \omega) \propto \langle |E(\mathbf{r}, \omega)|^2 \rangle$$

4) The current-density correlation function is given by the FD theorem

$$\langle j_m(\mathbf{r}_1) j_n^*(\mathbf{r}_2) \rangle = \frac{\omega}{\pi} \varepsilon_o \operatorname{Im} [\varepsilon(\omega)] \delta_{mn} \delta(\mathbf{r}_1 - \mathbf{r}_2) \frac{\hbar \omega}{\exp(\hbar \omega / k_B T) - 1}$$


Fluctuation

Absorption



Radiation of random currents = thermal radiation

$$\mathbf{E}(\mathbf{r}, \omega) = i \mu_o \omega \int \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) \cdot \mathbf{j}(\mathbf{r}', \omega) d\mathbf{r}'$$

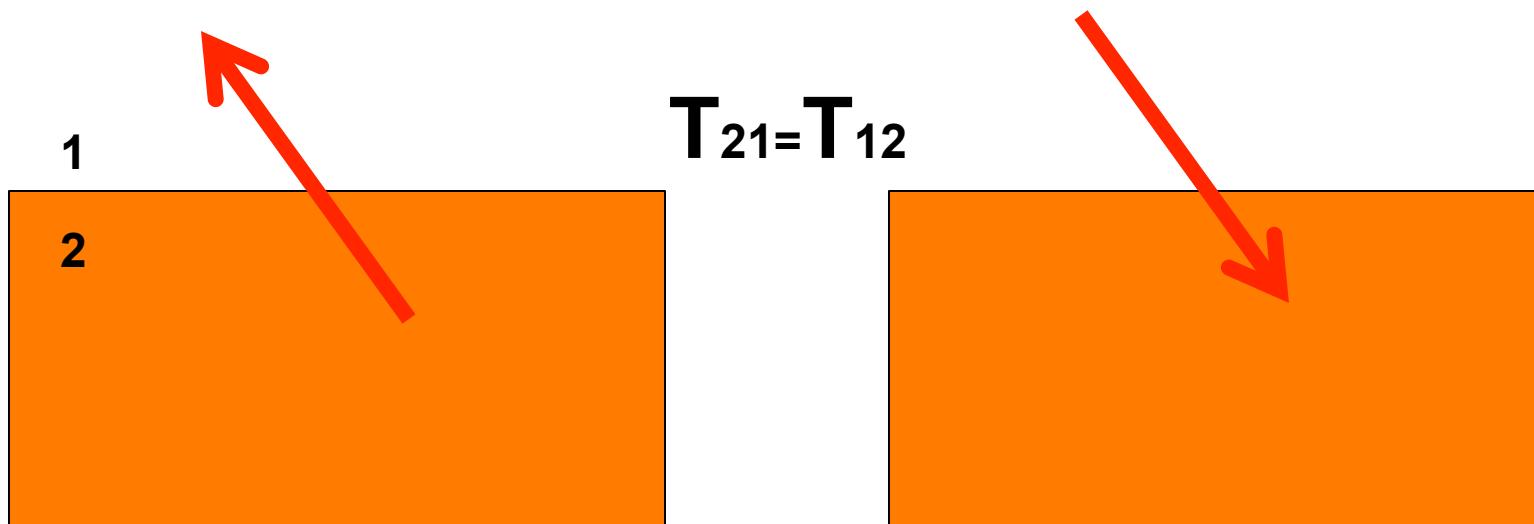
Emissivity= transmission factor

$$I_\omega^e(T) = \varepsilon I_\omega^{BB}(T) = T_F I_\omega^{BB}(T)$$

Physical meaning of emissivity and absorptivity

Kirchhoff's law:

$$\varepsilon = \alpha$$

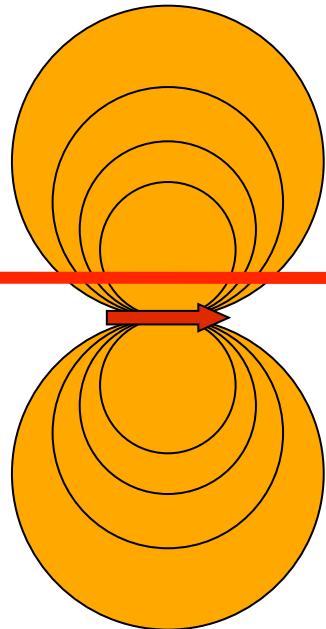


$$I_\omega^e(T) = \varepsilon I_\omega^{BB}(T) = T_F I_\omega^{BB}(T) = \alpha I_\omega^{BB}(T)$$

Basic concepts of near field

What is so special about near field ?

Back to basics : dipole radiation



$$\mathbf{E}_p = \frac{k_0^2}{4\pi\epsilon_0} \frac{e^{ik_0 r}}{r} \left[(\mathbf{I} - \mathbf{u}\mathbf{u}) + \left(\frac{i}{k_0 r} - \frac{1}{(k_0 r)^2} \right) (\mathbf{I} - 3\mathbf{u}\mathbf{u}) \right] \mathbf{p}$$

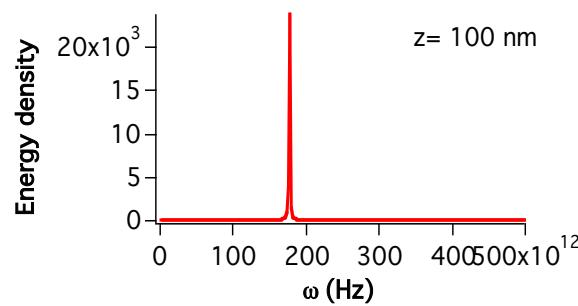
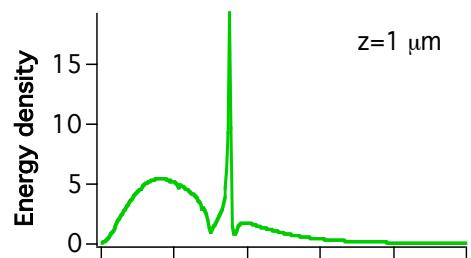
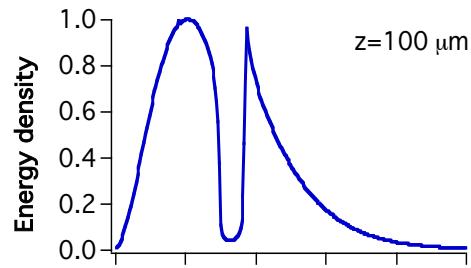
$1/r$ terms
Radiation

$1/r^2$ and $1/r^3$ terms
Near field

Near field: $k_0 r \ll 1$, $k_0 = 2\pi/\lambda$

Understanding near-field thermal emission

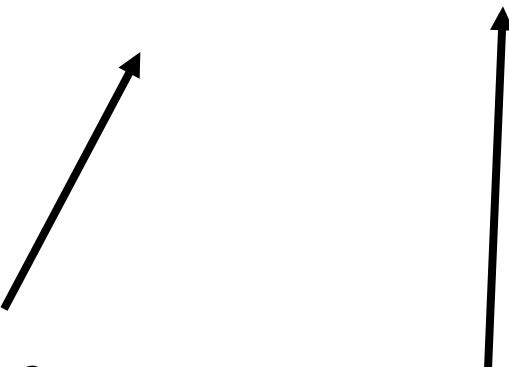
$$E_p = \frac{k_0^2}{4\pi\epsilon_0} \frac{e^{ik_0r}}{r} \left[(I - uu) + \left(\frac{i}{k_0r} - \frac{1}{(k_0r)^2} \right) (I - 3uu) \right] p$$



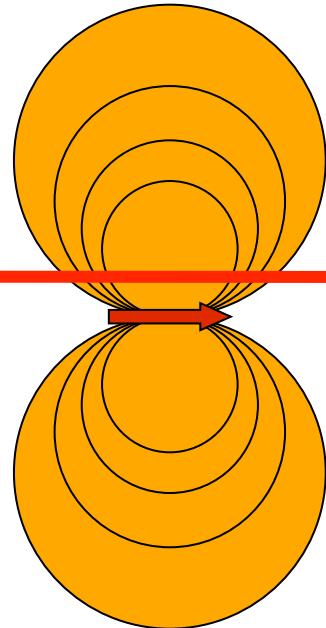
**1/r² and 1/r³ terms
Near field**

Dipole spectral resonances

Near field: $k_0 r \ll 1$, $k_0 = 2\pi/\lambda$



Back to basics : dipole radiation



$$\mathbf{E}_p = \frac{k_0^2}{4\pi\epsilon_0} \frac{e^{ik_0 r}}{r} \left[(\mathbf{I} - \mathbf{uu}) + \left(\frac{i}{k_0 r} - \frac{1}{(k_0 r)^2} \right) (\mathbf{I} - 3\mathbf{uu}) \right] \mathbf{p}$$

$1/r$ terms
Radiation

$1/r^2$ and $1/r^3$ terms
Near field

Near field: $k_0 r \ll 1$, $k_0 = 2\pi/\lambda$

$$\frac{\exp[ikR]}{R} = \frac{i}{2\pi} \iint \frac{dk_x dk_y}{k_z} \exp[i(k_x x + k_y y + k_z |z|)]$$

$$k_{\perp} < \frac{\omega}{c} \quad ; \quad k_z = \sqrt{\frac{\omega^2}{c^2} - k_{\perp}^2} \quad ; \quad k_{\perp} = (k_x, k_y)$$

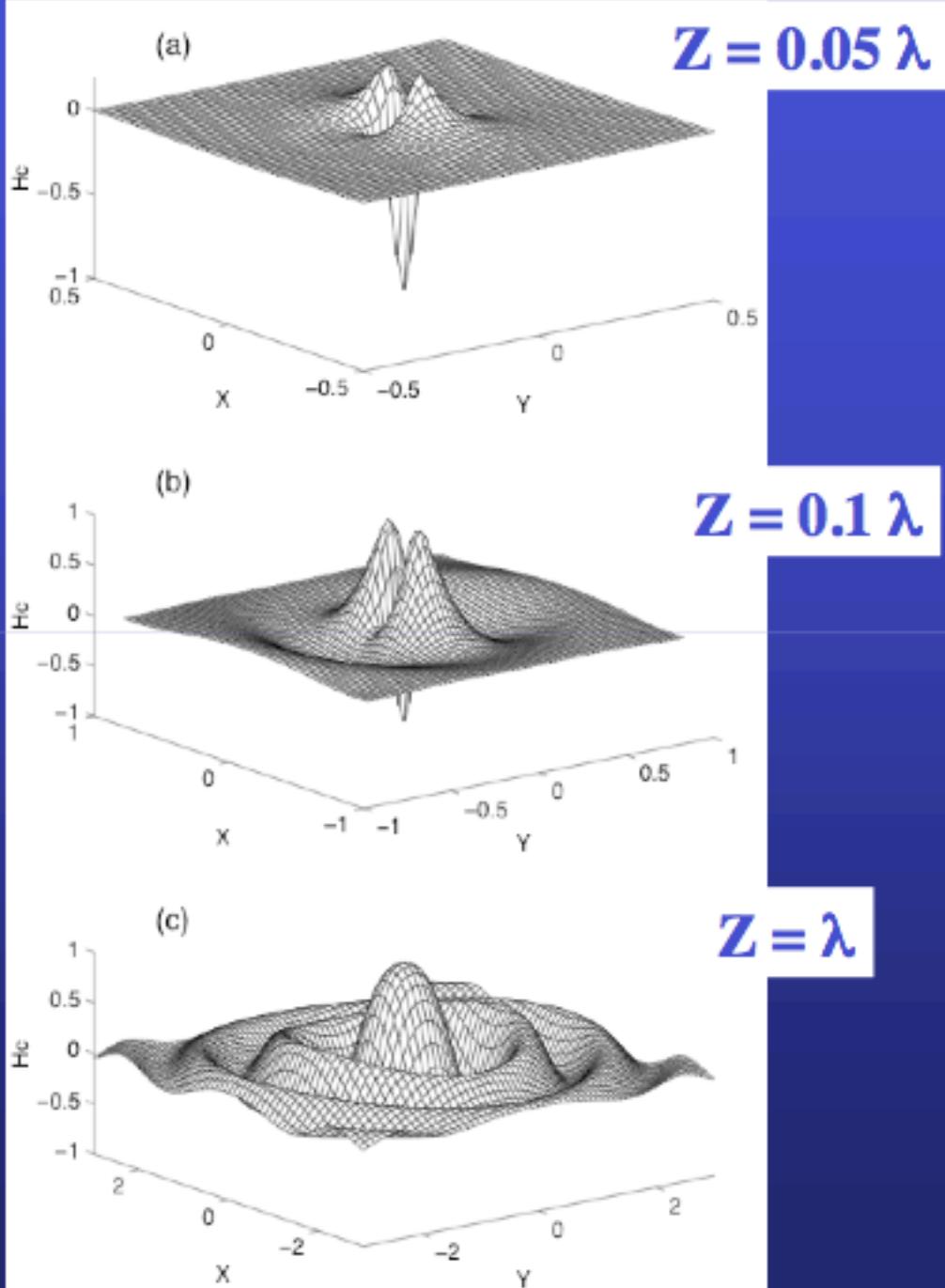
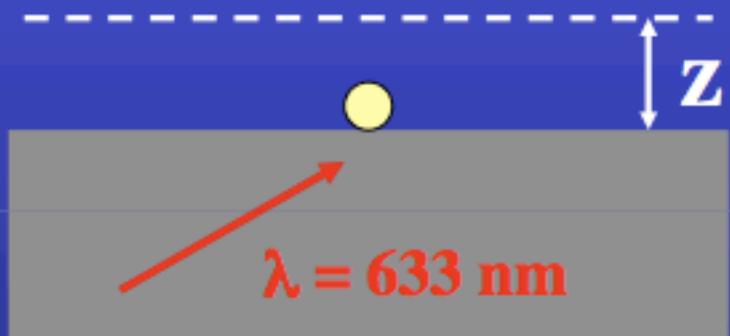
$$k_{\perp} > \frac{\omega}{c} \quad ; \quad k_z = i \sqrt{\left| \frac{\omega^2}{c^2} - k_{\perp}^2 \right|}$$

$$k_{\perp} \gg \frac{\omega}{c} \quad ; \quad k_z \approx ik_{\perp}$$

$$\exp[ik_z |z|] \approx \exp[-k_{\perp} z]$$

Take home message: large k are confined to distances $1/k$.

Evanescence waves filtering



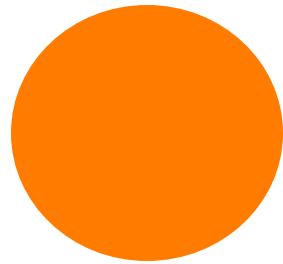
Energy density:
in vacuum,
close to a nanoparticle,
close to a surface

$$g(\omega) = \frac{\omega^2}{\pi^2 c^3}$$

$$U = \int_0^{\infty} \frac{\omega^2}{\pi^2 c^3} \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1} d\omega$$

Questions to be answered :

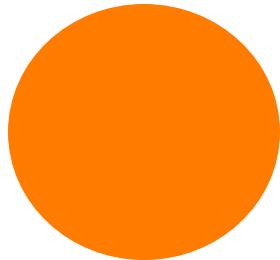
- Is the field orders of magnitude larger close to particles ?
- If yes, why ?
- Is the thermal field quasi monochromatic ? If yes, why ?



$$\mathbf{E}_p = \frac{k_0^2}{4\pi\epsilon_0} \frac{e^{ik_0r}}{r} \left[(\mathbf{I} - \mathbf{u}\mathbf{u}) + \left(\frac{i}{k_0r} - \frac{1}{(k_0r)^2} \right) (\mathbf{I} - 3\mathbf{u}\mathbf{u}) \right] \mathbf{p}$$

$$\langle p_m(\mathbf{r}_1)p_n^*(\mathbf{r}_2) \rangle = \frac{2}{\pi\omega} \varepsilon_o \text{Im}[\alpha(\omega)] \delta_{mn} \delta(\mathbf{r}_1 - \mathbf{r}_2) \frac{\hbar\omega}{\exp(\hbar\omega/k_B T) - 1}$$

Black body radiation close to a nanoparticle



$$\mathbf{E}_p = \frac{k_0^2}{4\pi\epsilon_0} \frac{e^{ik_0r}}{r} \left[(\mathbf{I} - \mathbf{u}\mathbf{u}) + \left(\frac{i}{k_0r} - \frac{1}{(k_0r)^2} \right) (\mathbf{I} - 3\mathbf{u}\mathbf{u}) \right] \mathbf{p}$$

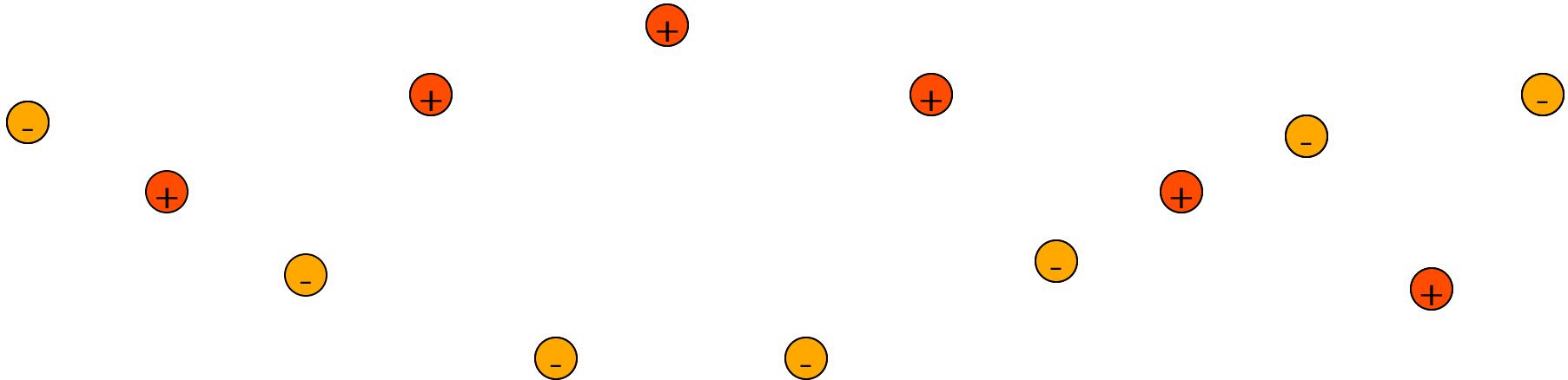
$$\langle p_m(\mathbf{r}_1)p_n^*(\mathbf{r}_2) \rangle = \frac{2}{\pi\omega} \epsilon_o \text{Im}[\alpha(\omega)] \delta_{mn} \delta(\mathbf{r}_1 - \mathbf{r}_2) \frac{\hbar\omega}{\exp(\hbar\omega/k_B T) - 1}$$

1. The particle is a random dipole.
2. The field increases close to the particle: electrostatic field !
3. The field may have a resonance, plasmon resonance.

Where are these (virtual) modes coming from ?

Strong coupling between material modes and photons produces

polaritons: Half a photon and half a phonon/exciton/electron.



Estimation of the number of electromagnetic modes in vacuum:

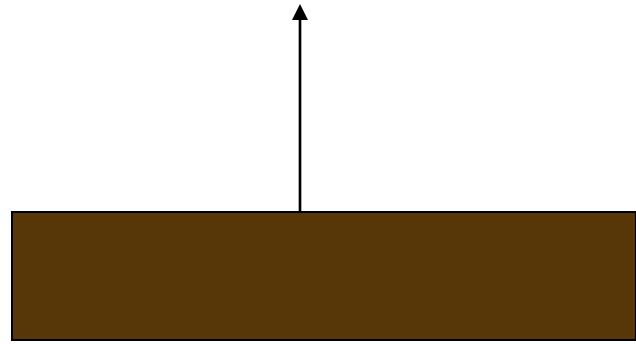
$$\frac{N}{V} = \int_0^{\omega} g(\omega') d\omega' = \frac{\omega^3}{3\pi^2 c^3} \quad N \approx \frac{V}{\lambda^3}$$

Estimation of the number of vibrational modes (phonons):

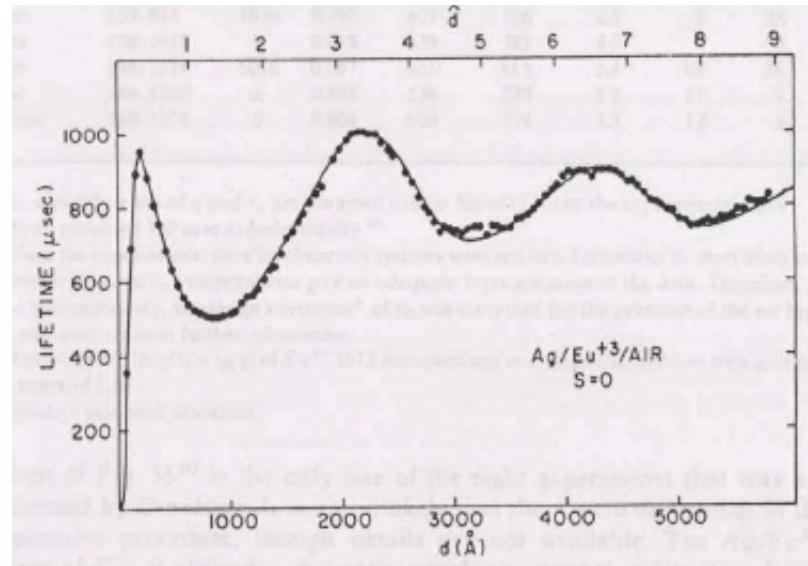
$$N \approx \frac{V}{a^3}$$

The electromagnetic field inherits the DOS of matter degrees of freedom

**Can we define a (larger and local)
electromagnetic density of states
close to a particle or a surface ?**

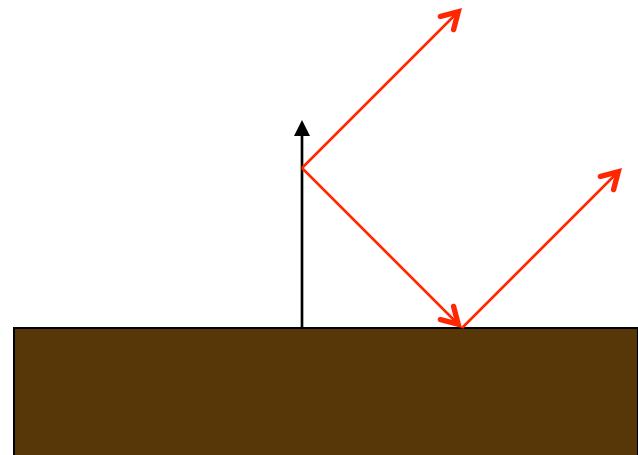


Lifetime of Europium 3+ above a silver mirror

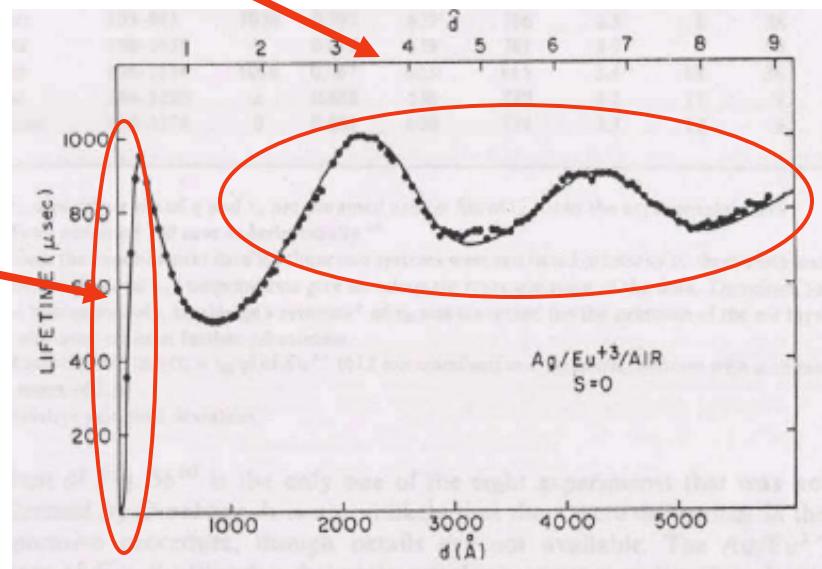


Drexhage, 1970

- Interference effects
- (microcavity type)



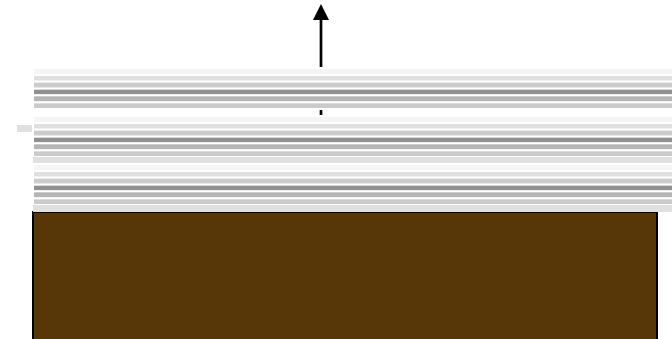
Near-field contribution



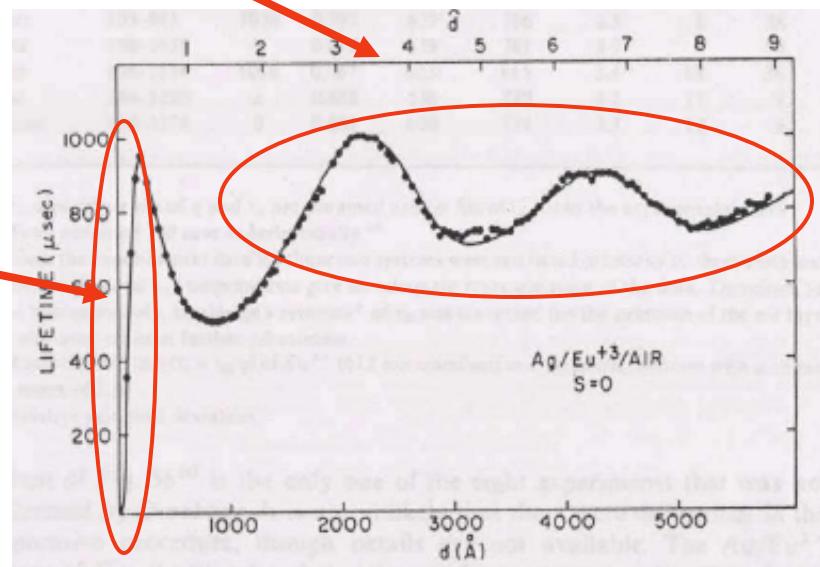
Drexhage, 1970

LDOS above a surface

- Interference effects
- (microcavity type)

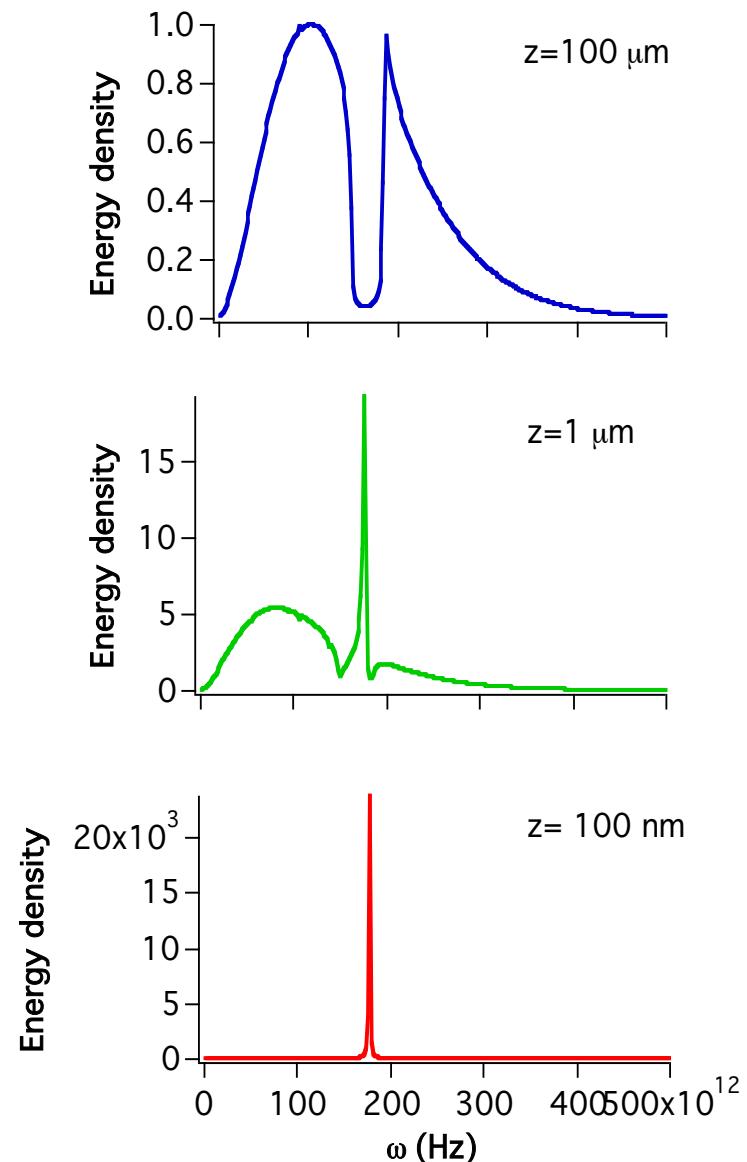
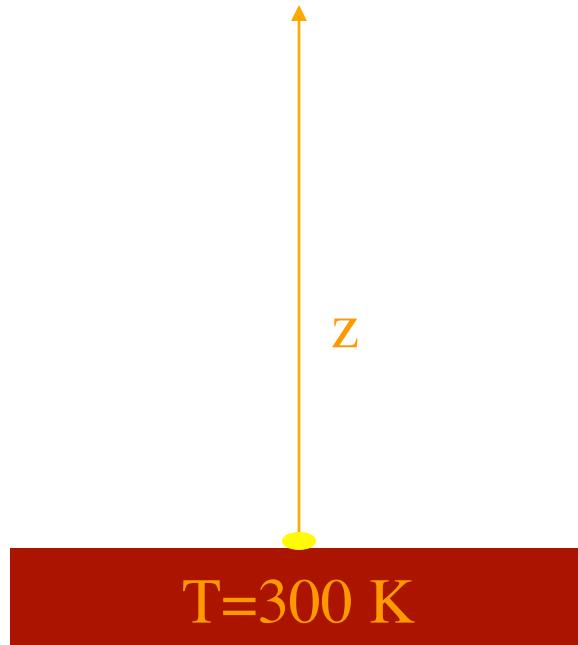


Near-field contribution



Drexhage, 1970

Density of energy near a SiC-vacuum interface



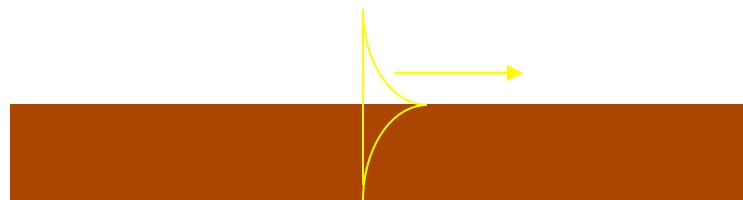
Energy density close to surfaces is orders of magnitude larger than blackbody energy density and monochromatic.

PRL, 85 p 1548 (2000)

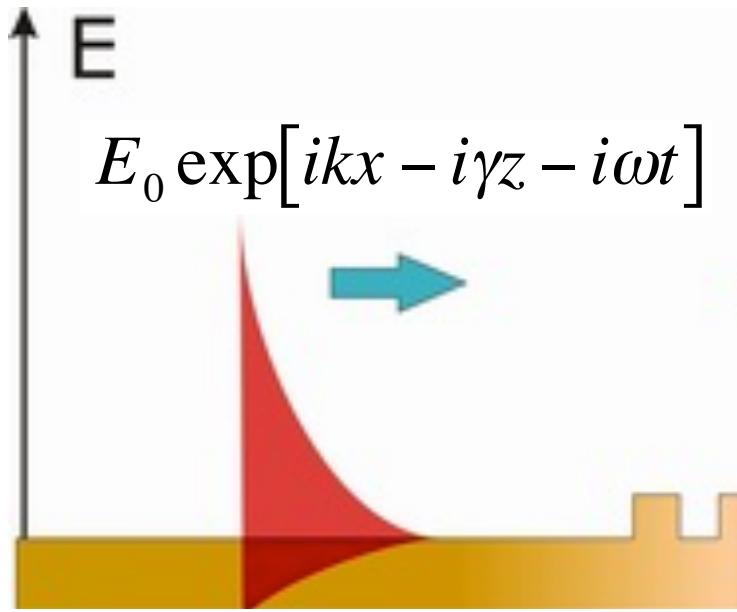
The density of energy is the product of

- the density of states,
- the energy $h\nu$
- the Bose Einstein distribution.

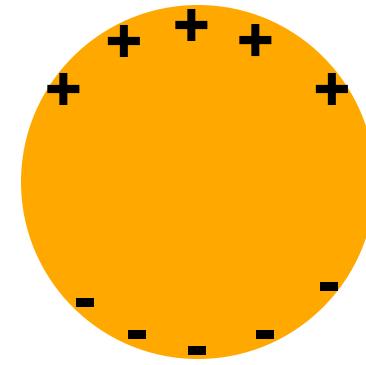
The density of states can diverge due to the presence of surface waves :
Surface **phonon**-polaritons.



What is a surface plasmon ?



$$E_0 \exp[ikx - i\gamma z - i\omega t]$$



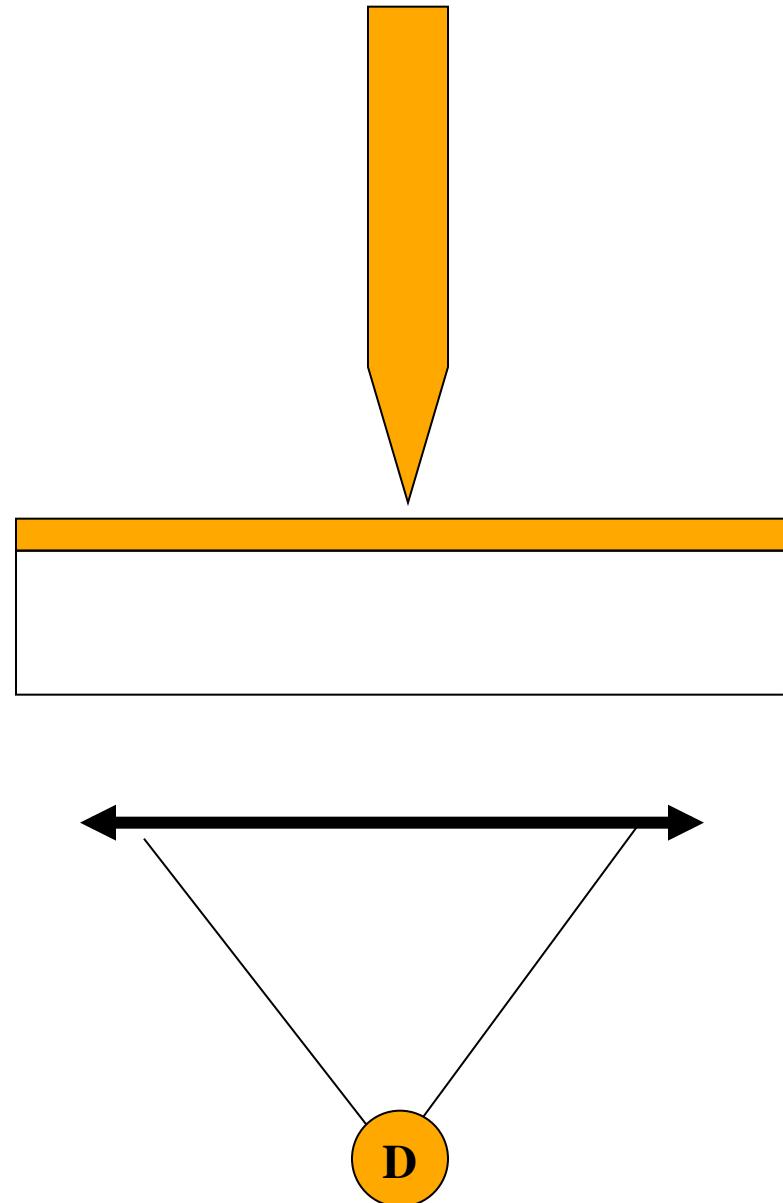
$$r_F = \frac{\varepsilon_2 k_{z1} - \varepsilon_1 k_{z2}}{\varepsilon_2 k_{z1} + \varepsilon_1 k_{z2}}$$

$$\alpha = 4\pi a^3 \frac{\varepsilon_m(\omega) - 1}{\varepsilon_m(\omega) + 2}$$

$$\varepsilon_2 k_{z1} + \varepsilon_1 k_{z2} = 0$$

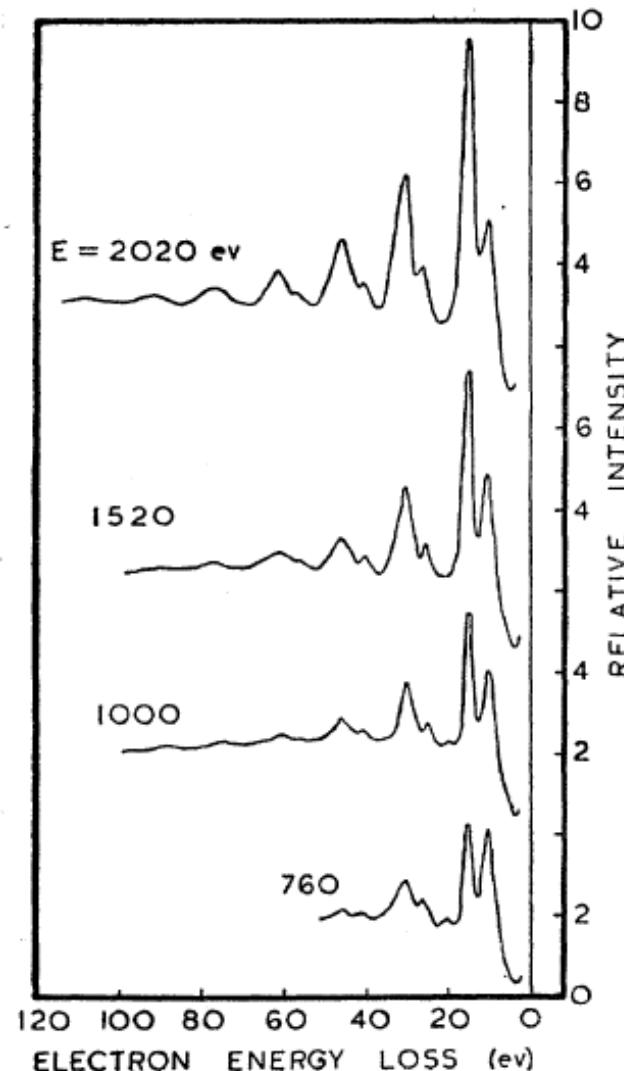
$$\varepsilon_m(\omega) + 2 = 0$$

Exciting a plasmon with a small aperture



Credit: Alexandre Bouhelier, D. Pohl

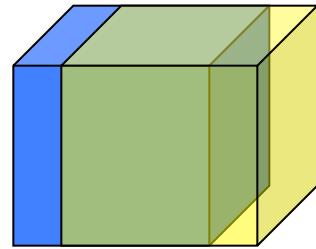
Surface plasmons



Qualitative introduction

Exemple : thin metallic film

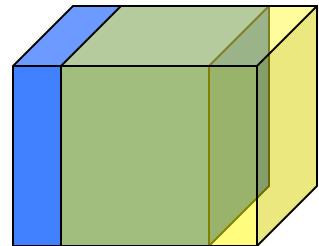
collective oscillation of the electrons



Qualitative introduction

Exemple : thin metallic film

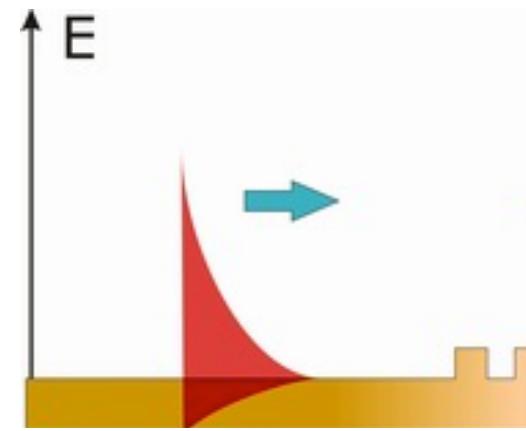
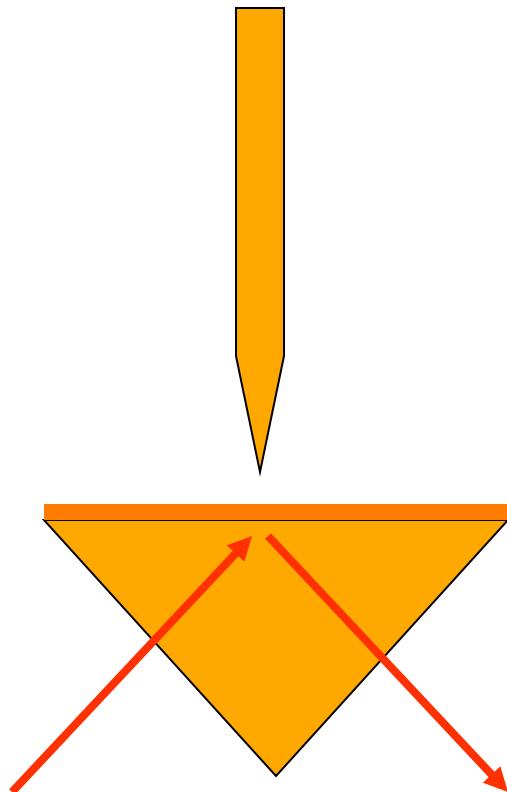
collective oscillation of the electrons



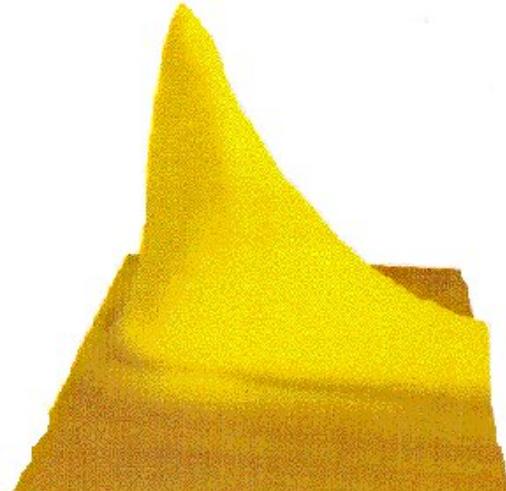
$$\omega_p^2 = \frac{ne^2}{m\epsilon_0}$$

$$m\ddot{x} = -eE - \gamma m\dot{x} = -n \frac{e^2}{\epsilon_0} x - \gamma m\dot{x}$$

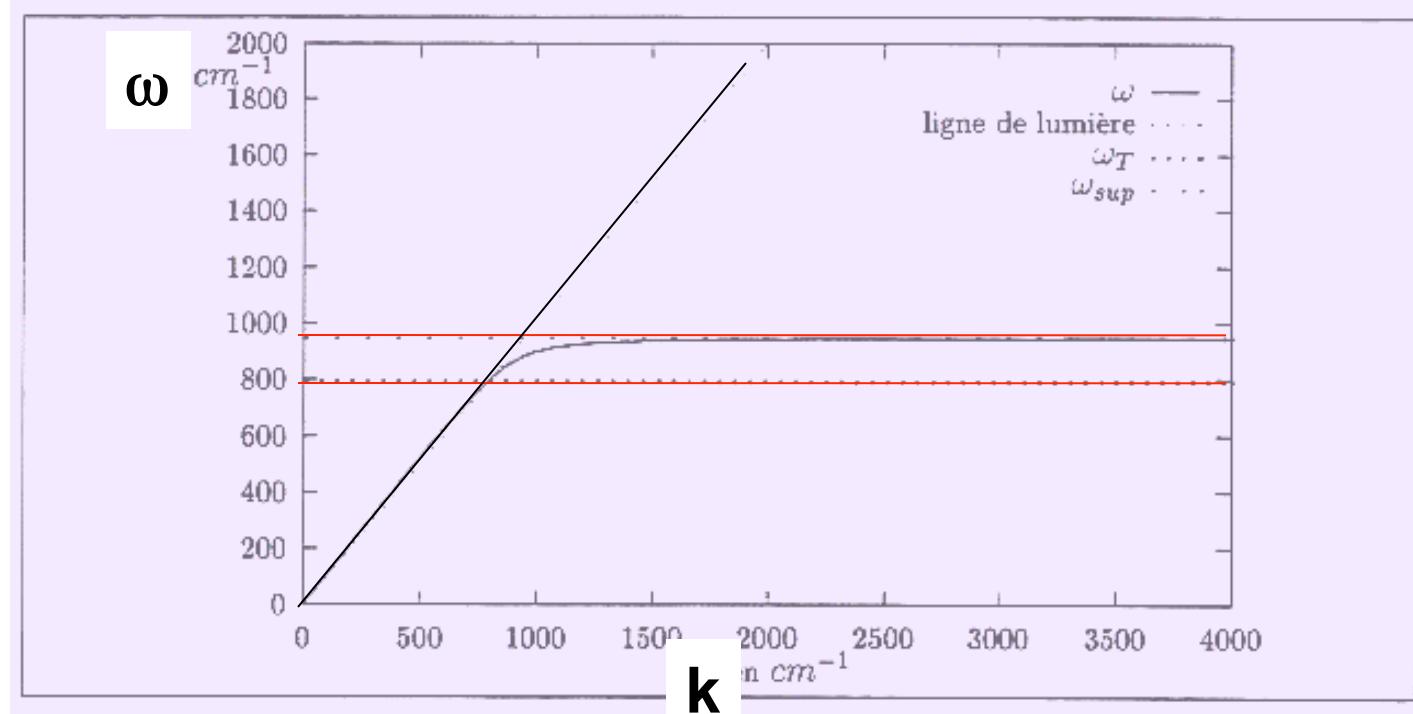
λ_i (μm)	0.633	1	10	36
δ_x (μm)	9.8	91.6	38 880	504 243
δ_{z1} (μm)	0.165	0.51	57.3	702.67
δ_{z2} (μm)	0.014	0.012	0.011	0.013



$$E_x \exp[ikx - i\gamma z - i\omega t]$$



$$k = \frac{\omega}{c} \sqrt{\frac{\epsilon(\omega)}{\epsilon(\omega) + 1}}$$



It is seen that the number of modes diverges for a particular frequency.

$$U(\mathbf{r}, \omega) = \rho(\mathbf{r}, \omega) \frac{\hbar\omega}{\exp(\hbar\omega/k_B T) - 1}.$$

$$\begin{aligned} U(\mathbf{r}, \omega) = & 4 \left((\epsilon_0/2) \sum_{i=1,3} \mathcal{E}_{ii}(\mathbf{r}, \mathbf{r}, \omega) \right. \\ & \left. + (\mu_0/2) \sum_{i=1,3} \mathcal{H}_{ii}(\mathbf{r}, \mathbf{r}, \omega) \right) \end{aligned}$$

LDOS is also used to deal with spontaneous emission using Fermi golden rule

$$U(\mathbf{r},\omega) = 4 \left((\epsilon_0/2) \sum_{i=1,3} \mathcal{E}_{ii}(\mathbf{r},\mathbf{r},\omega) + (\mu_0/2) \sum_{i=1,3} \mathcal{H}_{ii}(\mathbf{r},\mathbf{r},\omega) \right)$$

$$\begin{aligned} \mathcal{E}_{ij}(\mathbf{r},\mathbf{r}',t-t') &= \frac{1}{2\pi} \int d\omega \mathcal{E}_{ij}(\mathbf{r},\mathbf{r}',\omega) e^{-i\omega(t-t')} \\ &= \langle E_i(\mathbf{r},t) E_j^*(\mathbf{r}',t') \rangle, \end{aligned}$$

$$\begin{aligned} \mathcal{H}_{ij}(\mathbf{r},\mathbf{r}',t-t') &= \frac{1}{2\pi} \int d\omega \mathcal{H}_{ij}(\mathbf{r},\mathbf{r}',\omega) e^{-i\omega(t-t')} \\ &= \langle H_i(\mathbf{r},t) H_j^*(\mathbf{r}',t') \rangle. \end{aligned}$$

Problem: deriving the correlation function of the fields

Solution: FD theorem

Green tensor is (also) a linear response coefficient !

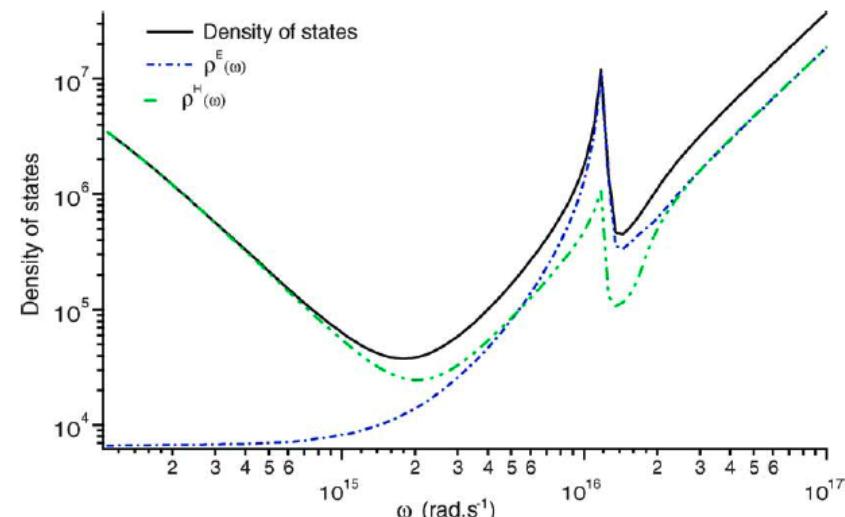
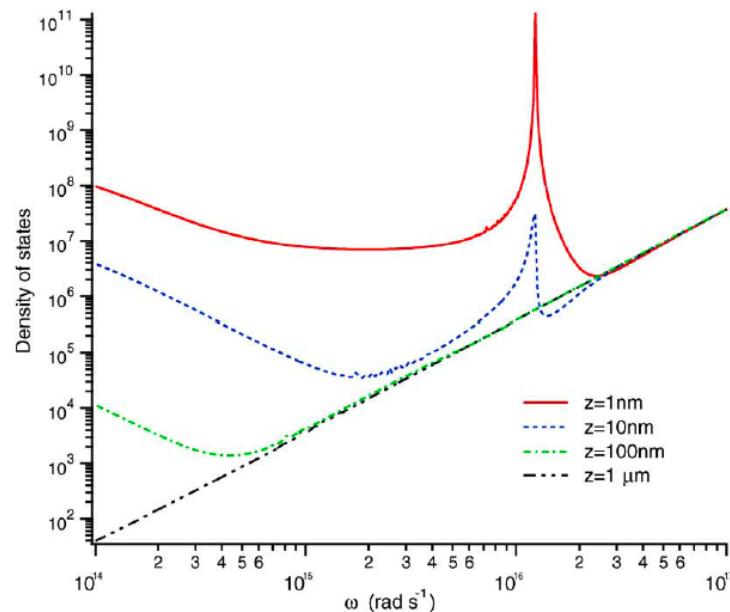
$$\mathbf{E}(\mathbf{r},\omega) = i\mu_0\omega \int \overleftrightarrow{\mathbf{G}}^E(\mathbf{r},\mathbf{r}',\omega) \cdot \mathbf{j}(\mathbf{r}') d^3\mathbf{r}'$$

Generalized response
Generalized force

$$\mathcal{E}_{ij}(\mathbf{r},\mathbf{r}',\omega) = \frac{\hbar\omega}{[\exp(\hbar\omega/k_B T) - 1]} \frac{\mu_0\omega}{2\pi} \text{Im } G_{ij}^E(\mathbf{r},\mathbf{r}',\omega)$$

$$\rho(\mathbf{r}, \omega) = \frac{\omega}{\pi c^2} \text{Im} \text{ Tr} \left[\overset{\leftrightarrow}{\mathbf{G}}^{EE}(\mathbf{r}, \mathbf{r}, \omega) + \overset{\leftrightarrow}{\mathbf{G}}^{HH}(\mathbf{r}, \mathbf{r}, \omega) \right] = \rho^E(\mathbf{r}, \omega) + \rho^H(\mathbf{r}, \omega).$$

This can be computed close to an interface !



$$U(\mathbf{r}, \omega) = \rho(\mathbf{r}, \omega) \frac{\hbar\omega}{\exp(\hbar\omega/k_B T) - 1}.$$

$$\rho^E(z, \omega) = \frac{\rho_v}{|\epsilon_2 + 1|^2} \frac{\epsilon_2''}{4k_0^3 z^3},$$

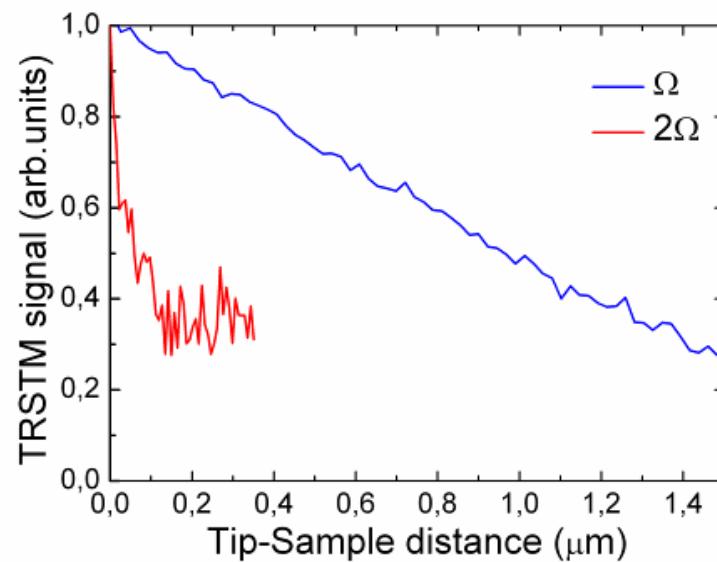
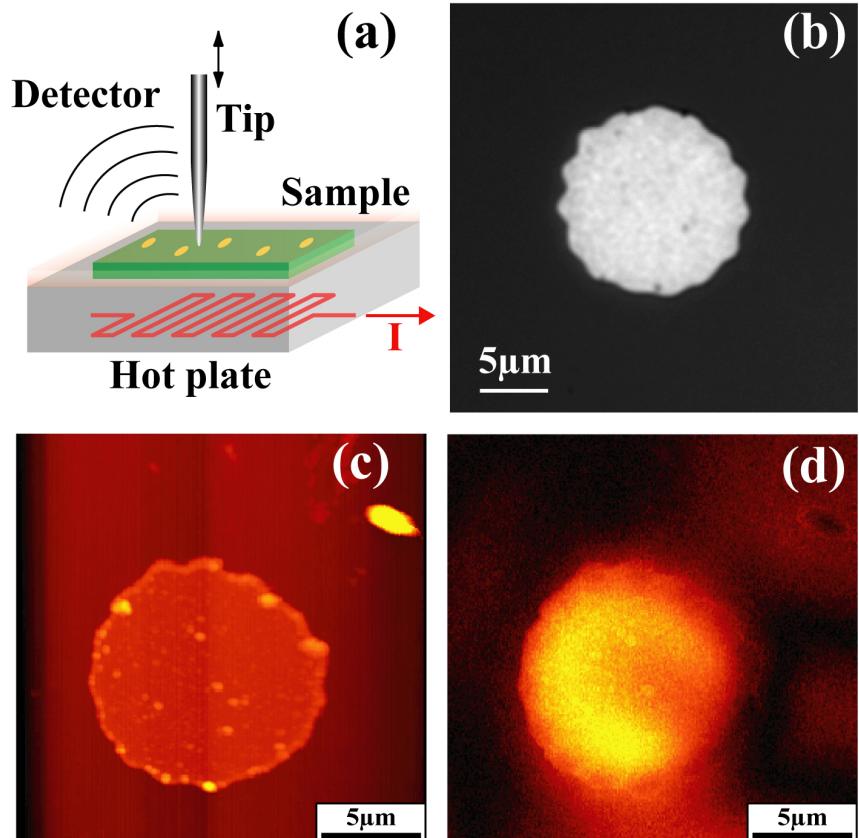
$$\rho^H(z, \omega) = \rho_v \left[\frac{\epsilon_2''}{8k_0 z} + \frac{\epsilon_2''}{2|\epsilon_2 + 1|^2 k_0 z} \right].$$

Plasmon resonance

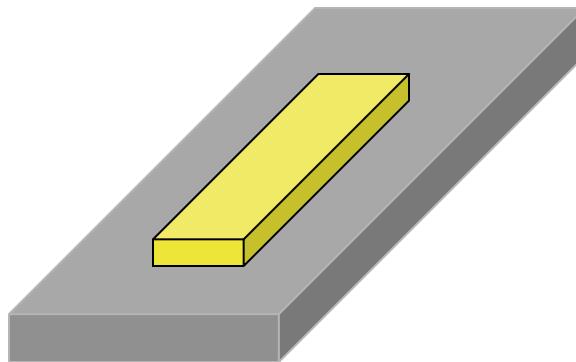
Electrostatic enhancement

Experimental evidence of thermally excited near fields

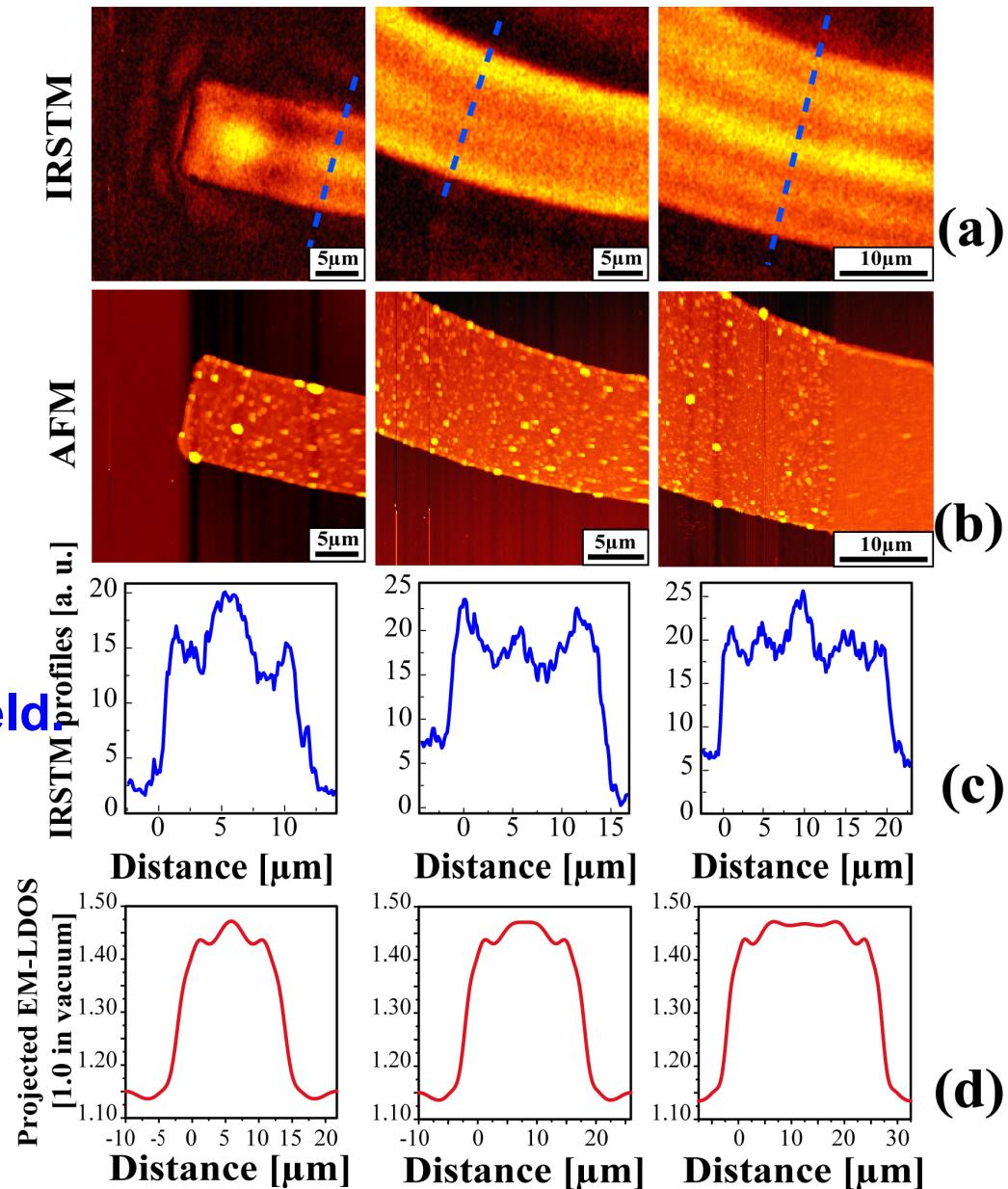
Direct experimental evidence

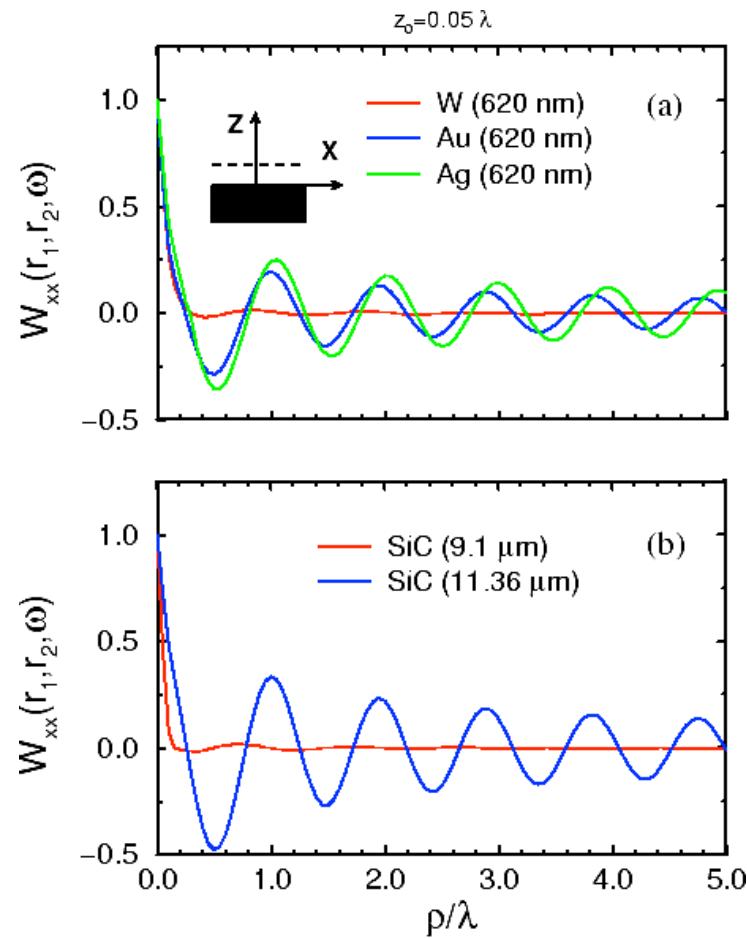
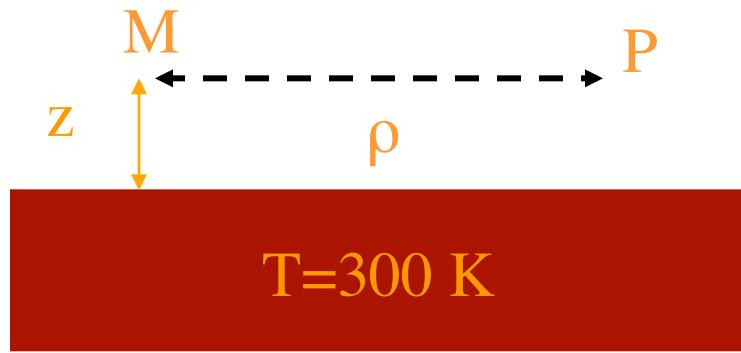


Imaging the LDOS of surface plasmons

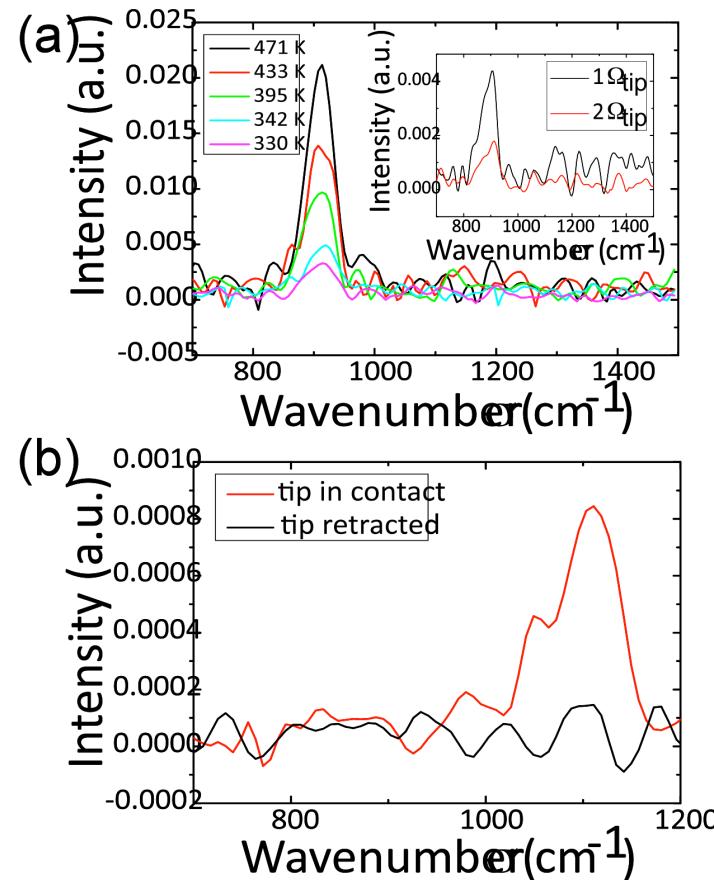
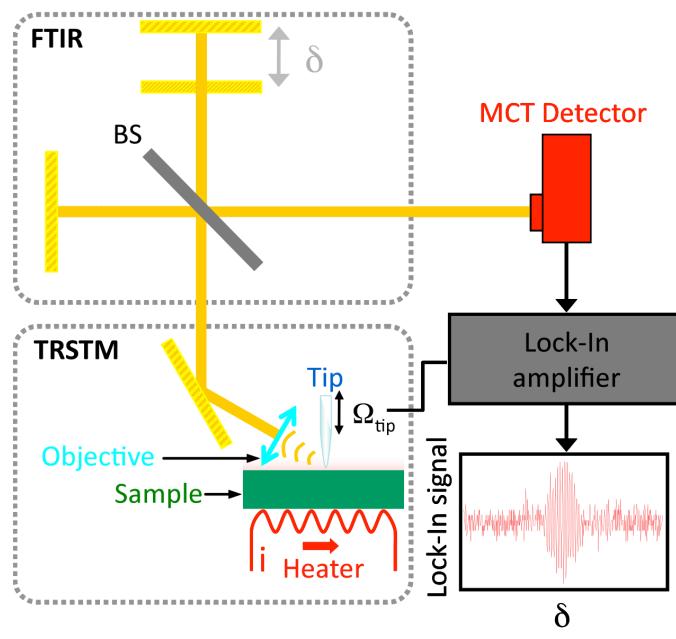


Thermally excited fields can be spatially coherent in the near field





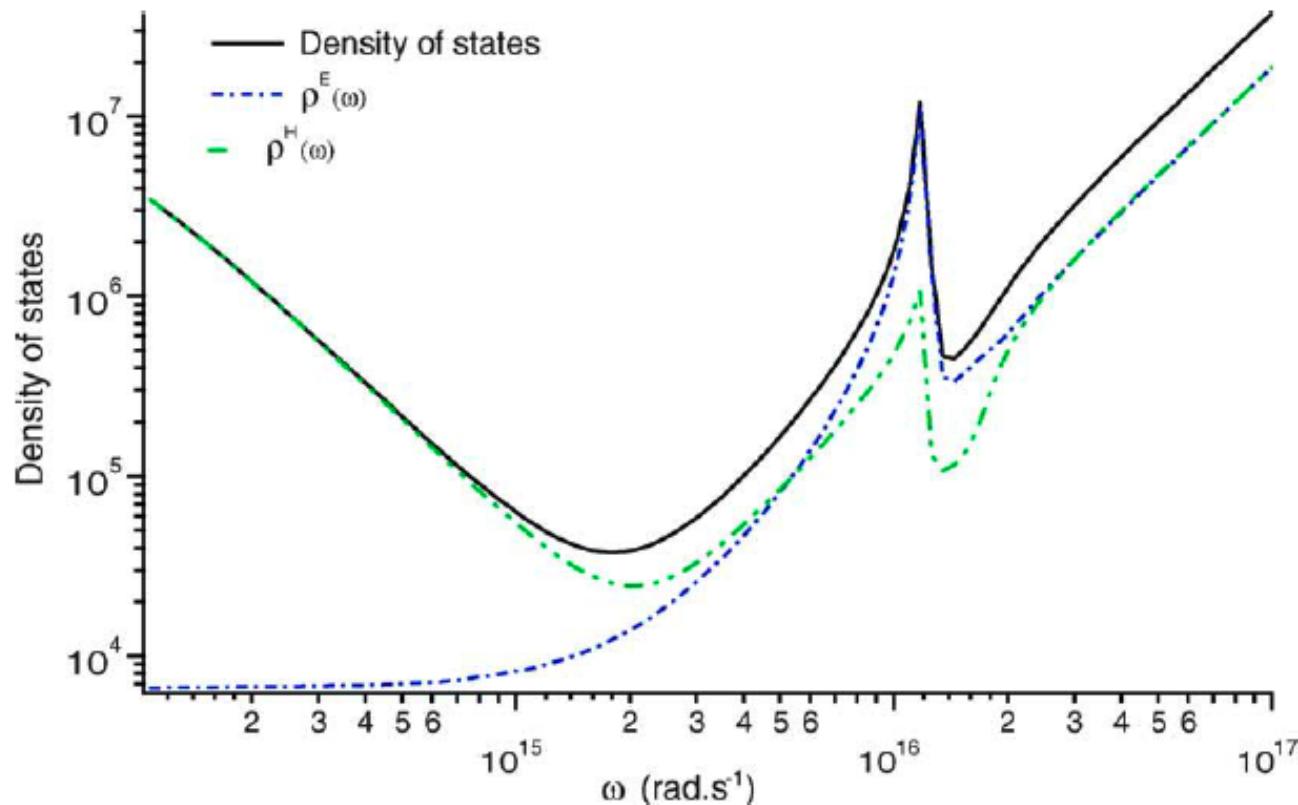
Spectrum of the thermal near field



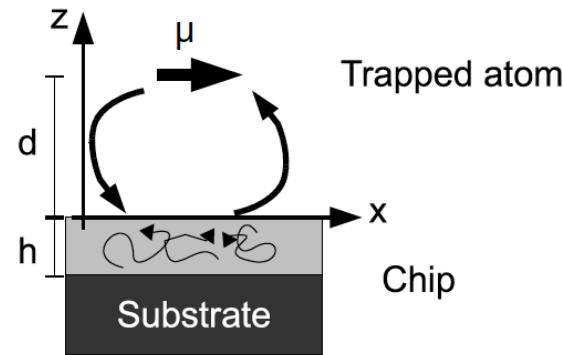
LDOS above a metallic surface

$$\rho^E(z, \omega) = \frac{\rho_v}{|\varepsilon_2 + 1|^2} \frac{\varepsilon''_2}{4k_0^3 z^3},$$

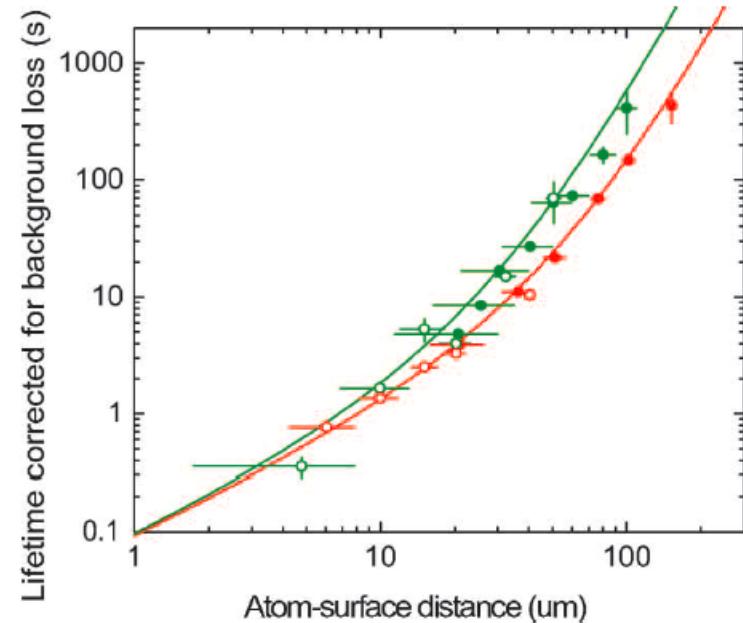
$$\rho^H(z, \omega) = \rho_v \left[\frac{\varepsilon''_2}{8k_0 z} + \frac{\varepsilon''_2}{2|\varepsilon_2 + 1|^2 k_0 z} \right].$$



Lifetime of an atom in a magnetic trap



Superconducting niobium



Copper 1.8 and 6.24 MHz

Lifetime in a magnetic trap versus distance to surface.

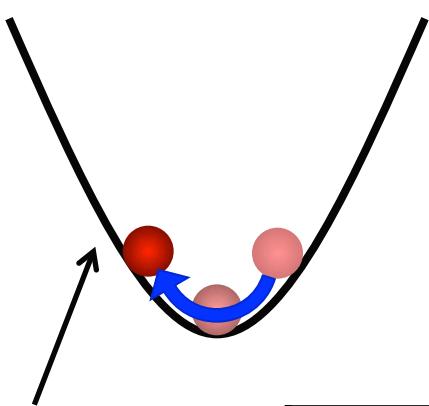
Thermal blackbody is increased by ten orders of magnitude.

D. Harber et al., *J.Low Temp.Phys.* **133**, 229 (2003);

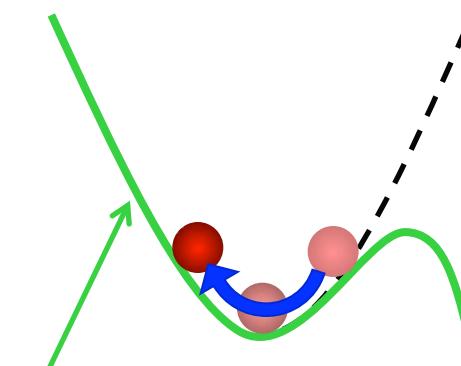
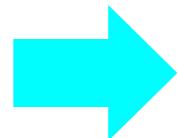
C. Henkel et al. *Appl.Phys.B* **69**, 379 (1999)

C. Roux et al., *EPL* **87**, 13002, (2009)

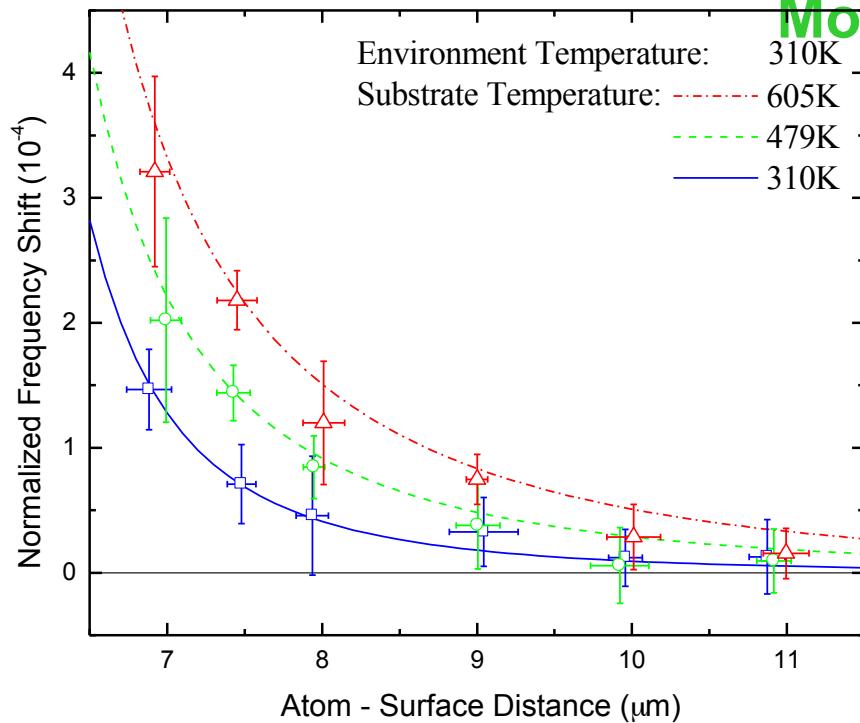
Casimir-Polder force measurement



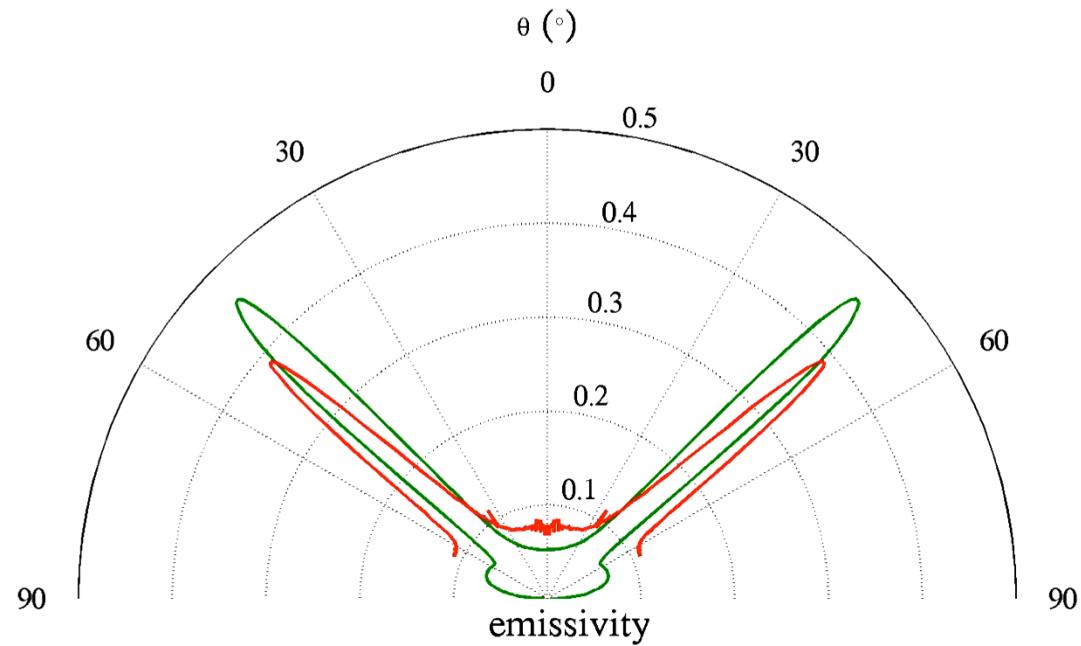
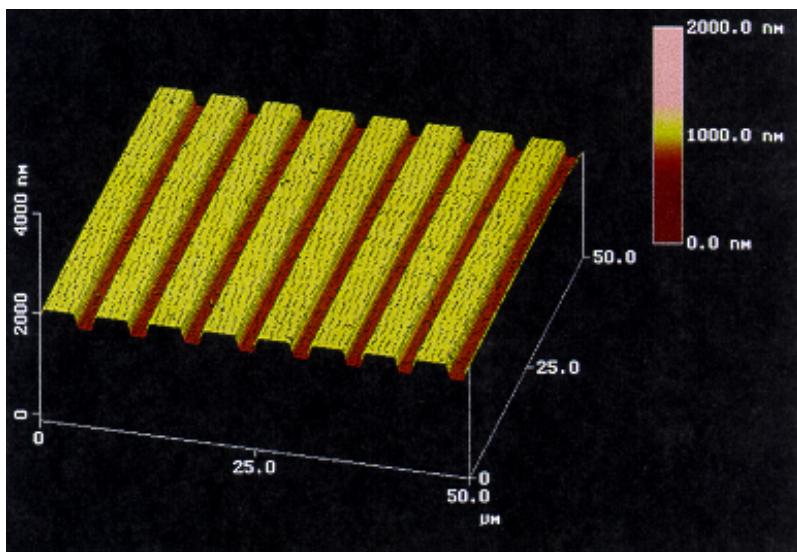
Move near
the surface



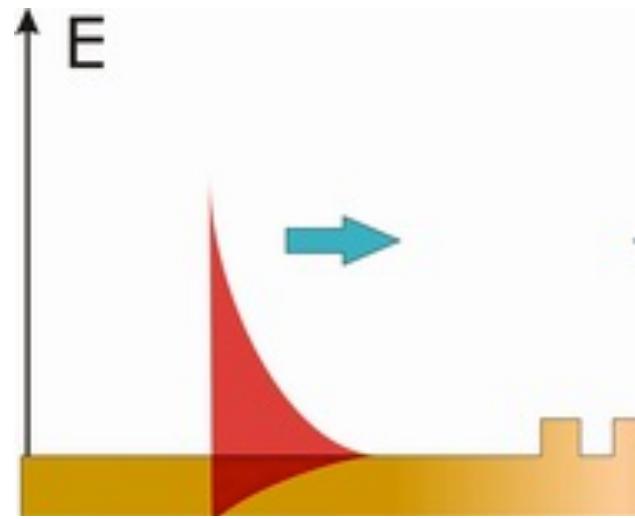
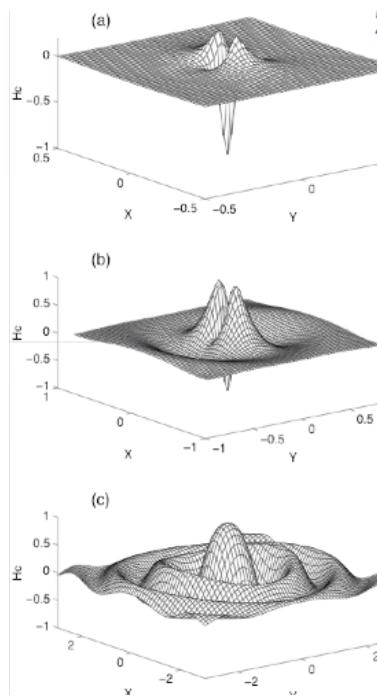
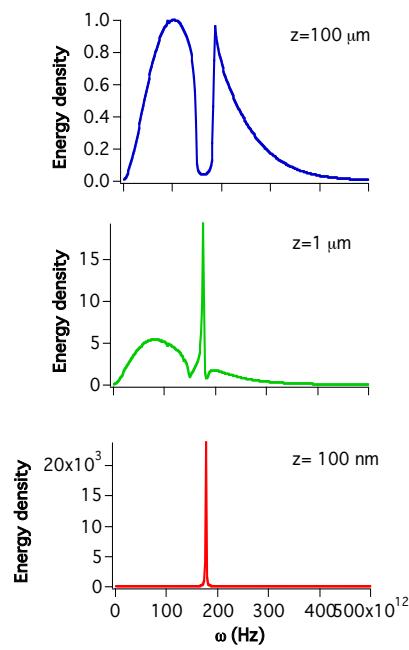
Modified trap, ω_x



Proof of spatial coherence of thermal radiation



Take Home messages



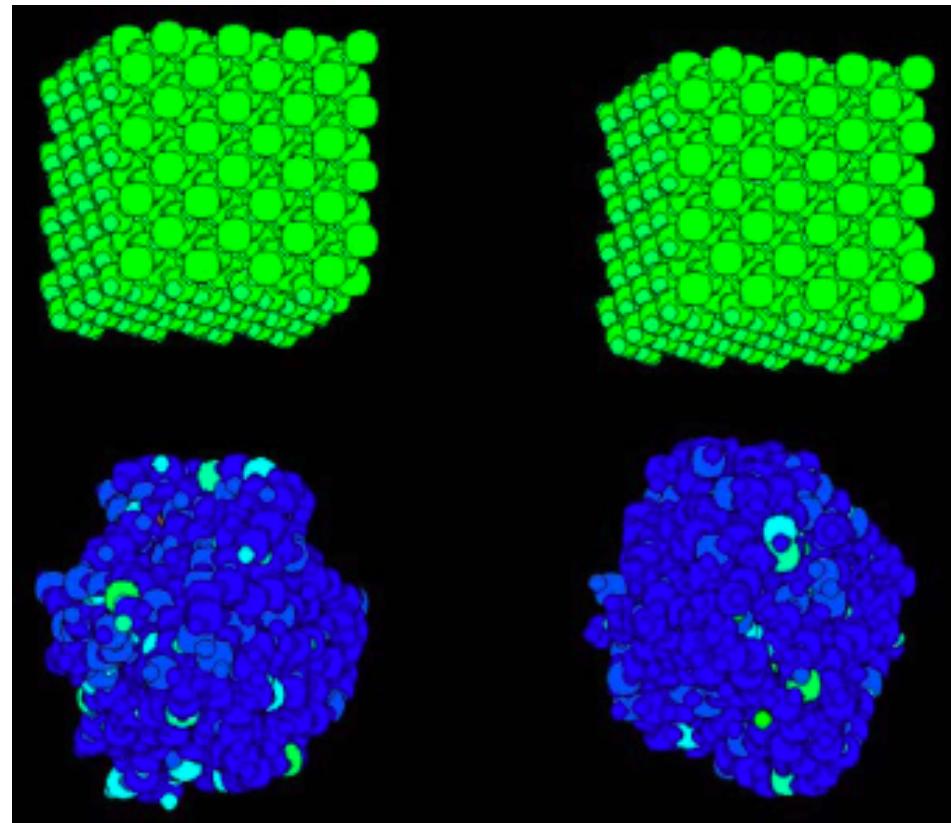
$$\rho(\mathbf{r}, \omega) = \frac{\omega}{\pi c^2} \text{Im} \text{ Tr} \left[\overset{\leftrightarrow}{\mathbf{G}}^{EE}(\mathbf{r}, \mathbf{r}, \omega) + \overset{\leftrightarrow}{\mathbf{G}}^{HH}(\mathbf{r}, \mathbf{r}, \omega) \right]$$

Second Lecture

Radiative heat transfer at the nanoscale

Heat transfer between two nanoparticles

Heat transfer between two nanoparticles



Heat conductance

$$\frac{Q_{12}(\omega)}{T_0} = G_{12}^*(\omega) \Delta T(\omega),$$

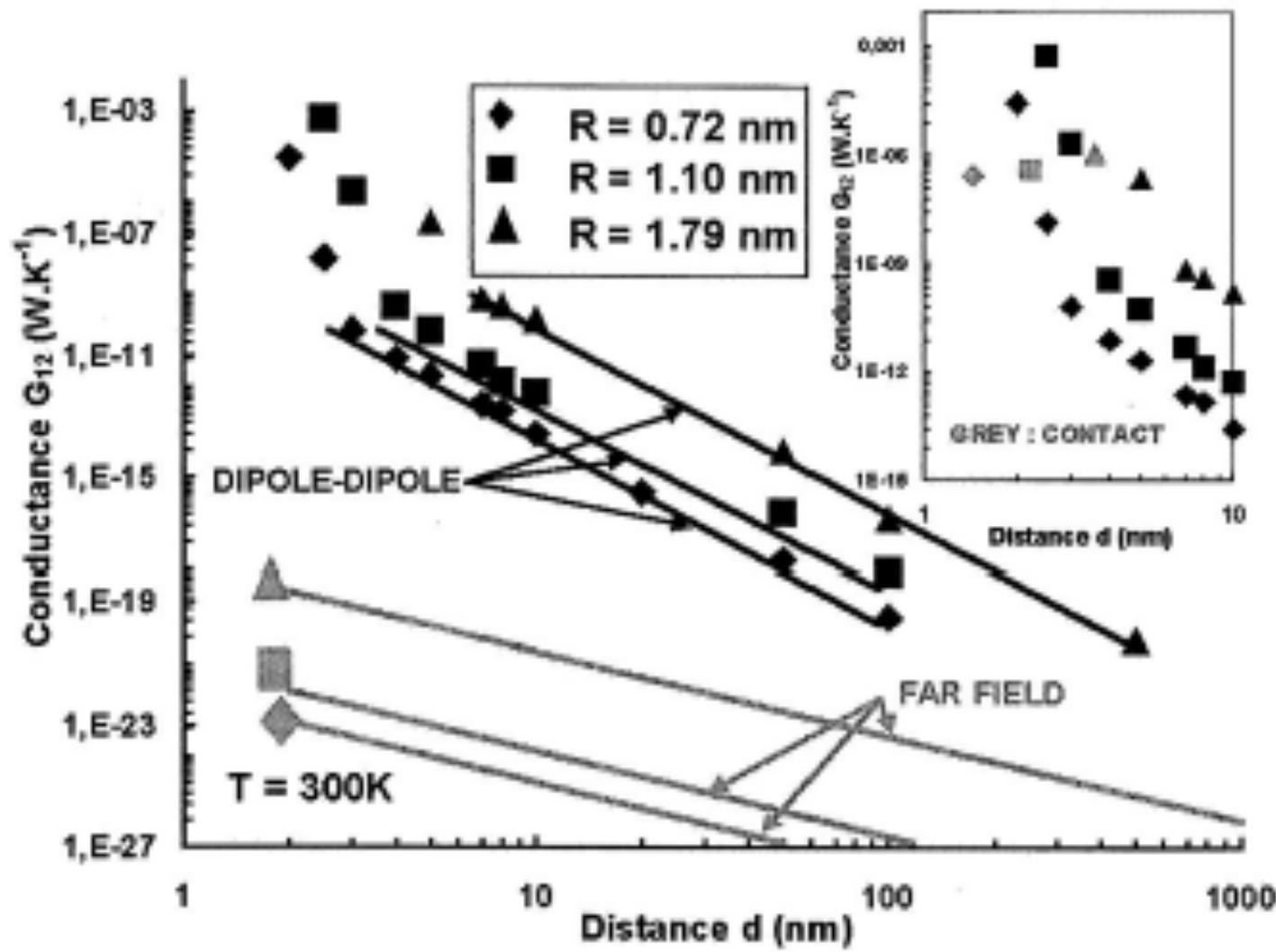
**Fluctuations power
spectral density**

$$P_{\Delta T}(\omega) = \frac{P_{Q_{12}}(\omega)}{|G_{12}^* T_0|^2}.$$

**Fluctuation-Dissipation
theorem**

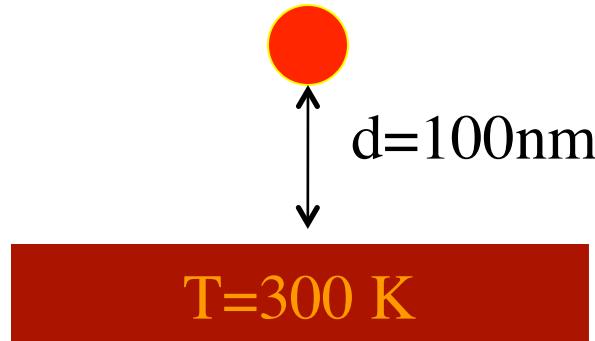
$$P_{\Delta T}(\omega) = \frac{\text{Re}(G_{12}^*)}{|G_{12}^*(\omega)|^2} \Theta(\omega, T_0),$$

$$Q_{12} = \sum_{\substack{i \in \text{NP1} \\ j \in \text{NP2}}} \mathbf{f}_{ij} \cdot \mathbf{v}_j - \sum_{\substack{i \in \text{NP1} \\ j \in \text{NP2}}} \mathbf{f}_{ji} \cdot \mathbf{v}_i.$$

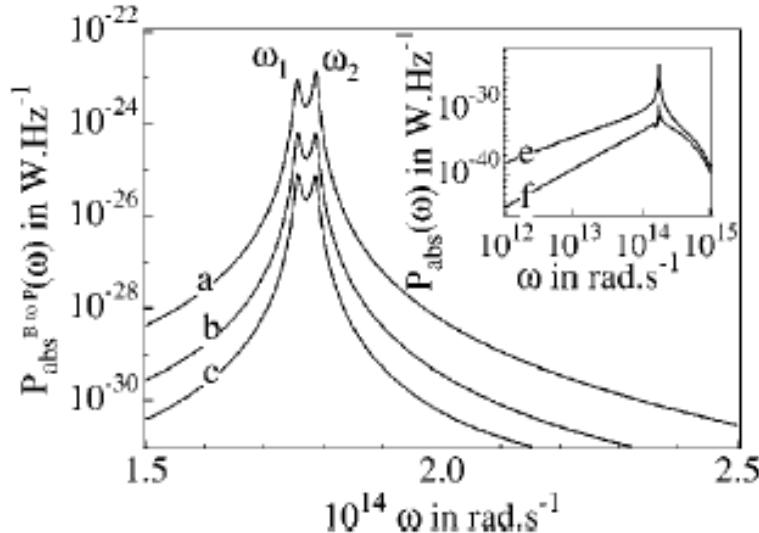
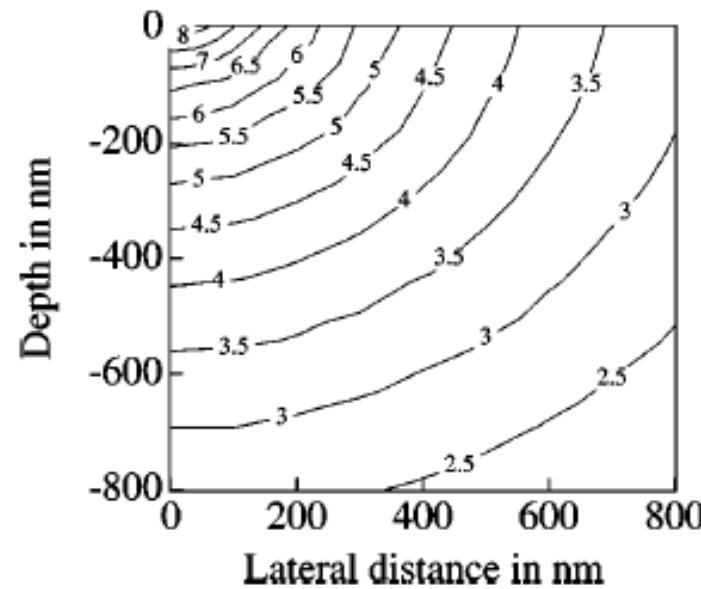


Heat transfer between a particle and a dielectric (SiC) surface

$$\varepsilon_B(\omega) = \varepsilon_p(\omega) = \varepsilon(\omega) = \varepsilon_\infty \left(1 + \frac{\omega_L^2 - \omega_T^2}{\omega_T^2 - \omega^2 - i\Gamma\omega} \right)$$



Take home message:
For source at distance d ,
the power is dissipated within
a distance $2 d$.



$$P_{\text{abs}}^{B \rightarrow P}(d, \omega) \sim \frac{1}{4\pi^2 d^3}$$

Key feature 1

$$\times 4\pi a^3 \underbrace{\frac{3\epsilon''(\omega)}{|\epsilon(\omega)+2|^2}}_{\text{particle}} \underbrace{\frac{\epsilon''(\omega)}{|\epsilon(\omega)+1|^2} \Theta(\omega, T_B)}_{\text{bulk}}$$



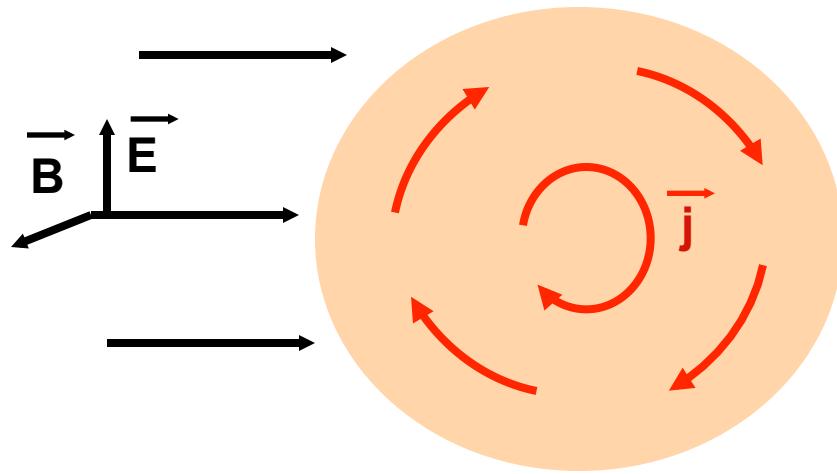
$T=300\text{ K}$

Key feature 2

Microscopic resonance if $\epsilon(\omega)+2=0$

Take home message: The flux is dramatically enhanced by the surface waves resonances

Heat transfer between a particle and a metallic surface



The magnetic field induces eddy currents that produce Joule losses.
Losses are given by the product of two terms :

$$P_E = 2\omega \text{Im}(\alpha_E) \frac{\langle \vec{\varepsilon}_0 | \vec{E} |^2 \rangle}{2} + \boxed{2\omega \text{Im}(\alpha_M) \frac{\langle |\vec{B}|^2 \rangle}{2\mu_0}}$$

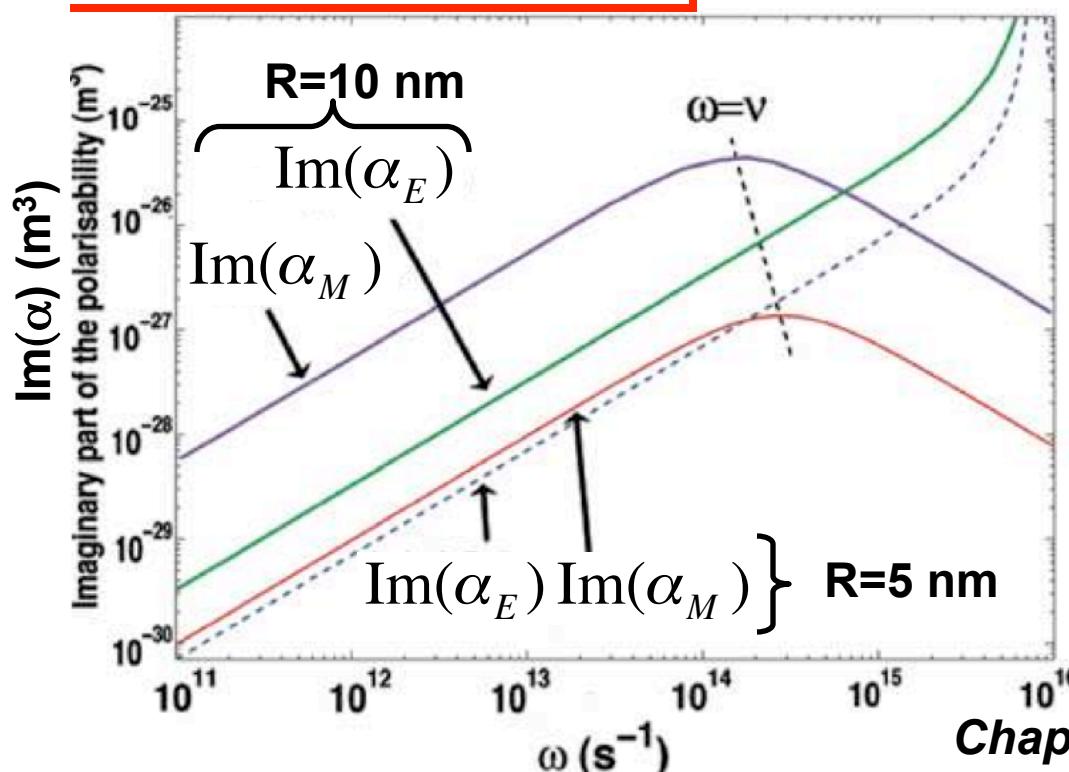
Magnetic losses versus electric losses

- Electric dipole →
- Magnetic polarisability

$$|\alpha_M| \ll |\alpha_E| \quad \text{for } R \rightarrow 0$$

$$\alpha_E = 4\pi R^3 \frac{\epsilon - 1}{\epsilon + 2}$$

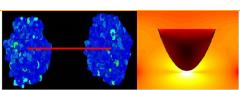
$$\alpha_M = \frac{2\pi}{15} R^3 \frac{R^2}{\lambda^2} (\epsilon - 1)$$



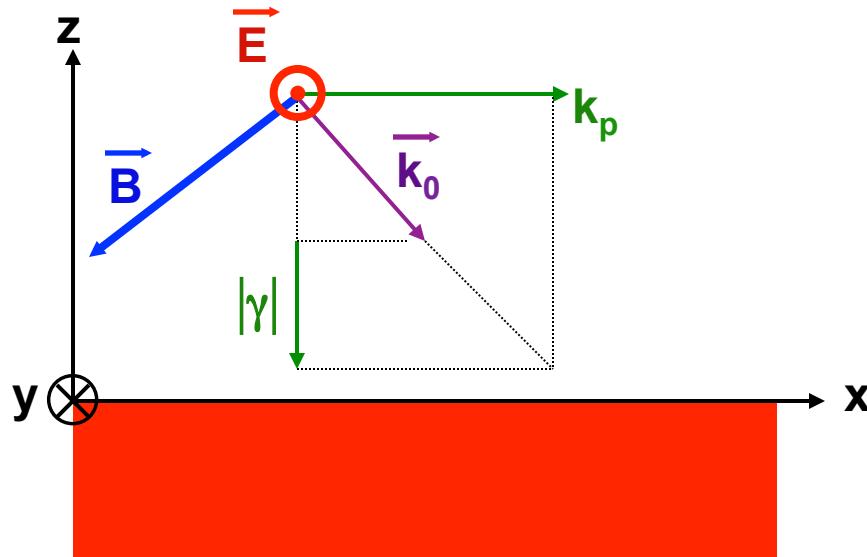
$|\epsilon| \gg 1$

for a metal at
low frequencies

Chapuis, Phys.Rev.B 125402, 2008



Electric and magnetic energy density above an interface :
an unusual property

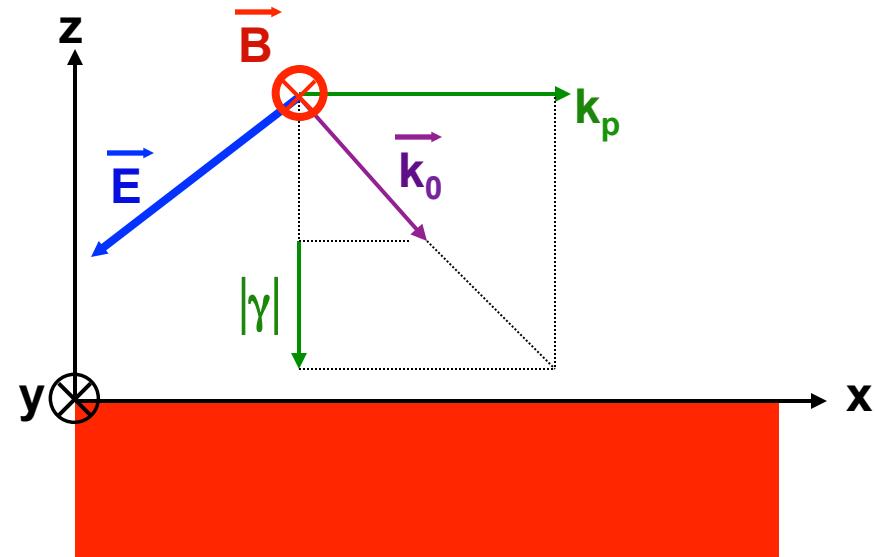


s polarisation:magnetostatics
propagating evanescent

$$B = \frac{E}{c}$$

Magnetostatics

$$B = \frac{E}{c} \sqrt{2 \left(\frac{k_p}{k_0} \right)^2 - 1}$$

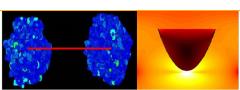


p polarisation: electrostatics
propagating evanescent

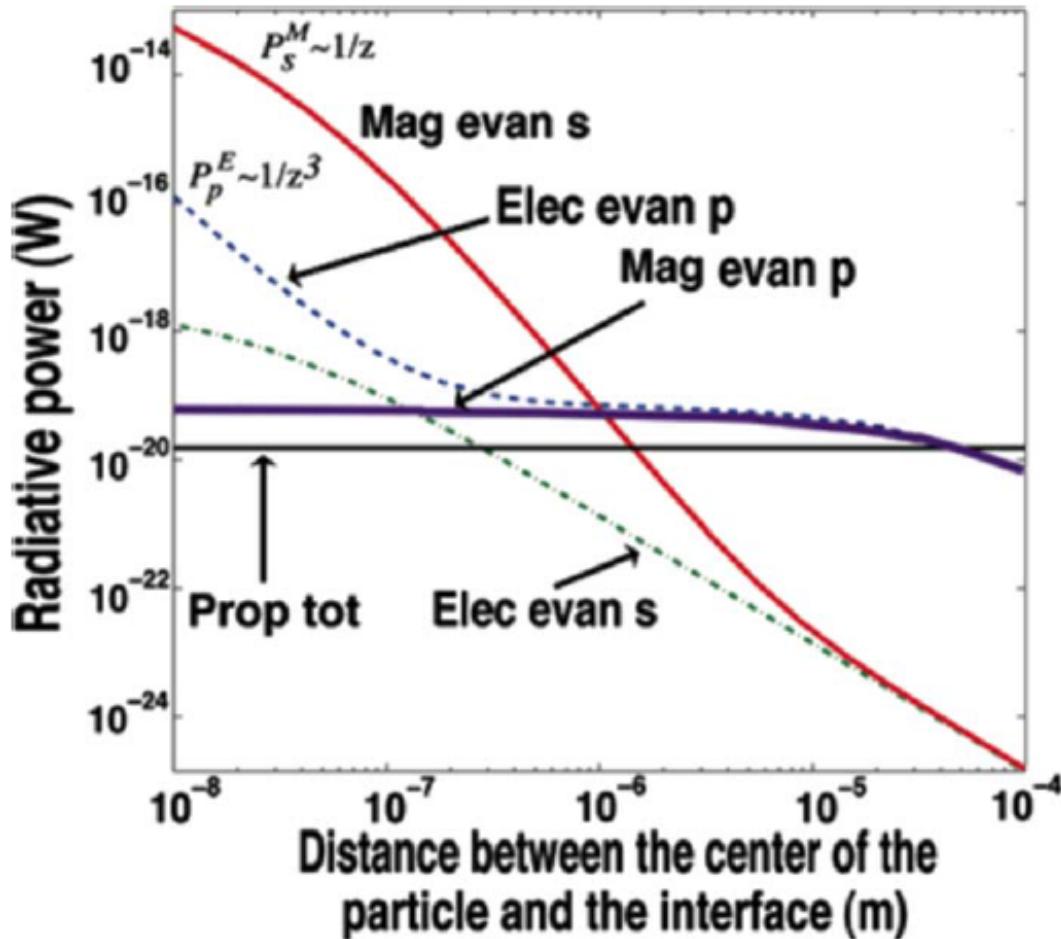
$$E = cB$$

$$E = cB \sqrt{2 \left(\frac{k_p}{k_0} \right)^2 - 1}$$

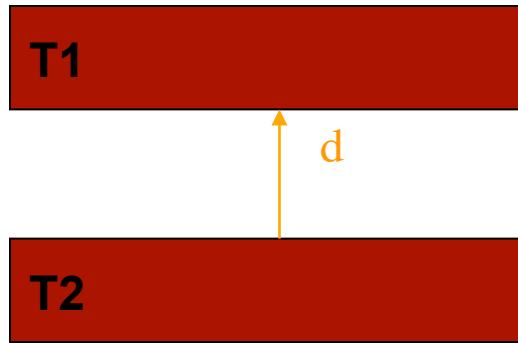
Chapuis, Phys.Rev.B 125402, 2008



Radiative heat transfer between a particle and a surface



Radiative heat transfer between two surfaces

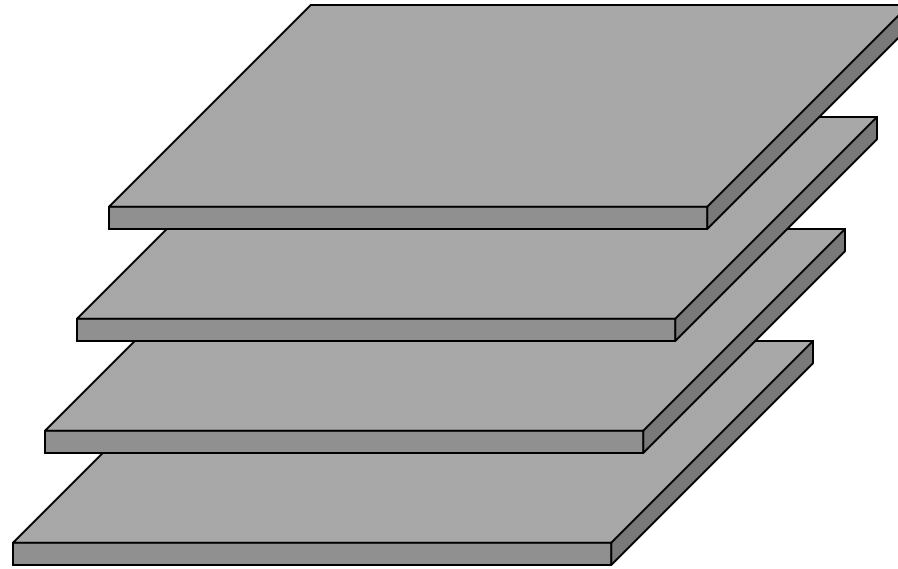


$$\Phi < \sigma(T_1^4 - T_2^4)$$

Theme : influence of surface waves on the mesoscopic energy transfer

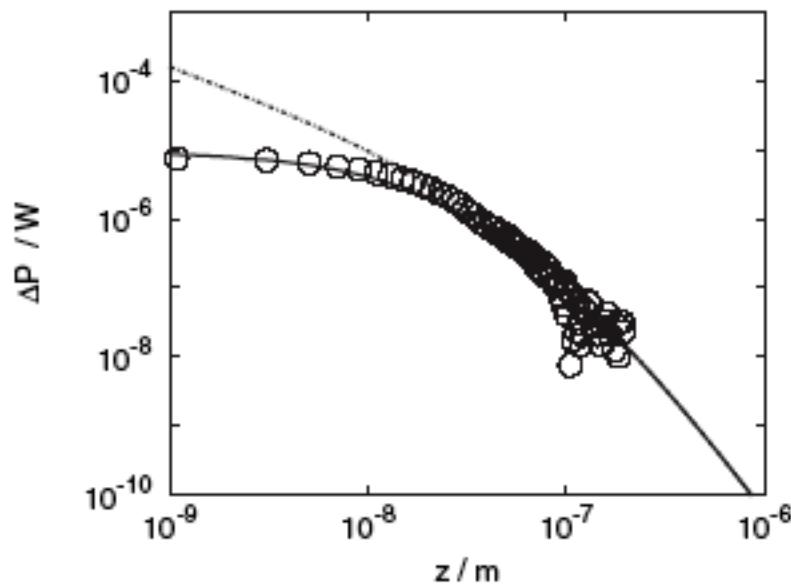
Anomalous heat transfer

Radiation shield

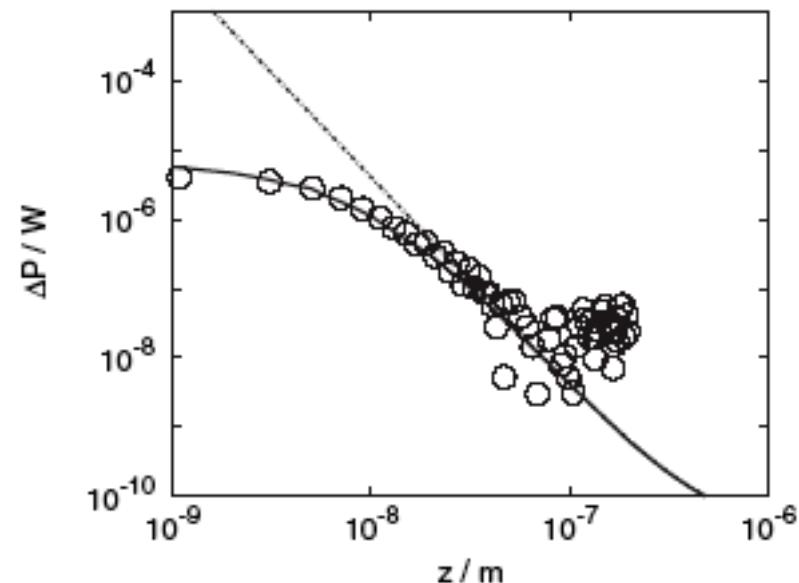


Hargreaves, 1969 *Anomalous heat transfer*

- 1) Measurements at low temperature, Hargreaves (69),
- 2) Measurements between metals in the infra red, Xu: inconclusive,
- 3) Measurements in the nm regime with metal (Oldenburg, 2005).



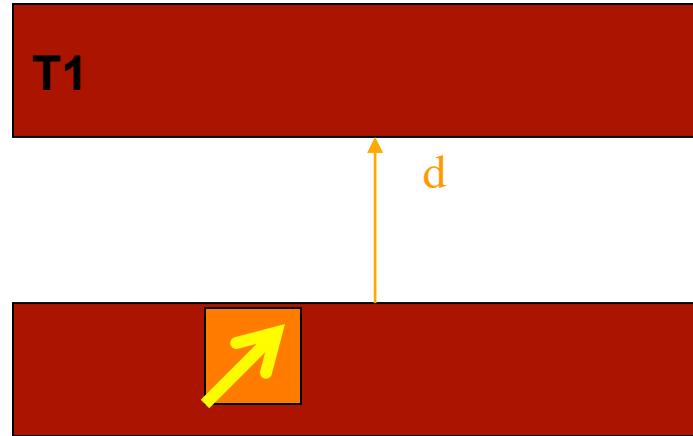
Au



GaN

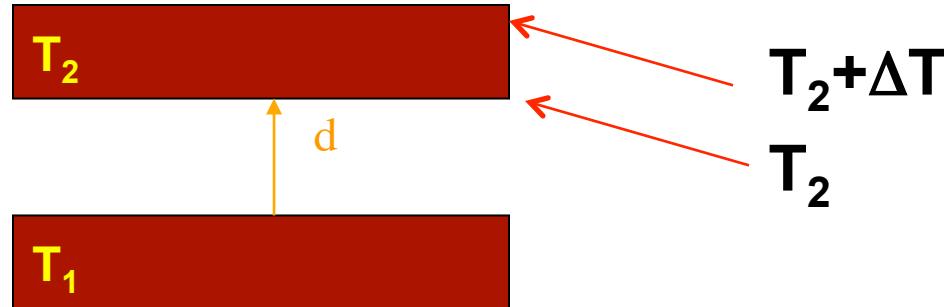
Radiative heat transfer at the nanoscale

Radiometric approach



$$q(\omega) = \int_0^{2\pi} \cos \theta d\Omega \int_0^{\infty} d\omega \frac{\varepsilon'_{1\omega} \varepsilon'_{2\omega}}{1 - \rho'_{1\omega} \rho'_{2\omega}} [I_{\omega}^0(T_1) - I_{\omega}^0(T_2)],$$

$$\langle j_m(\mathbf{r}_1) j_n^*(\mathbf{r}_2) \rangle = \frac{\omega}{\pi} \varepsilon_o \operatorname{Im} [\varepsilon(\omega)] \delta_{mn} \delta(\mathbf{r}_1 - \mathbf{r}_2) \frac{\hbar\omega}{\exp(\hbar\omega/k_B T) - 1}$$



What about the temperature gradient ?
Can we assume the temperature to be uniform ?

$$h(T_2 - T_1) = -k \frac{\partial T}{\partial z}$$

$$\Delta T \approx \frac{\partial T}{\partial z} d \approx \frac{h(T_2 - T_1)}{k} d$$

$$\frac{\Delta T}{(T_2 - T_1)} \approx \frac{1000}{1} 10^{-7} \approx 10^{-4}$$

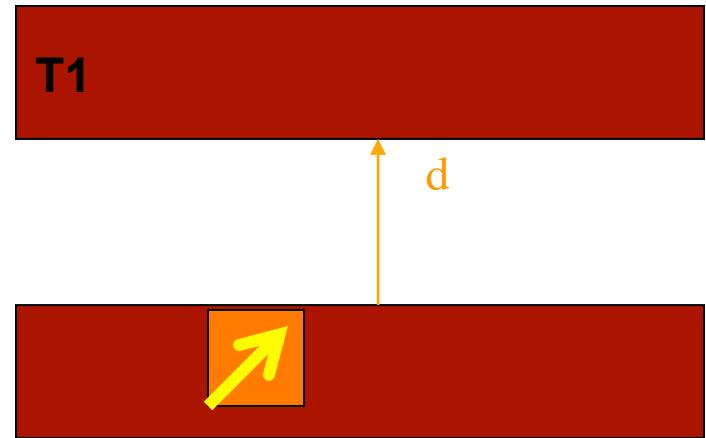
Radiative heat transfer at the nanoscale

Theory : Polder, van Hove, PRB 1971
 (Tien, Caren, Rytov, Pendry)

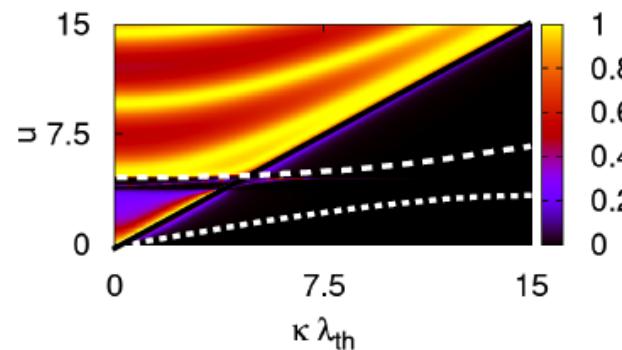
$$\phi = \int_{\omega=0}^{+\infty} d\omega [I_\omega^0(T_1) - I_\omega^0(T_2)]$$

$$\times \sum_{\alpha=s,p} \left[\int_0^{\omega/c} \frac{KdK}{\omega^2/c^2} \frac{(1 - |r_{31}^\alpha|^2)(1 - |r_{32}^\alpha|^2)}{|1 - r_{31}^\alpha r_{32}^\alpha e^{2i\gamma_3 d}|^2} \right.$$

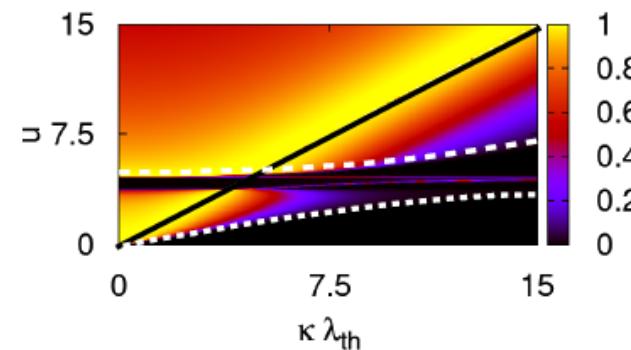
$$\left. + \int_{\omega/c}^{\infty} \frac{KdK}{\omega^2/c^2} \frac{4 \operatorname{Im}(r_{31}^\alpha) \operatorname{Im}(r_{32}^\alpha) e^{-2\gamma_3'' d}}{|1 - r_{31}^\alpha r_{32}^\alpha e^{-2\gamma_3'' d}|^2} \right],$$



Transmission factor

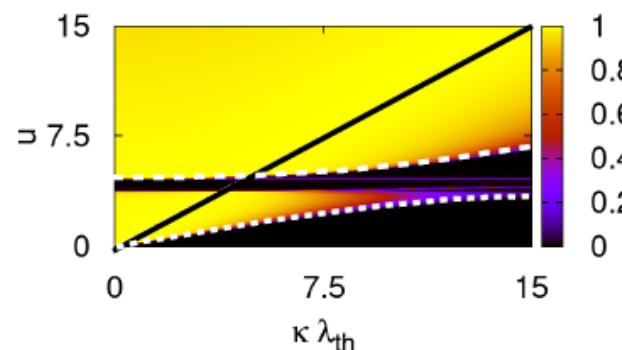


(a) $d = 5 \mu\text{m}$

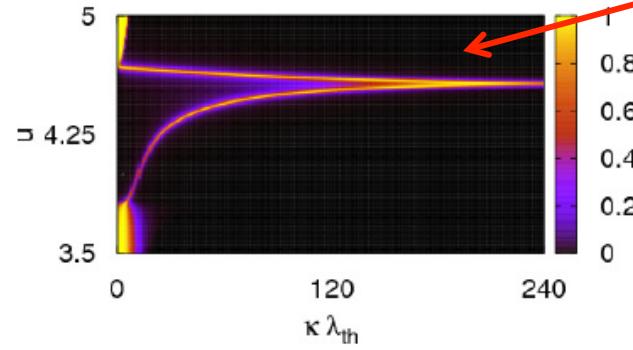


(b) $d = 500 \text{ nm}$

Enhancement due to surface waves

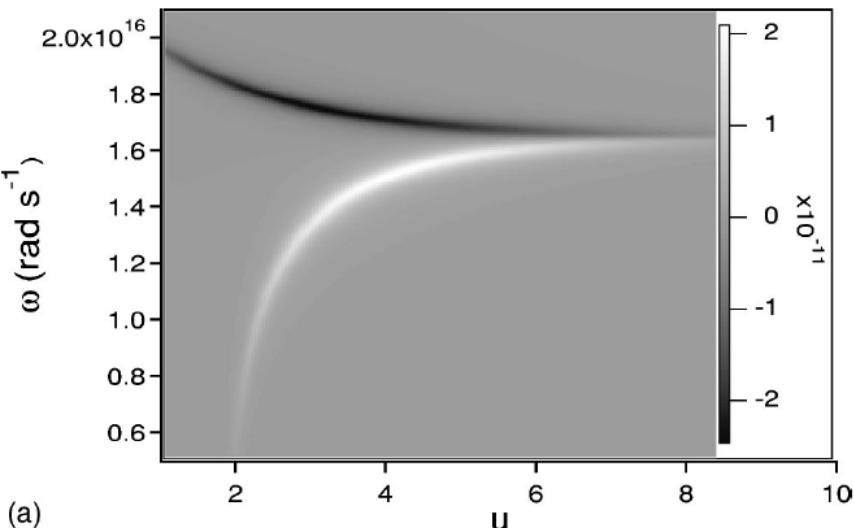


(c) $d = 100 \text{ nm}$



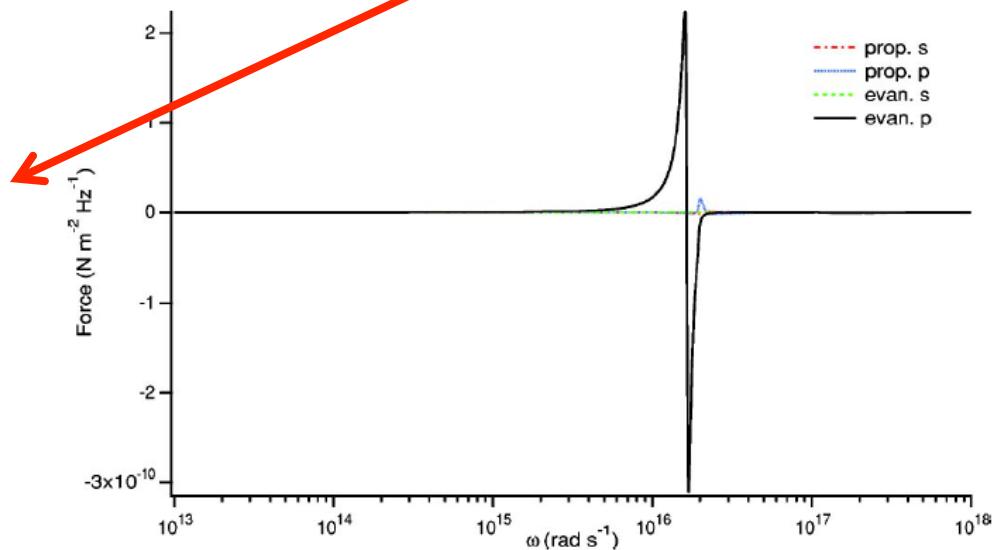
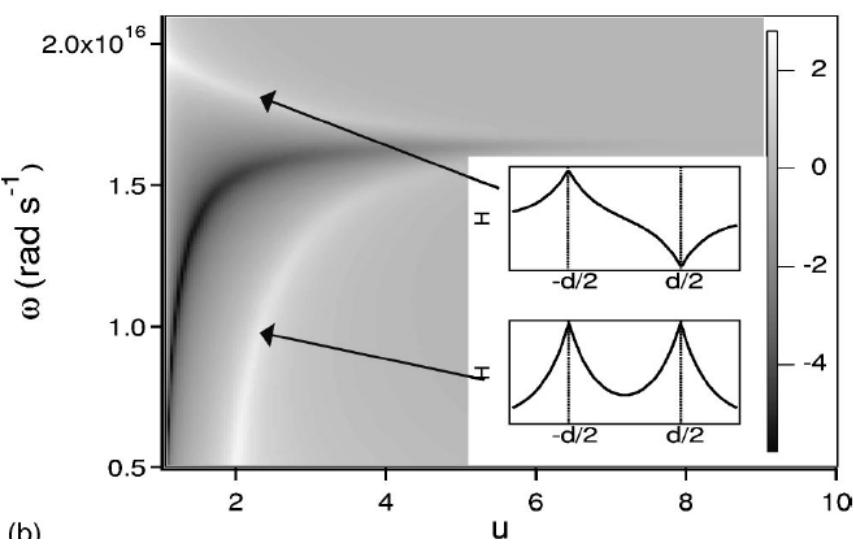
(d) $d = 100 \text{ nm}$

Analogy with Casimir force

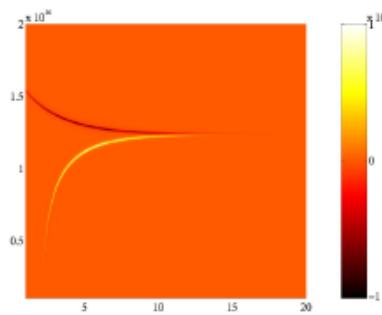


$$F = \int_0^\infty \frac{d\omega}{2\pi} \int_0^\infty \frac{du}{2\pi} F(u, \omega),$$

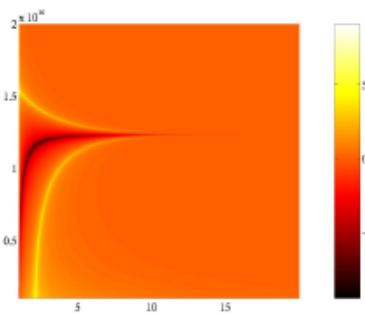
$$F(u, \omega) = \frac{2\hbar\omega^3 u}{c^3} \text{Im} \left(v \sum_{\mu=s,p} \frac{r_\mu^2(u, \omega) e^{-2\omega v d}}{1 - r_\mu^2(u, \omega) e^{-2\omega v d/c}} \right)$$



Casimir force for aluminum

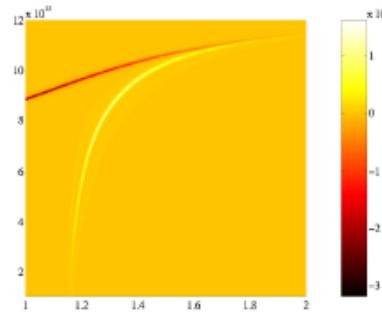


(a) $d = 10 \text{ nm}$

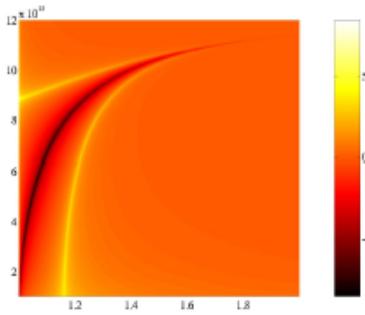


(b) $d = 10 \text{ nm}$

10 nm

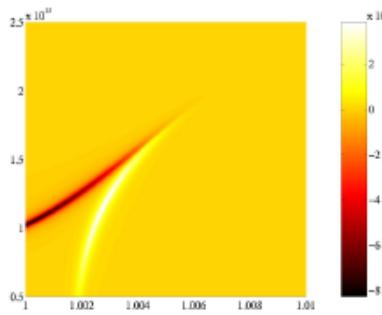


(c) $d = 100 \text{ nm}$

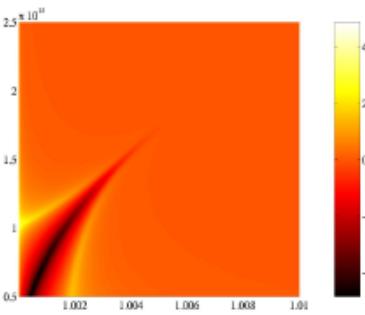


(d) $d = 100 \text{ nm}$

100 nm



(e) $d = 10 \mu\text{m}$



(f) $d = 10 \mu\text{m}$

10 μm

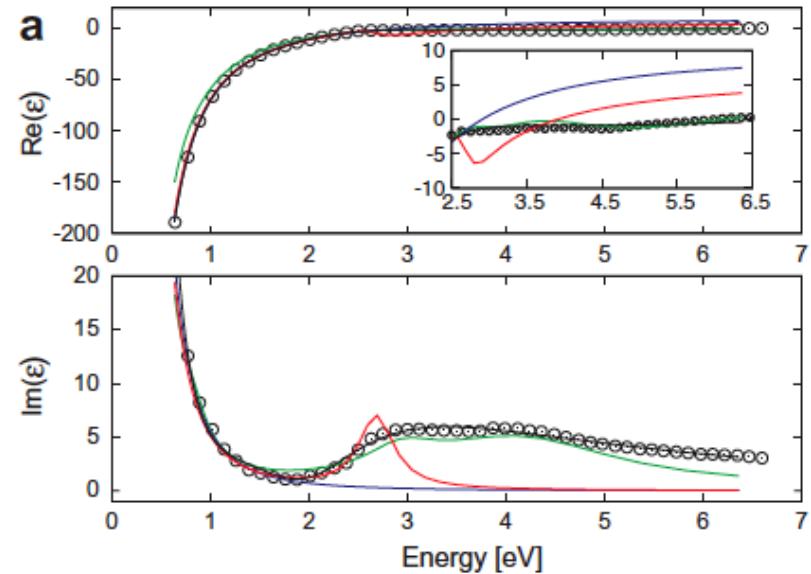
Drude model is not accurate

Nordlander's model

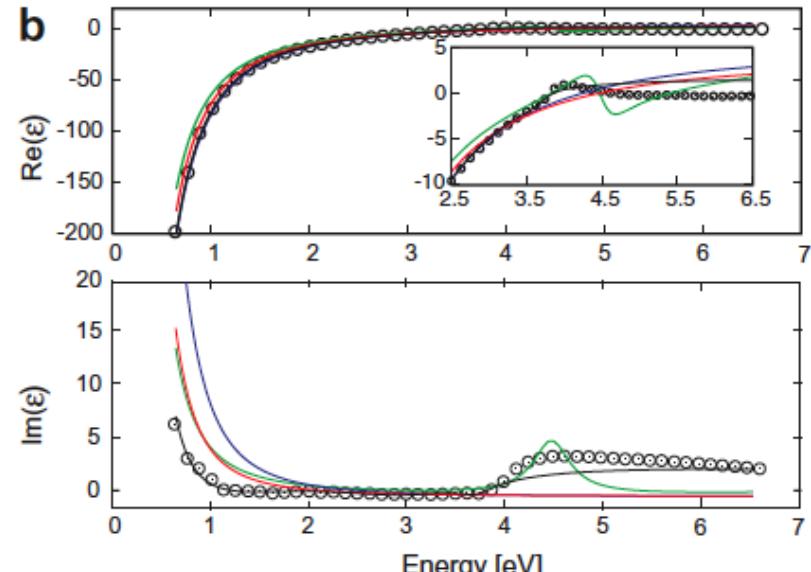
$$\epsilon(\omega) = \epsilon_{\infty} + \frac{\sigma/\epsilon_0}{j\omega} + \sum_{p=1}^P \frac{C_p}{\omega^2 + A_p j\omega + B_p}.$$

4 Lorentzian terms are used

Gold



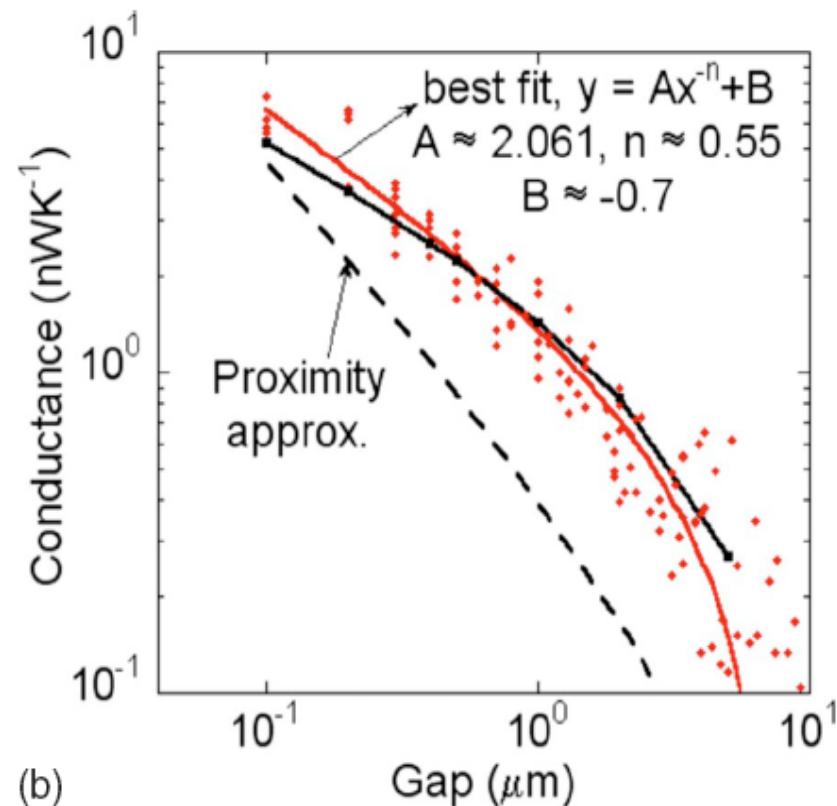
Silver



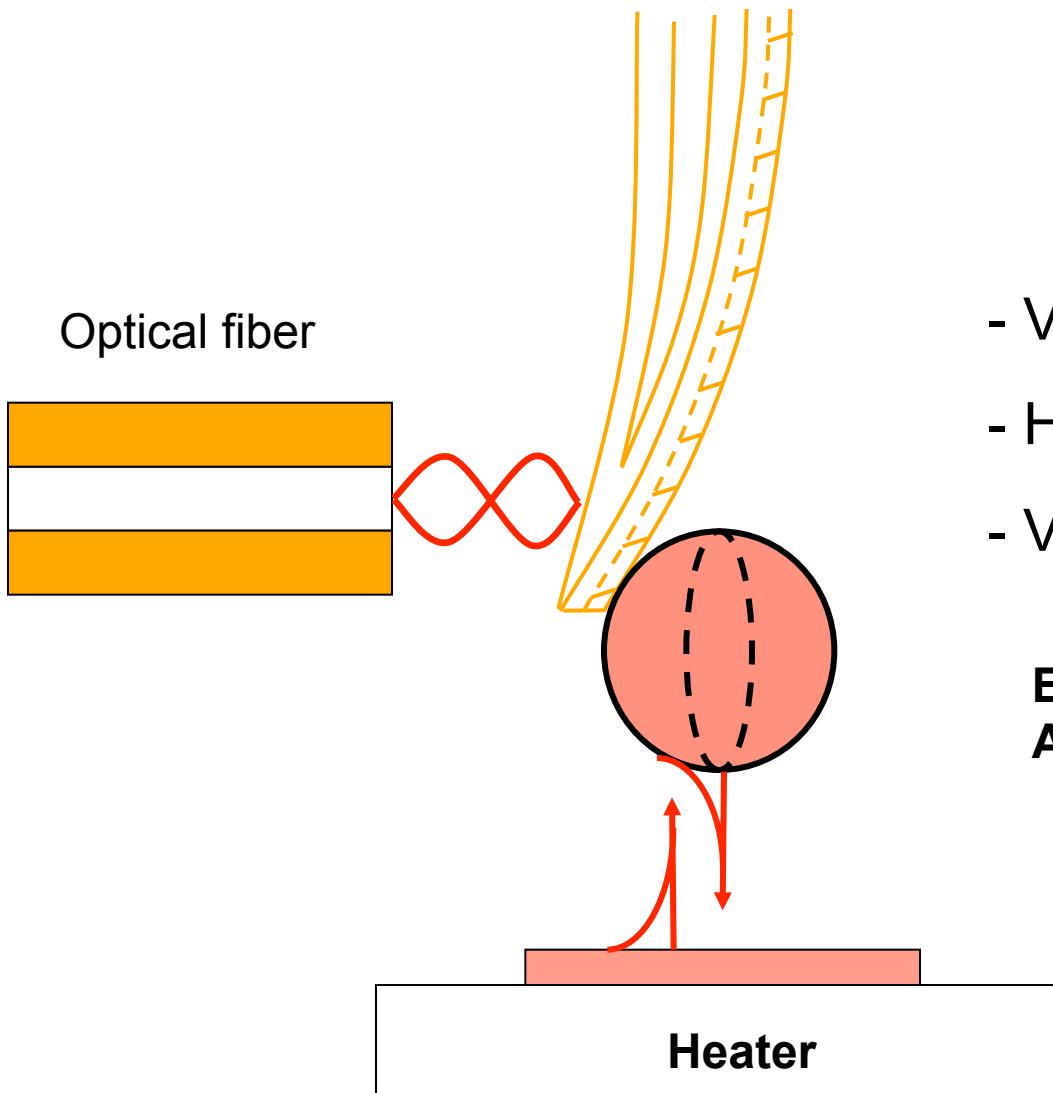
Circle: Johnson and Christy
Blue : Drude model

Recent experimental results

Measurements with silica



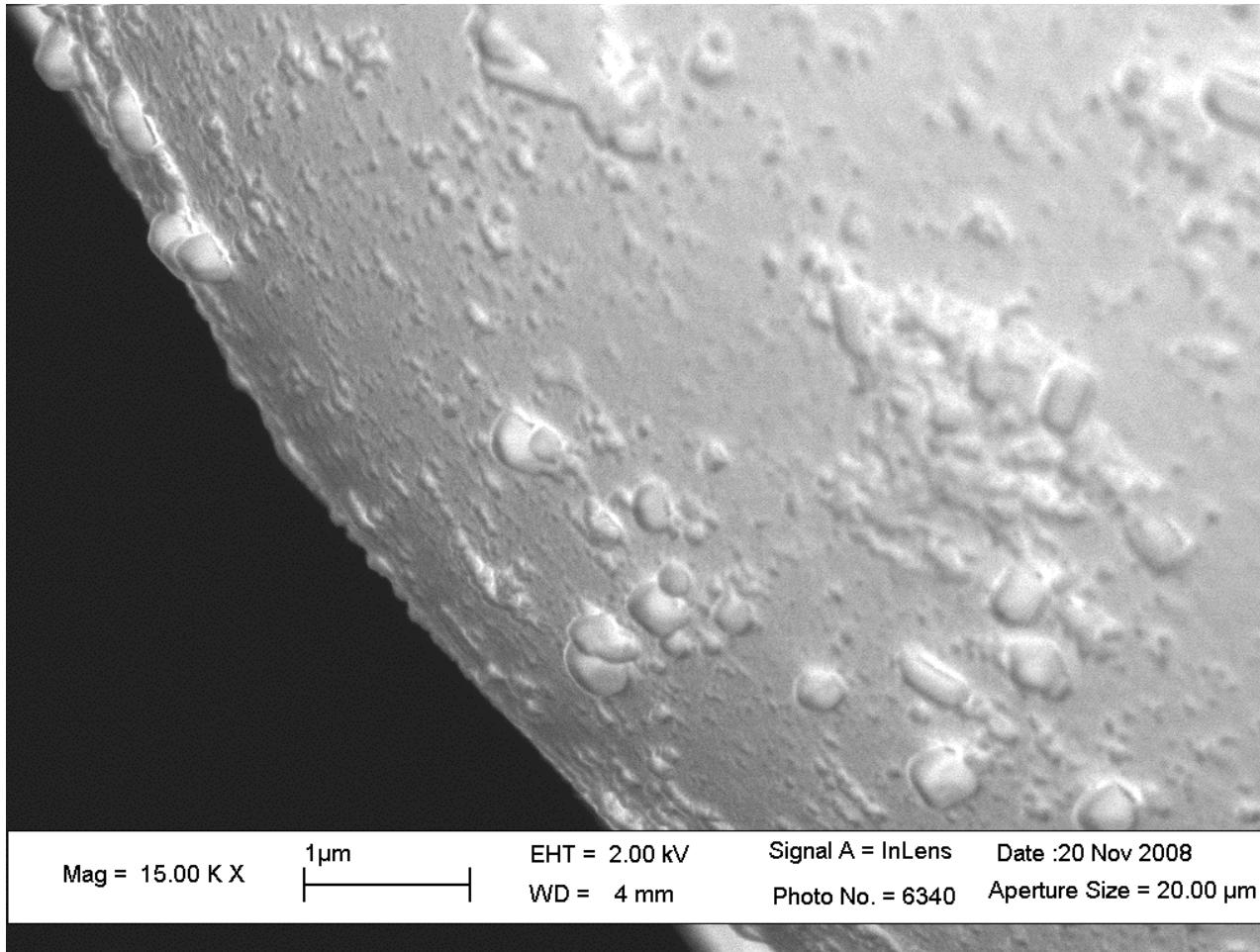
Detection scheme



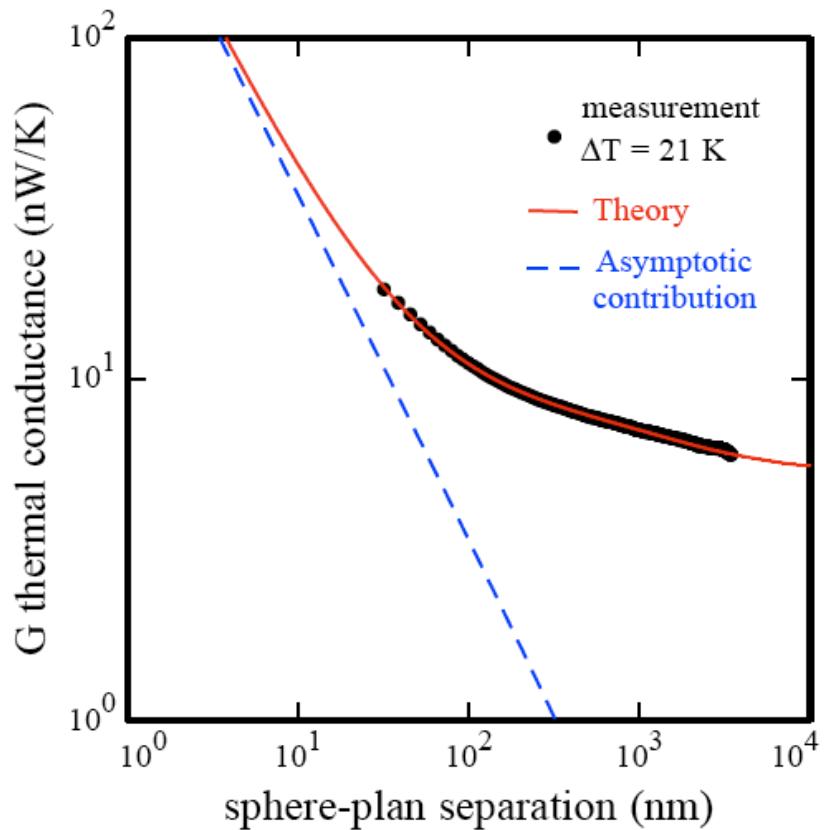
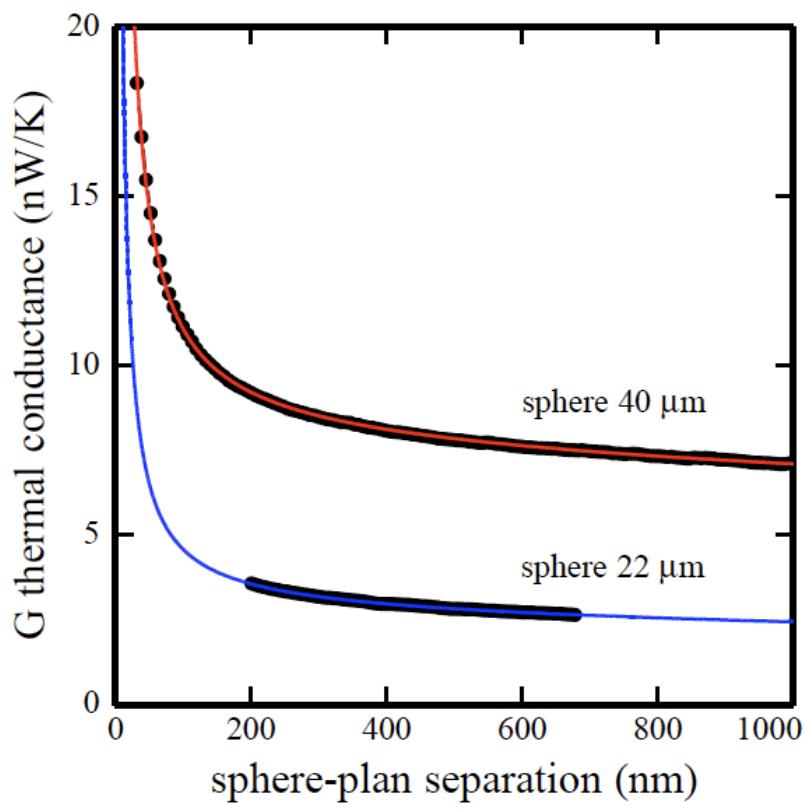
- Vertical configuration
- Heating of the sample
- Vacuum $P \sim 10^{-6}$ mbar

**E. Rousseau
A. Siria, J. Chevrier**

Defects size



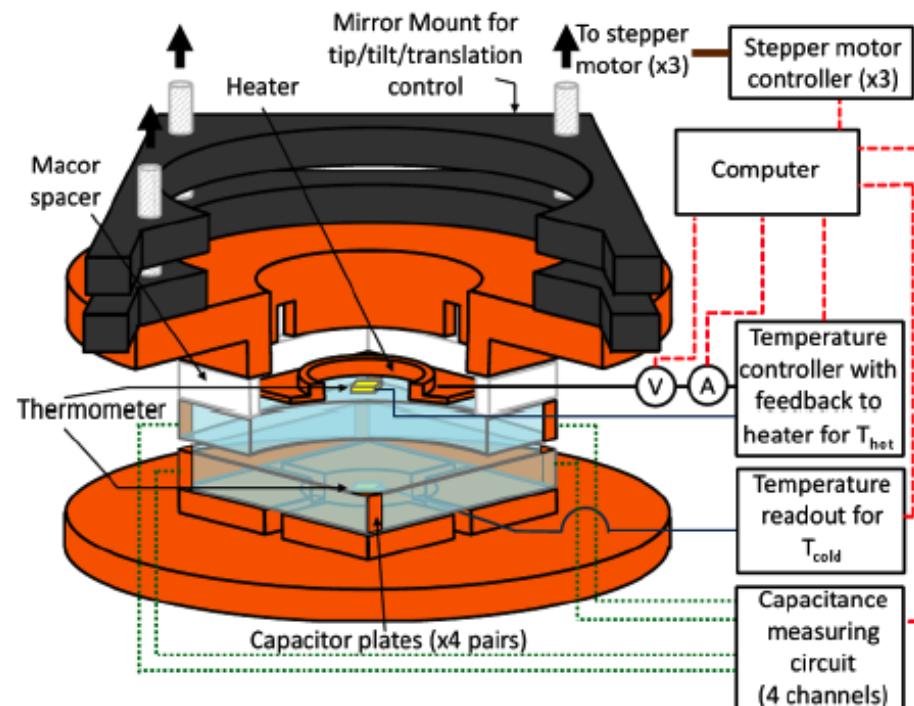
Comparison theory/data

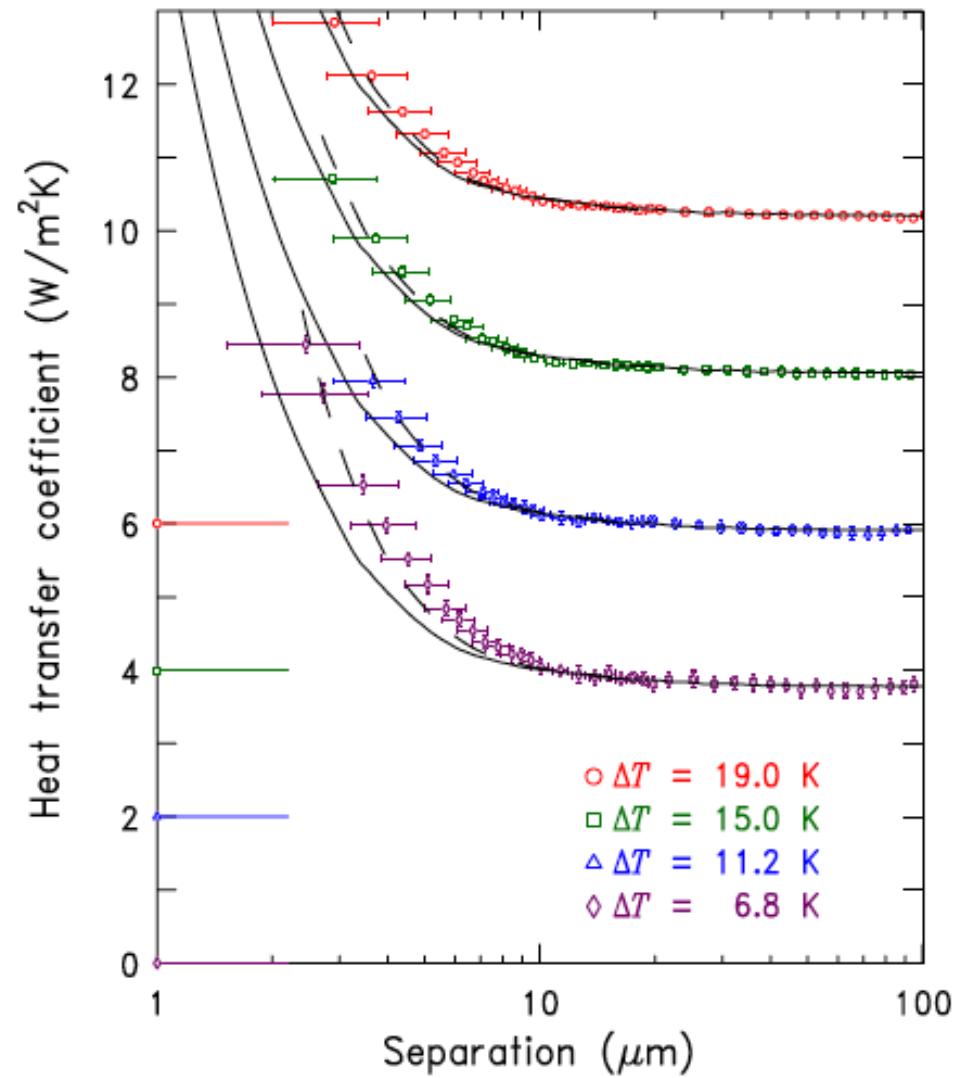
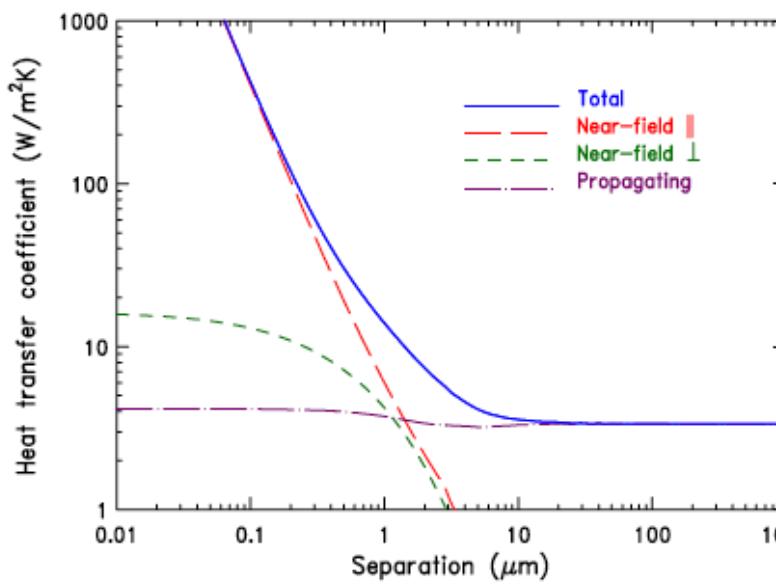




Near-Field Radiative Heat Transfer between Macroscopic Planar Surfaces

R. S. Ottens,¹ V. Quetschke,² Stacy Wise,^{1,*} A. A. Alemi,^{1,†} R. Lundock,^{1,‡} G. Mueller,¹
D. H. Reitze,¹ D. B. Tanner,¹ and B. F. Whiting¹



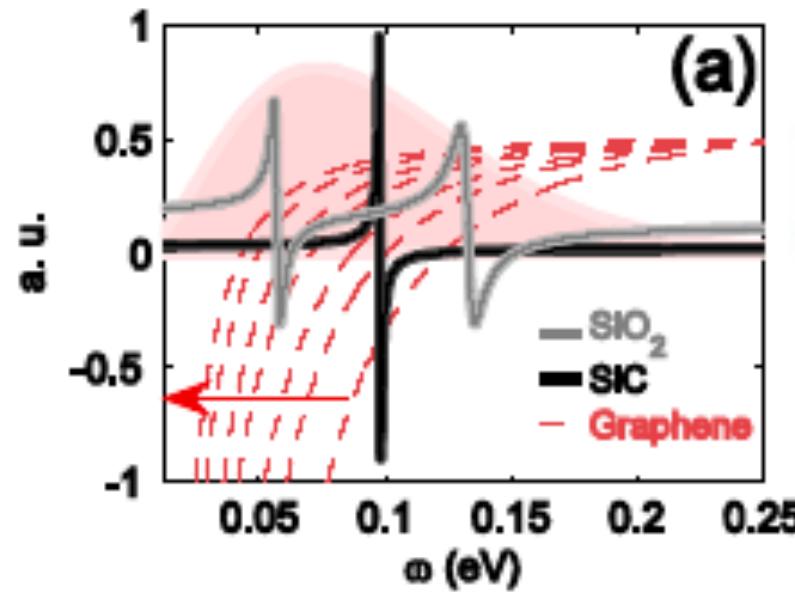


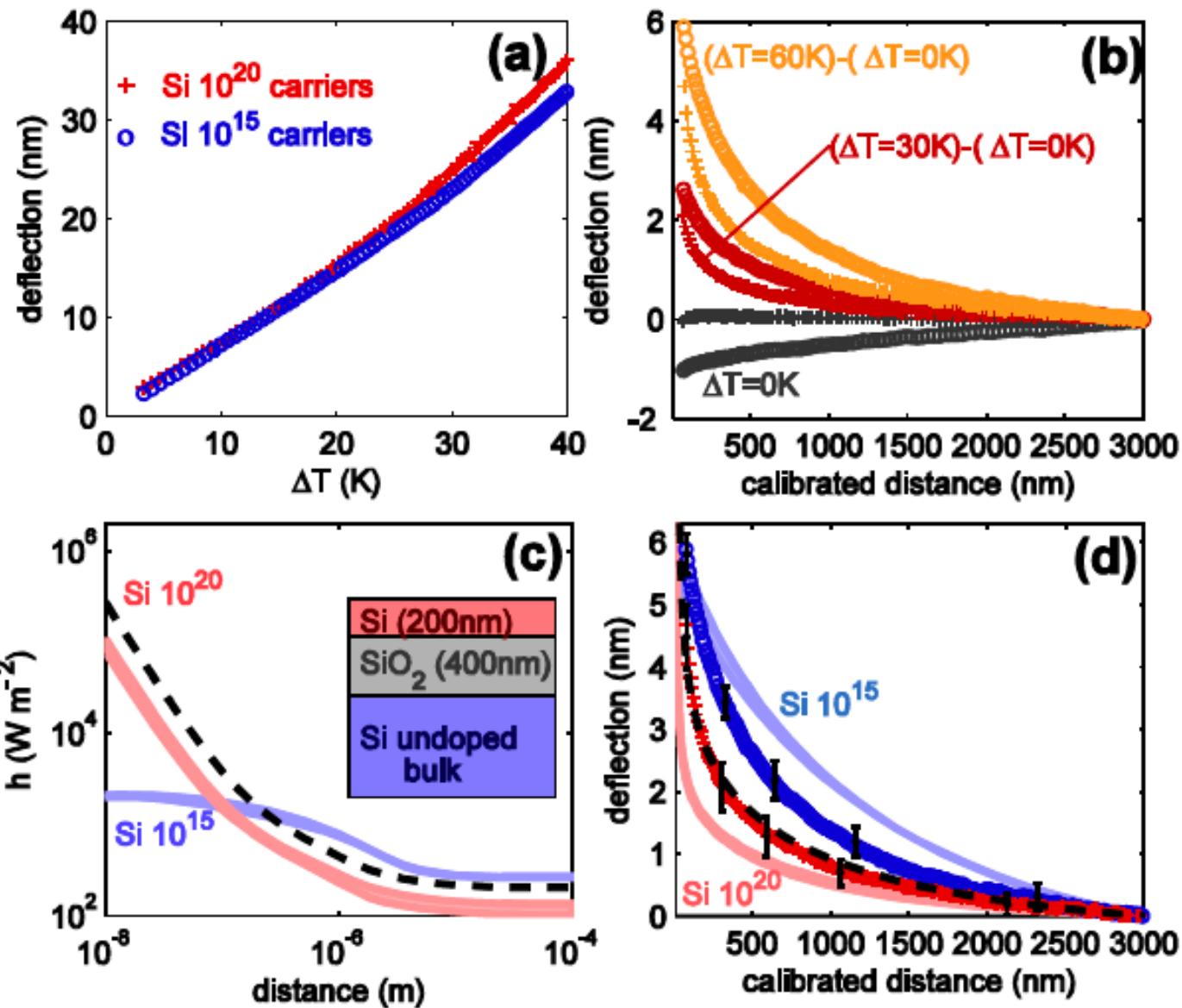
Observation of enhanced nanoscale radiative heat flow due to surface plasmons in graphene and doped silicon

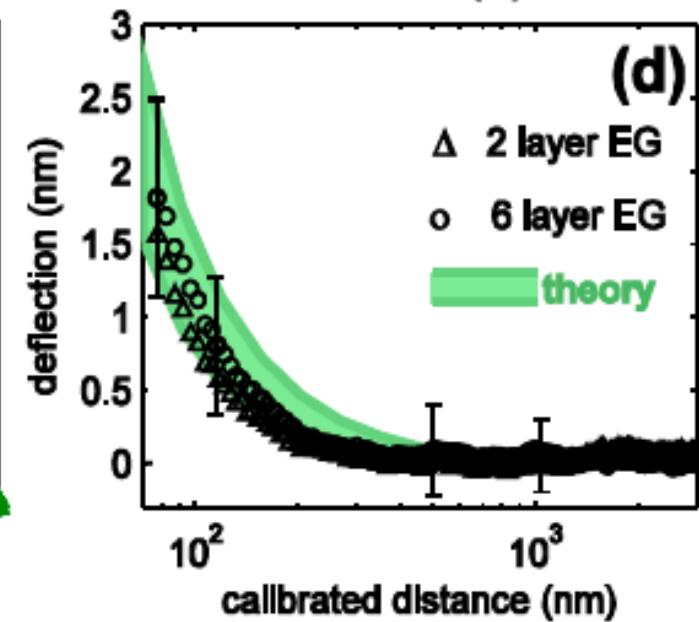
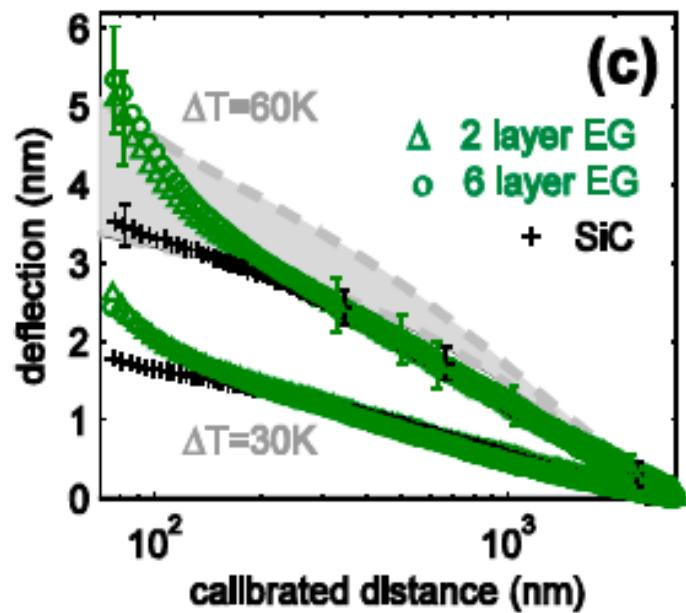
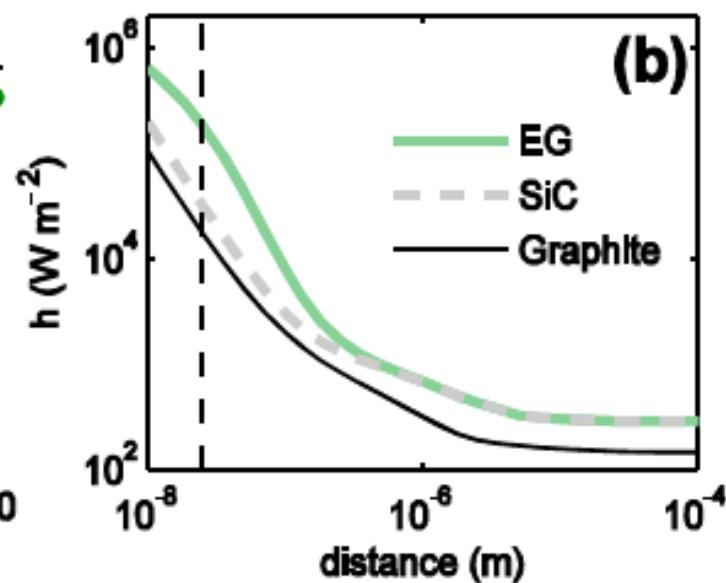
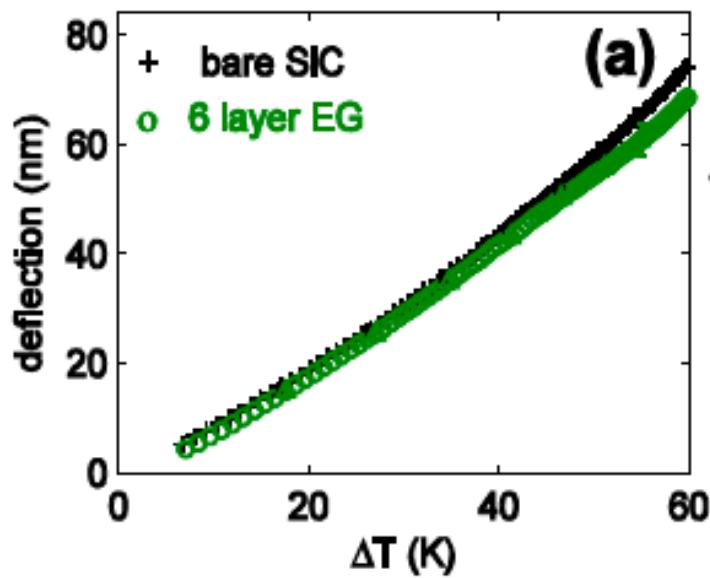
P.J. van Zwol¹, S. Thiele¹, C. Berger^{1,2}, W. A. de Heer², J. Chevrier¹

¹ Institut Néel, CNRS and Université Joseph Fourier Grenoble, BP 166 38042 Grenoble Cedex 9, France

² School of Physics, Georgia Institute of Technology, Atlanta, GA 30332, USA.



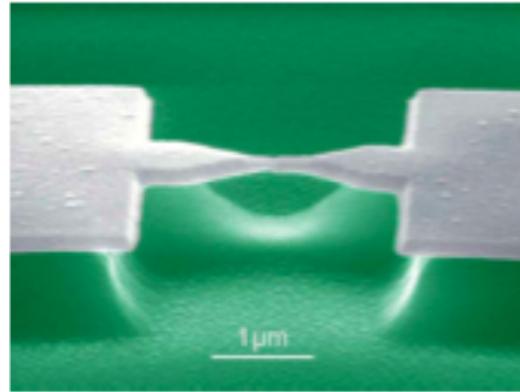




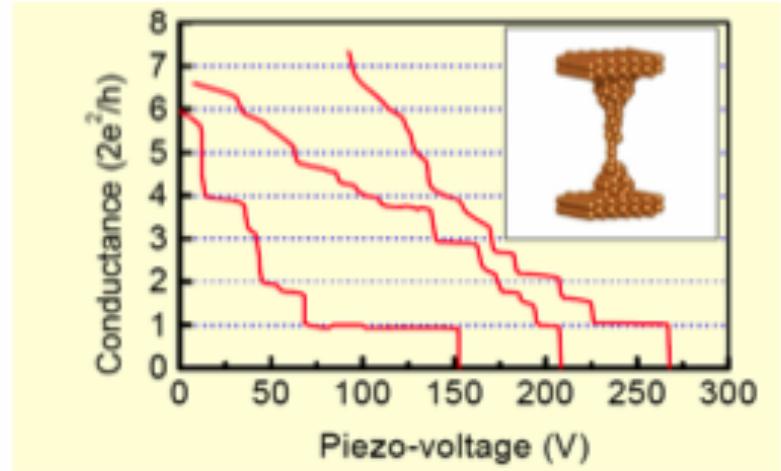
Introduction to the concept of quantum conductance Mesoscopic analysis of heat transfer at the nanoscale

$$\begin{aligned}\phi = & \int_{\omega=0}^{+\infty} d\omega [I_\omega^0(T_1) - I_\omega^0(T_2)] \\ & \times \sum_{\alpha=s,p} \left[\int_0^{\omega/c} \frac{KdK}{\omega^2/c^2} \frac{(1 - |r_{31}^\alpha|^2)(1 - |r_{32}^\alpha|^2)}{|1 - r_{31}^s r_{32}^\alpha e^{2i\gamma_3 d}|^2} \right. \\ & \left. + \int_{\omega/c}^{\infty} \frac{KdK}{\omega^2/c^2} \frac{4 \operatorname{Im}(r_{31}^\alpha) \operatorname{Im}(r_{32}^\alpha) e^{-2\gamma_3'' d}}{|1 - r_{31}^\alpha r_{32}^\alpha e^{-2\gamma_3'' d}|^2} \right],\end{aligned}$$

1 Experimental observation



contact ponctuel atomique



2 Landauer formalism



$$I = \Gamma V$$

$$\Gamma = \sum_n \left[\frac{2e^2}{h} \right] T_n$$

Fundamental limits to radiative heat transfer at the nanoscale

Sum over modes

$$\Phi \Delta t L^2 = \int_0^\infty \frac{d\omega}{(2\pi/\Delta t)} [\Theta(\omega, T_1) - \Theta(\omega, T_2)] \sum_{j=\{s,p\}} \int \frac{d^2\kappa}{(2\pi/L)^2} \mathcal{T}_j(\omega, \kappa; d).$$

$$\int_0^\infty \frac{d\omega}{2\pi} [\Theta(\omega, T_1) - \Theta(\omega, T_2)] \approx g_0 [T_1 - T_2],$$

$$g_0 = \int_0^\infty \frac{d\omega}{2\pi} \frac{\partial \Theta}{\partial T} = \frac{\pi^2 k_B^2 T}{3h},$$

A mesoscopic formulation of radiative heat transfer at nanoscale

$$\phi = \int_{\omega=0}^{+\infty} d\omega [I_\omega^0(T_1) - I_\omega^0(T_2)]$$

$$\times \sum_{\alpha=s,p} \left[\int_0^{\omega/c} \frac{KdK}{\omega^2/c^2} \frac{(1-|r_{31}^\alpha|^2)(1-|r_{32}^\alpha|^2)}{|1-r_{31}^s r_{32}^\alpha e^{2i\gamma_3 d}|^2} \right.$$

$$\left. + \int_{\omega/c}^{\infty} \frac{KdK}{\omega^2/c^2} \frac{4 \operatorname{Im}(r_{31}^\alpha) \operatorname{Im}(r_{32}^\alpha) e^{-2\gamma_3'' d}}{|1-r_{31}^\alpha r_{32}^\alpha e^{-2\gamma_3'' d}|^2} \right],$$

$$I = \sum_n T_n \left[\frac{2e^2}{h} \right] V$$

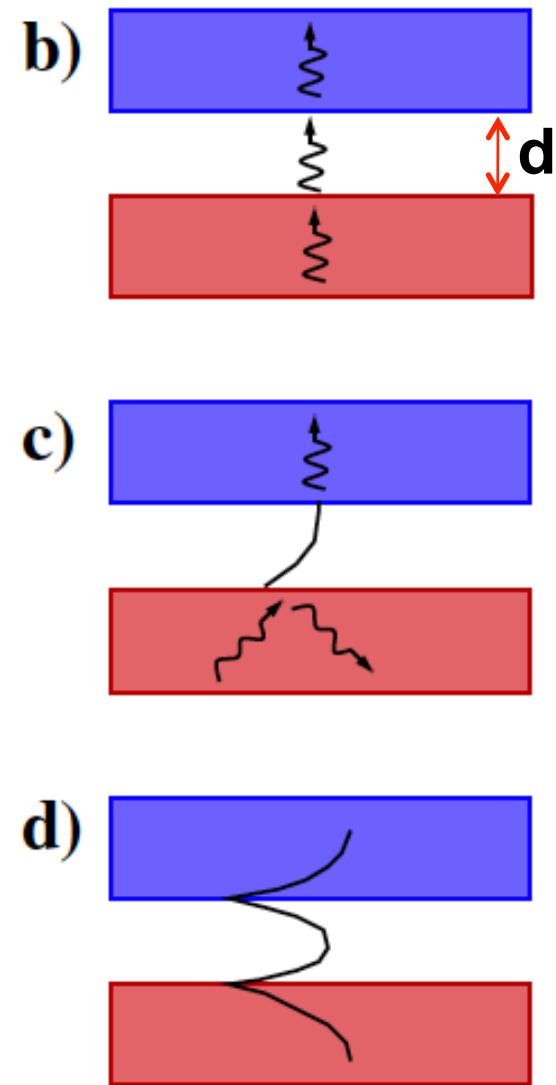
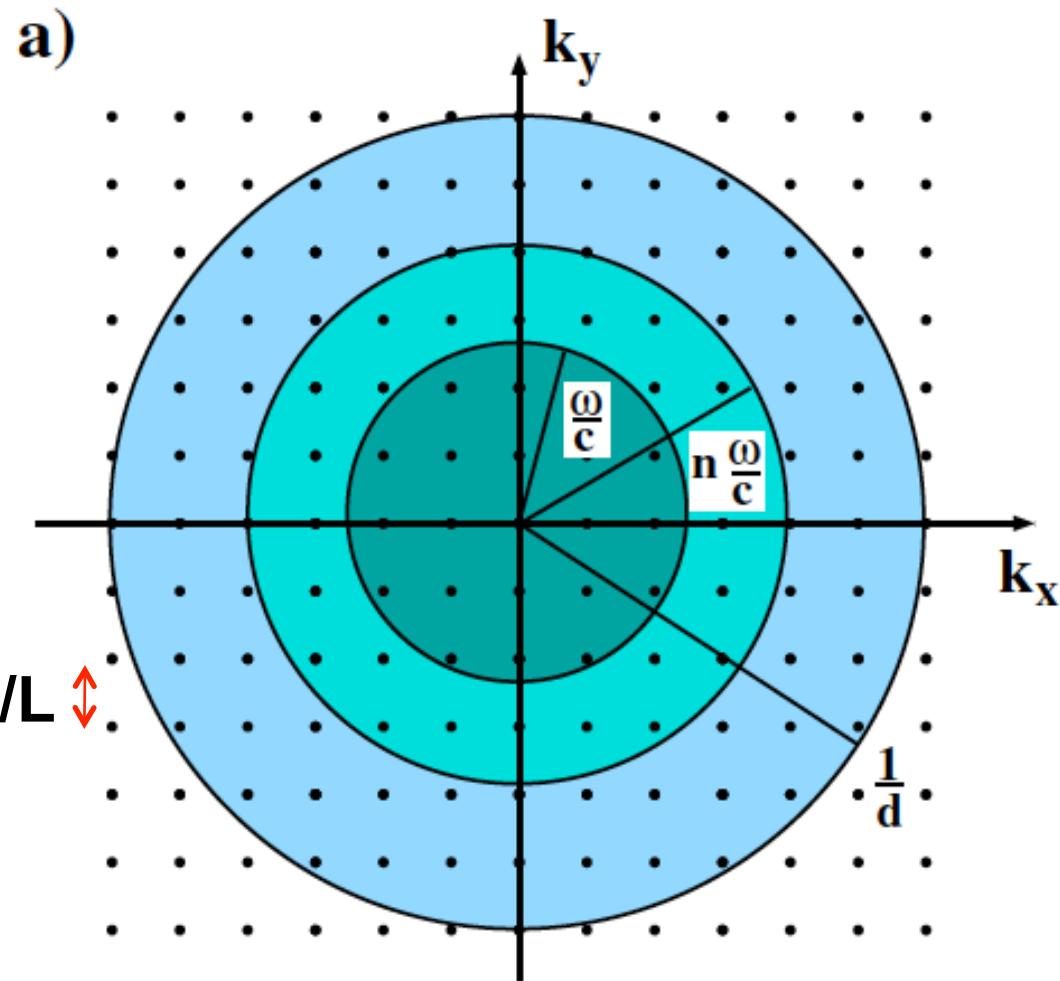
$$\Phi = \iint \frac{d^2\mathbf{k}}{4\pi^2} \overline{T(\mathbf{k})} \left[\frac{\pi^2}{3} \frac{k_B^2 T}{h} \right] \Delta T$$

↗ **Transmission factor** ↑ **Thermal quantum conductance**

$$T_j^{12}(\omega, \kappa, d) = \begin{cases} \frac{(1-|r_j^1|^2)(1-|r_j^2|^2)}{|D_j|^2} & , \kappa \leq k_0 \\ \frac{4\operatorname{Im}(r_j^1)\operatorname{Im}(r_j^2)e^{-2\gamma d}}{|D_j|^2} & , \kappa > k_0 \end{cases}$$

$$\Phi = g_0 \left(\sum_{j=s,p} \int \frac{d^2\kappa}{(2\pi)^2} \overline{\mathcal{T}}_j \right) [T_1 - T_2]$$

$$\overline{\mathcal{T}}_j = \frac{\int_0^\infty du f(u) \mathcal{T}_j(u, \kappa; d)}{\int_0^\infty du f(u)}$$

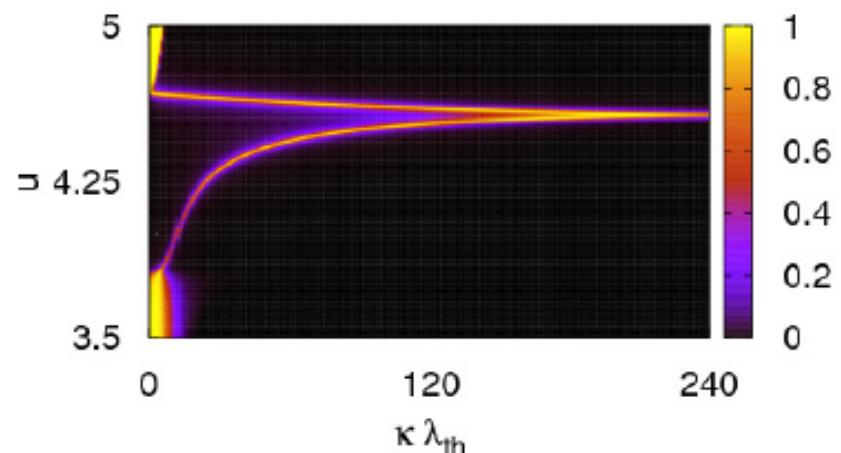
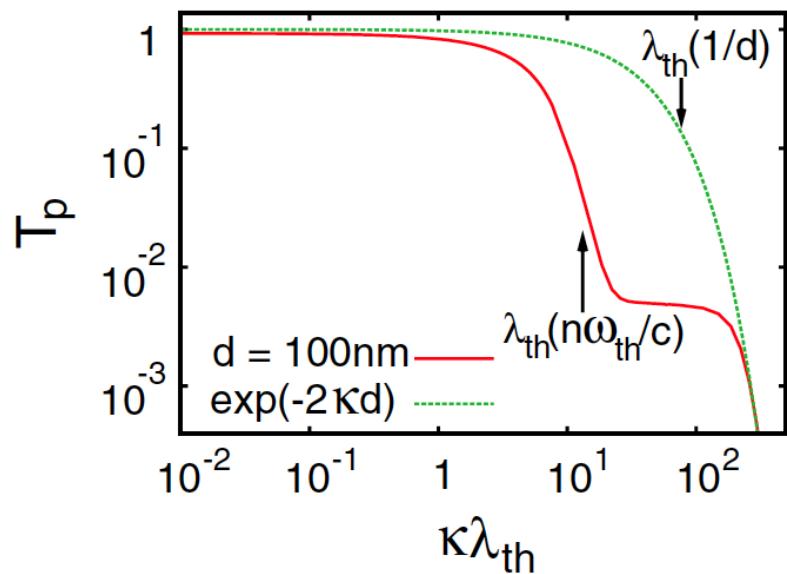


$$\Phi_{\text{BB}} = \int_0^{\infty} [\Theta(\omega, T_1) - \Theta(\omega, T_2)] \left(\frac{\omega^2}{c^3 \pi^2} \right) \frac{c}{4} = \sigma_{\text{BB}} (T_1^4 - T_2^4)$$

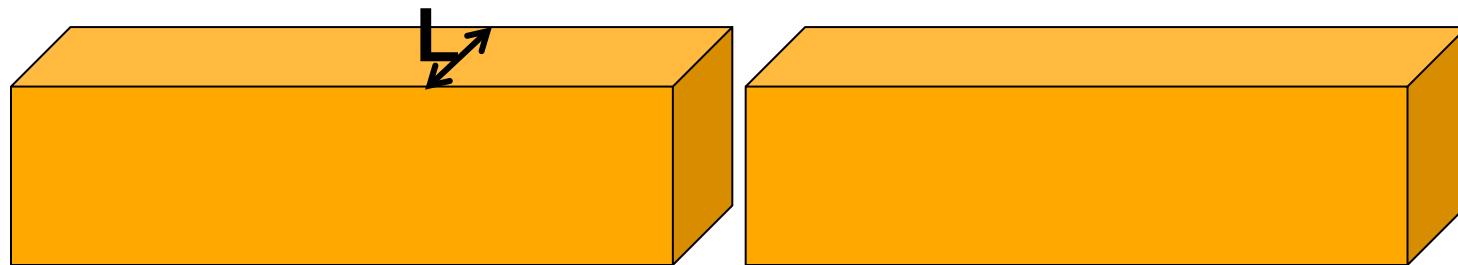
$$\Phi_{\text{BB}} = g_0 \frac{2\pi}{5\lambda_T^2} (T_1 - T_2). \quad \lambda_T = \hbar c / (k_B T)$$

$$\Phi_{\text{max}} = g_0 \frac{k_c^2}{2\pi} (T_1 - T_2) = g_0 \frac{2}{\pi d^2} (T_1 - T_2)$$

Are we there yet ?



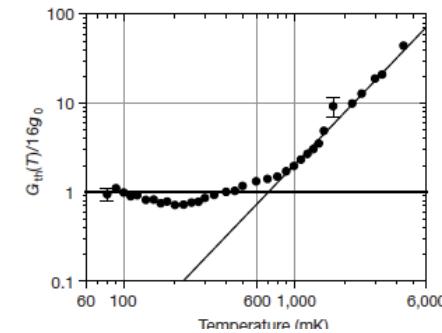
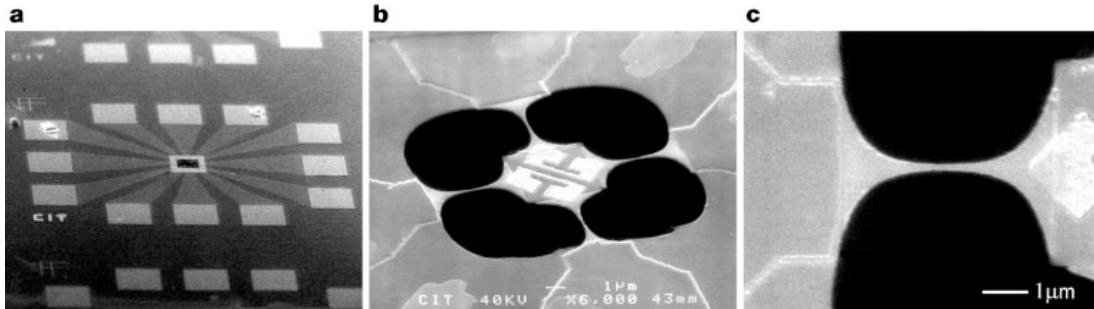
Quantized radiative thermal conductance



If L is on the order of the thermal wavelength,

$$\iint \frac{d^2\mathbf{k}}{4\pi^2} \rightarrow \sum_n$$

Biehs et al., Phys.Rev.Lett. 105, 234301 (2010)



Measurement of the quantum of thermal conductance

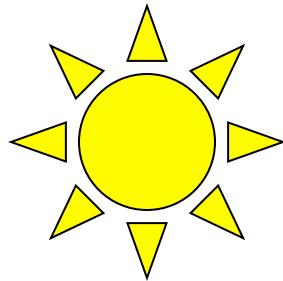
Applications

Thermophotovoltaics

Application : thermophotovoltaics

Photovoltaics

T= 6000K



PV cell

T= 300 K

Thermophotovoltaics

thermal source

T= 2000 K



TPV cell

T= 300 K

Near-field thermophotovoltaics

thermal source

T= 2000 K

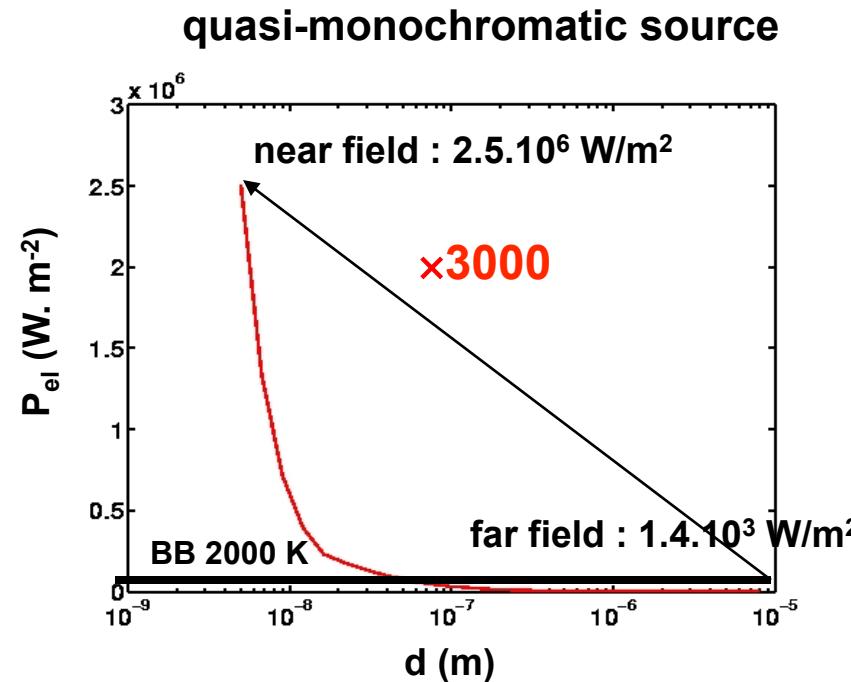
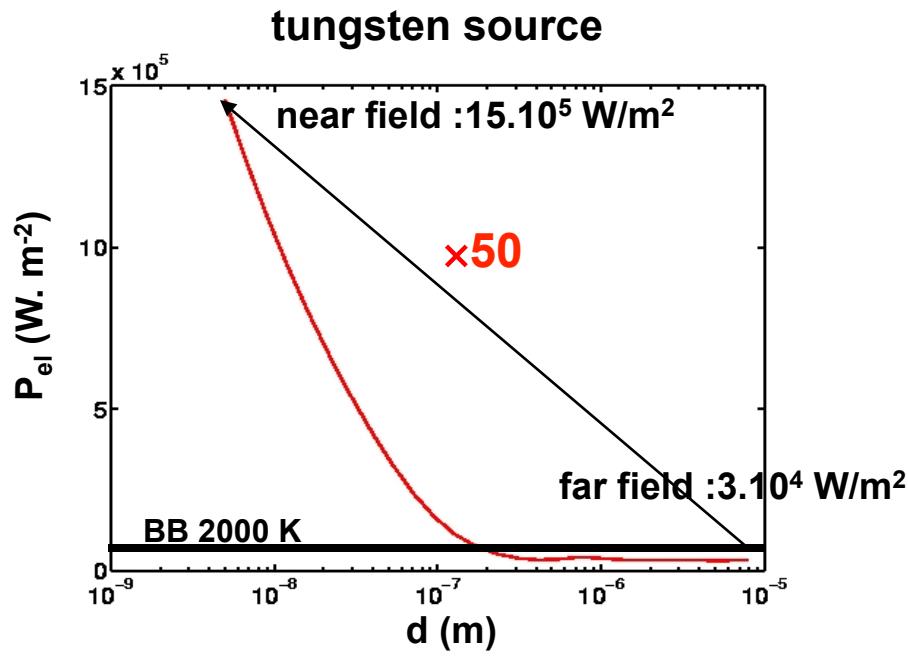


TPV cell

T= 300 K



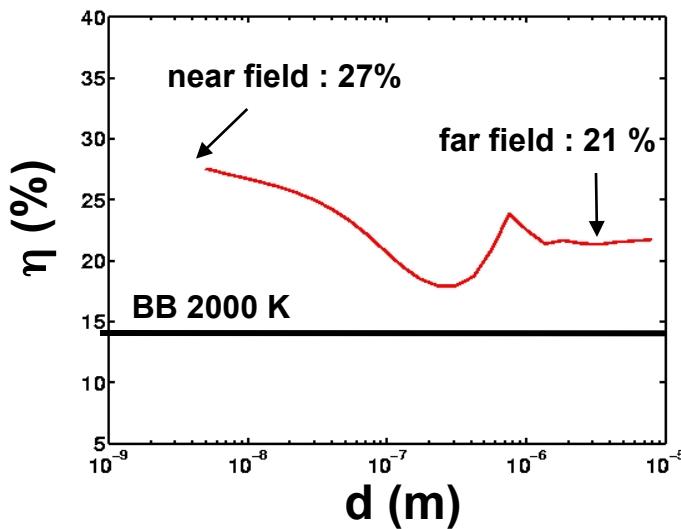
Near-field output electric power



output electric power enhanced
by at least one order of magnitude

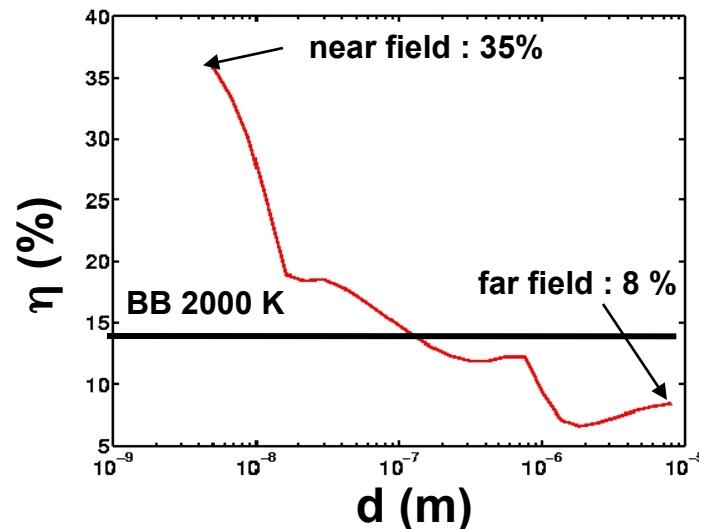
Near-field TPV converter efficiency

tungsten source



$$\eta = \frac{P_{el}}{P_{rad}}$$

quasi-monochromatic source



significant increase of the efficiency

Modulators

Modulator with phase change material

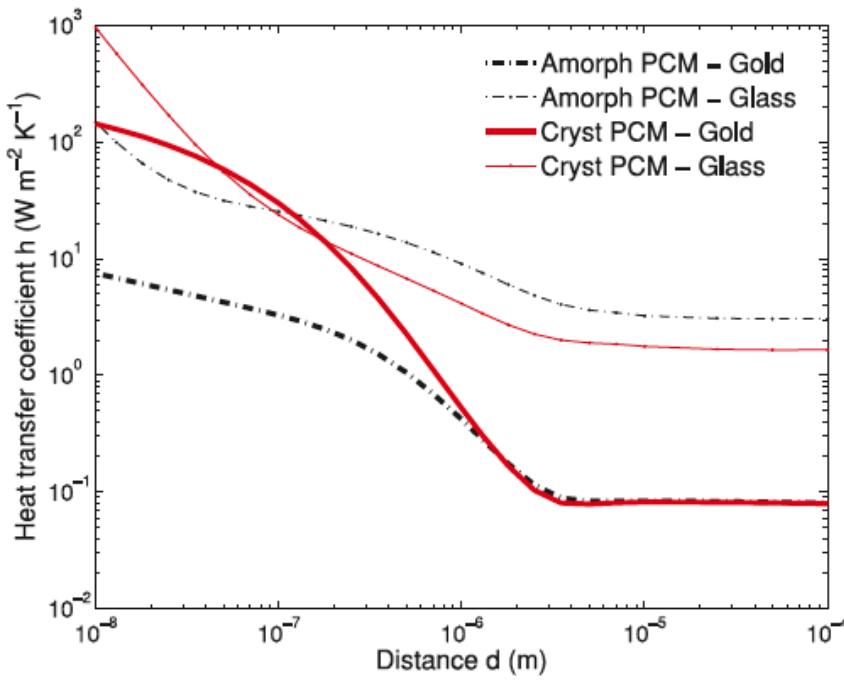


FIG. 3. (Color online) Radiative heat transfer for PCM glass and PCM gold.

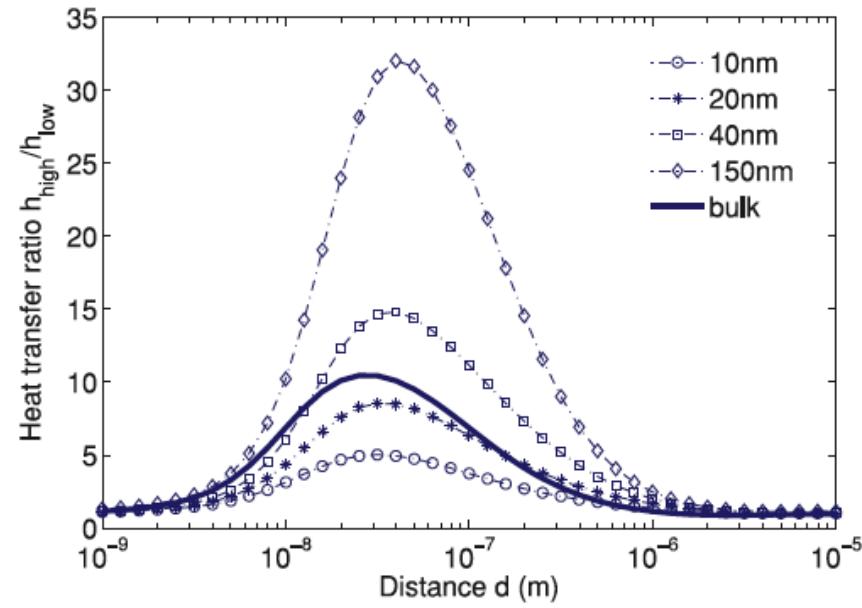
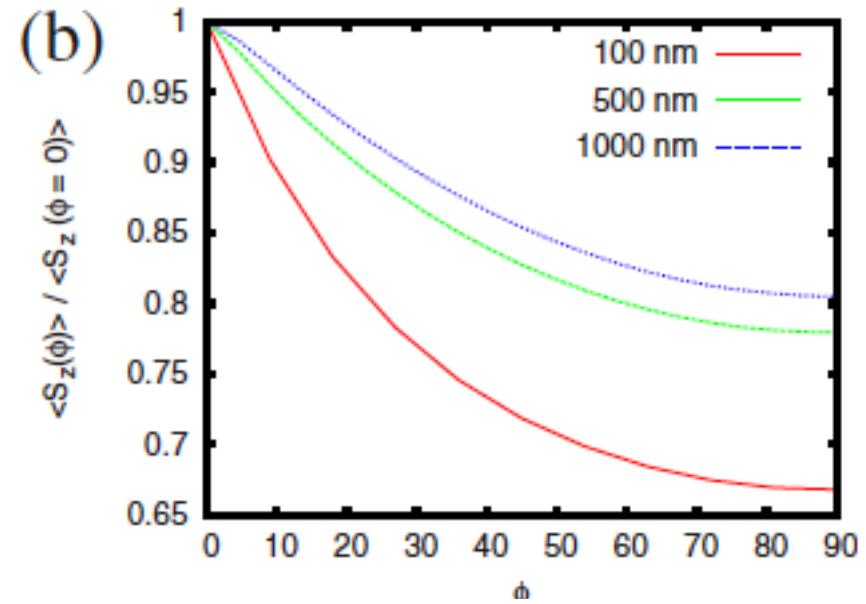
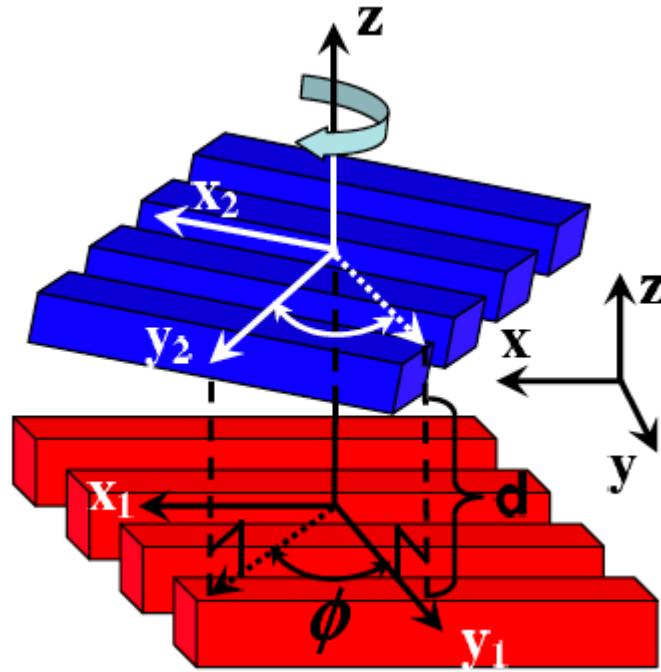


FIG. 4. (Color online) Modulation of heat transfer in vacuum between a gold surface and a switchable PCM (AIST) layer on a glass substrate protected by a 5 nm glass top layer. Data are shown for different thicknesses of the PCM material.



APPLIED PHYSICS LETTERS 98, 243102 (2011)

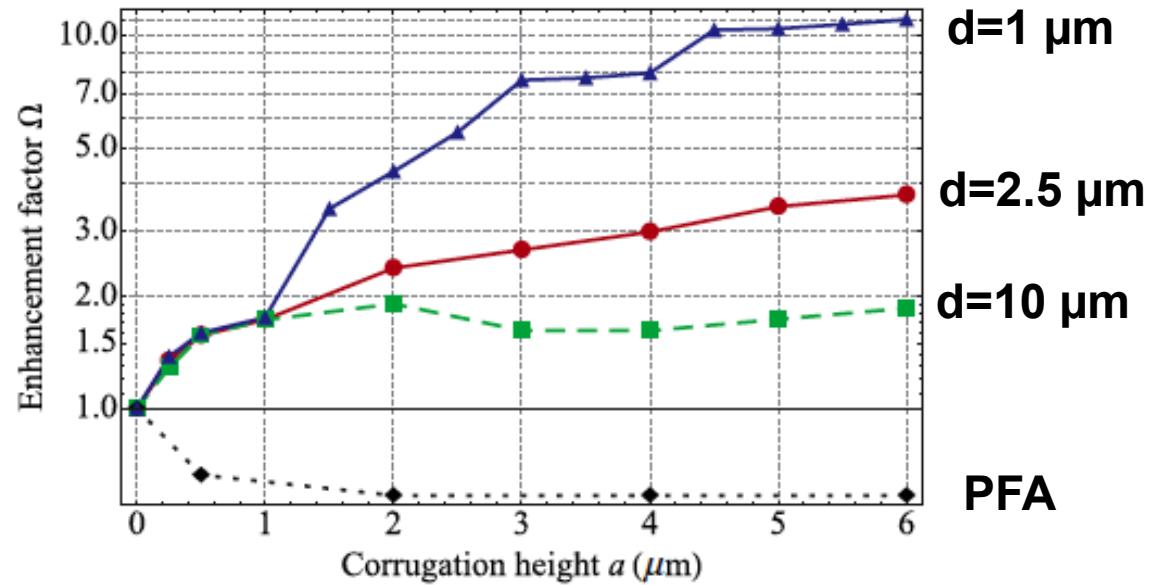
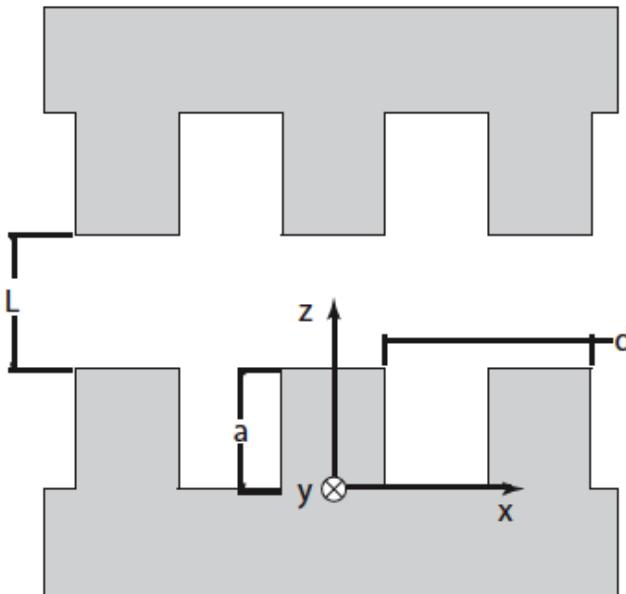
Modulation of near-field heat transfer between two gratings

S.-A. Biehs,^{a)} F. S. S. Rosa, and P. Ben-Abdallah

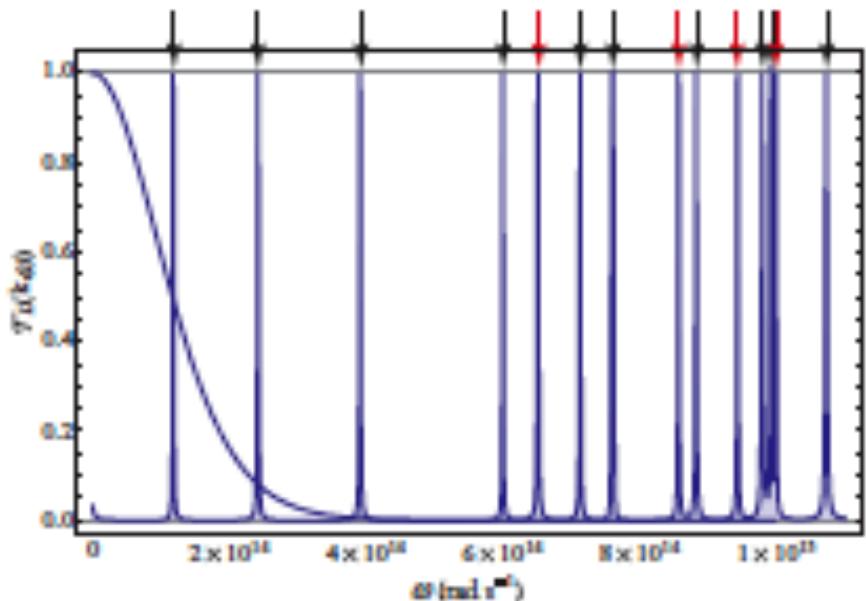
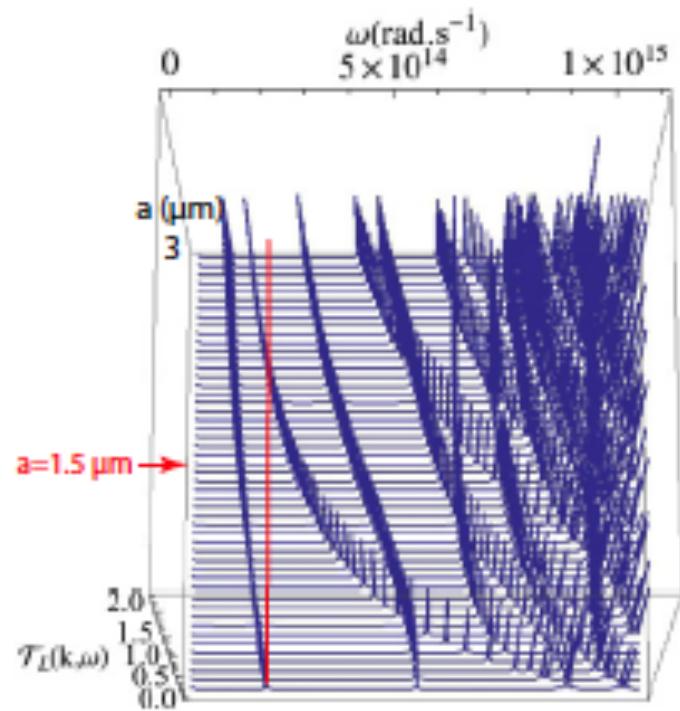
Laboratoire Charles Fabry, Institut d'Optique, CNRS, Université Paris-Sud, Campus Polytechnique, RD128, 91127 Palaiseau Cedex, France

Textured surfaces

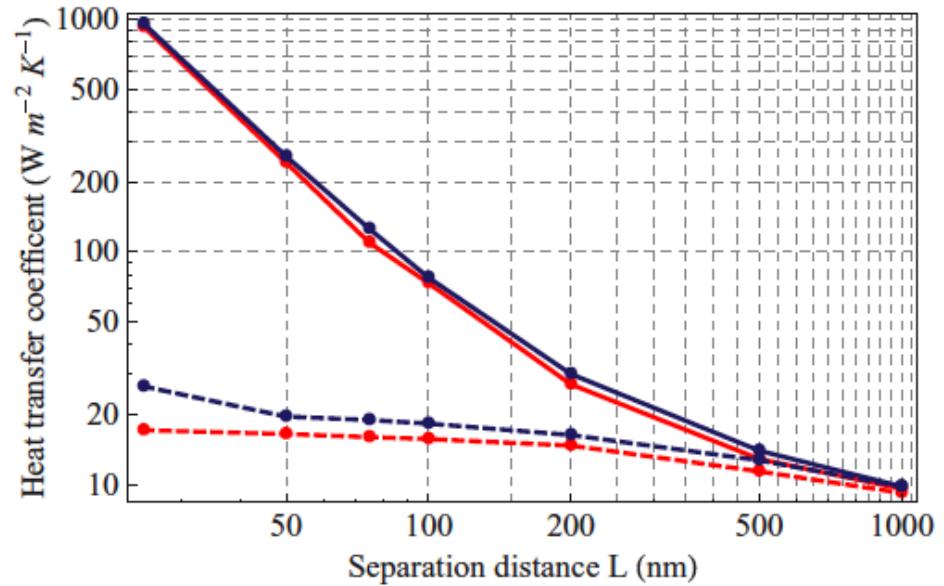
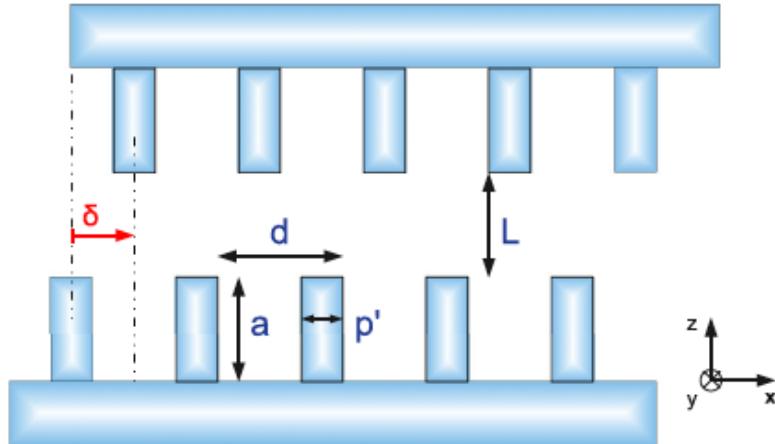
Heat transfer between gold gratings



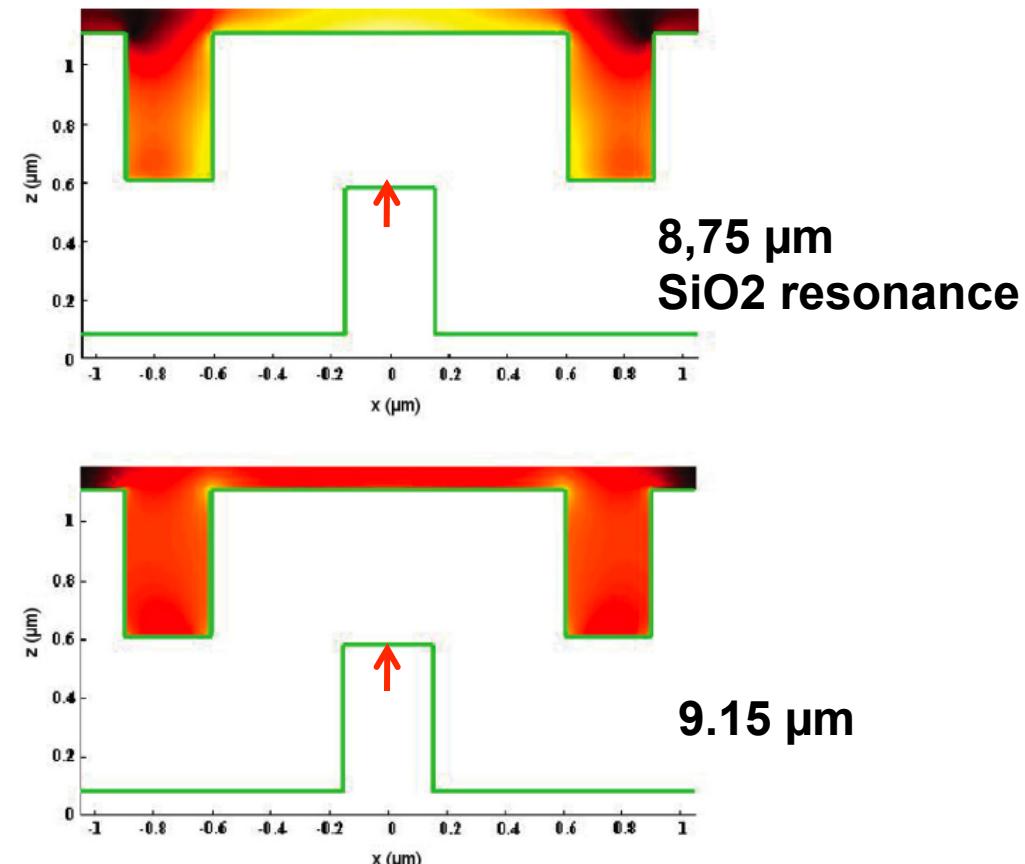
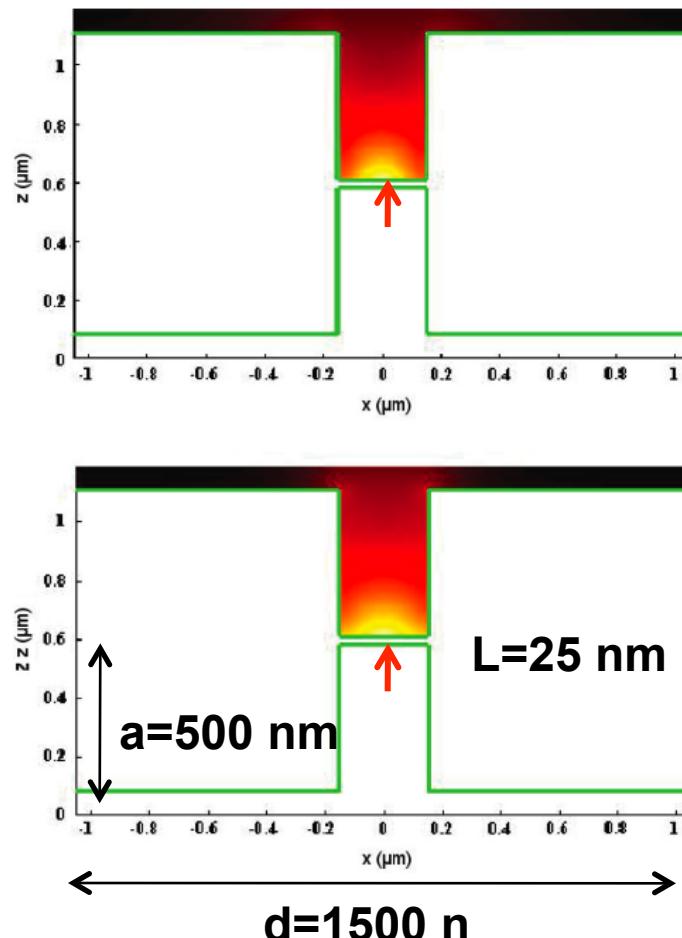
Textured surfaces



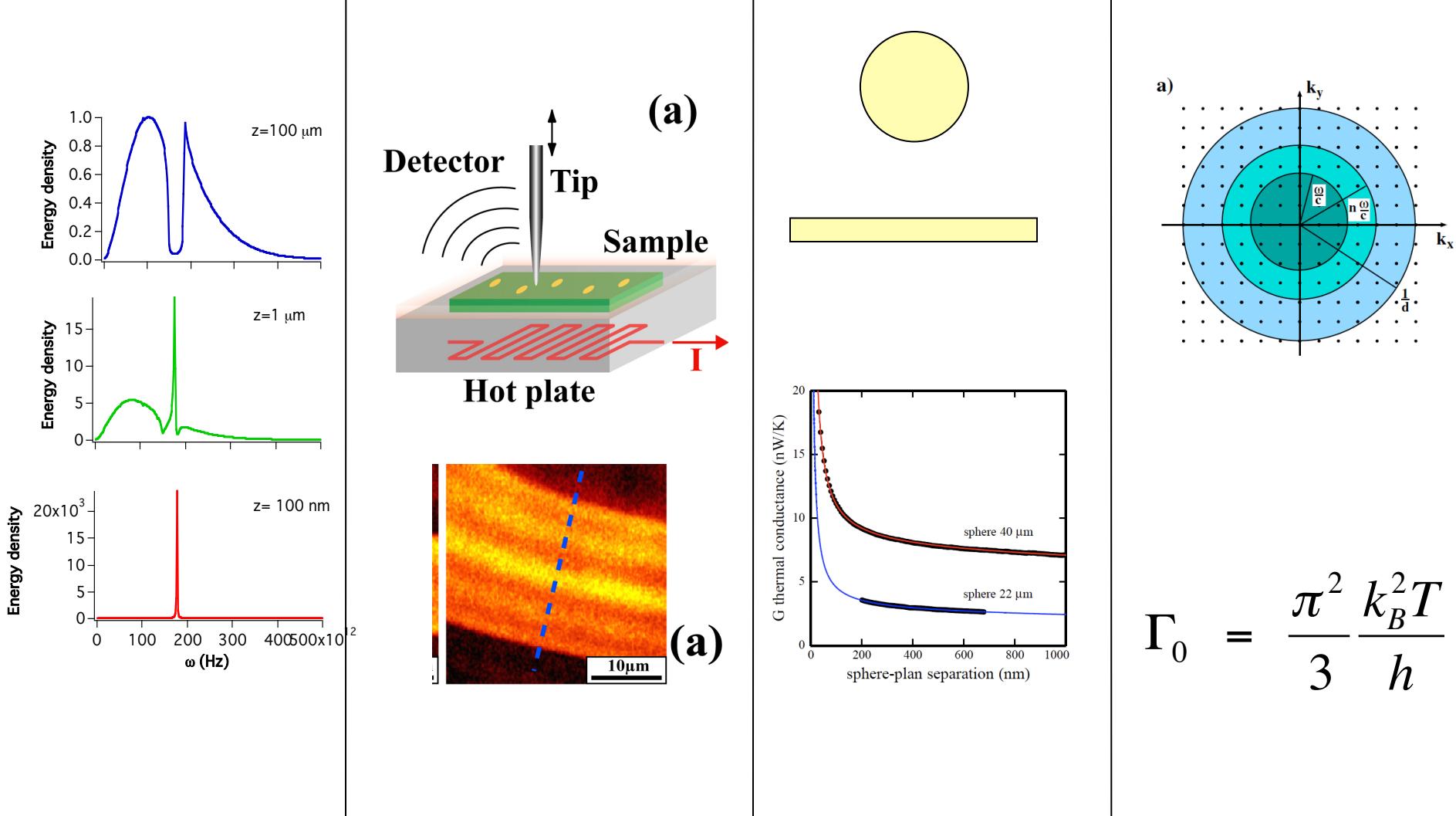
Heat transfer between SiC gratings



Heat transfer between two SiO₂ gratings: Physical mechanism



Summary



$$\Gamma_0 = \frac{\pi^2}{3} \frac{k_B^2 T}{h}$$